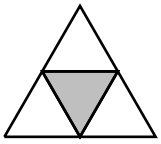
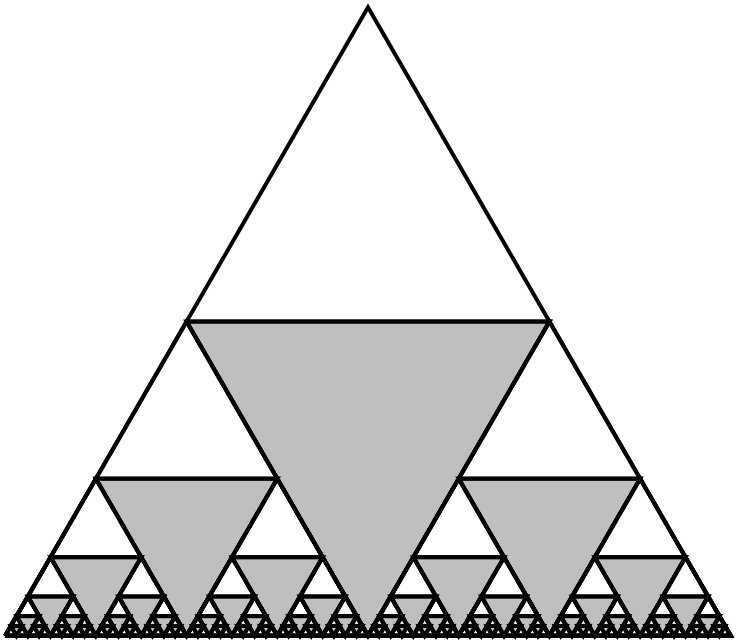
Challenge Problem 1: Shaded Triangles

A triangle is divided into fourths, and the center triangle is shaded.



Then the process is repeated over and over with each of the downward pointing sub-triangles touching the bottom line as pictured below. What portion of the original triangle is shaded?



Challenge Problem 2: Inventor Paradox

Three friends are talking about Ivan.[1]

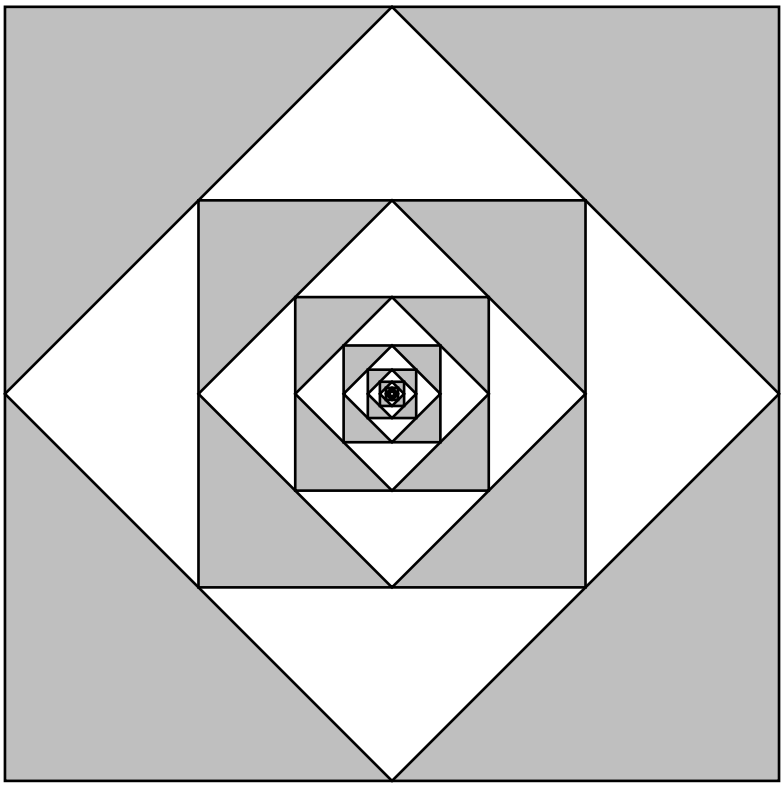
"Ivan has invented at least 100 toys," Gerry said.
"No he hasn't," George said. "He has invented fewer than that."
"Well, he has invented at least one toy." Anita said.

Only one of those statements is true. How many toys has Ivan invented?

[1]This problem is from Playthinks by Ivan Moscovich.

Challenge Problem 3: A Shaded Square

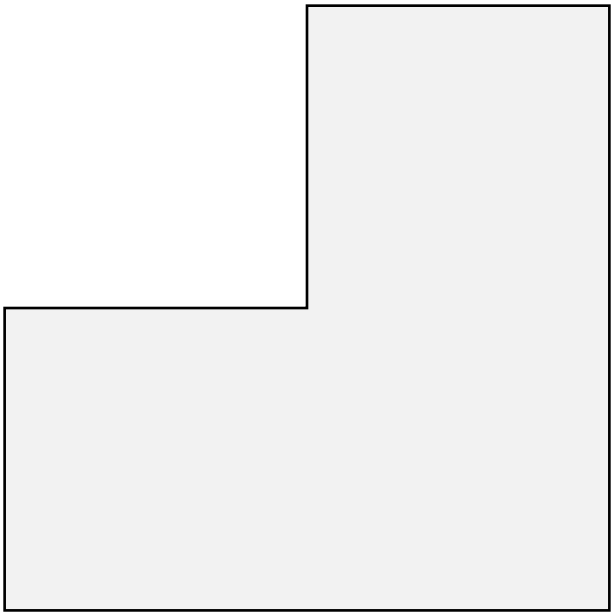
A square is subdivided by joining the midpoints of sides and then shading the outermost corners. Another square is drawn inside the the unshaded portion, and the process is repeated as pictured.



What portion of the big square is shaded?

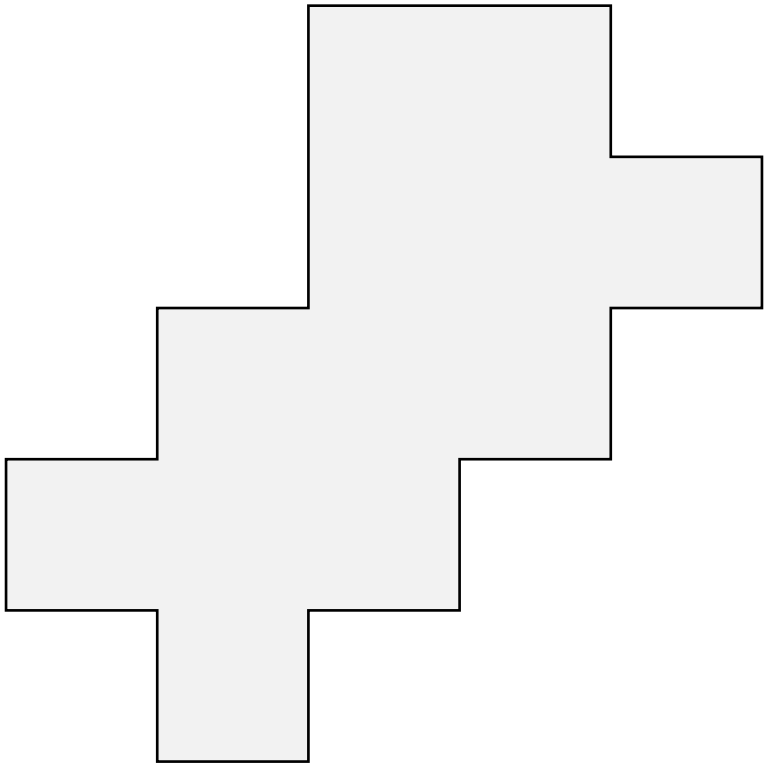
Challenge Problem 4: Dissecting an "L"

Cut the region into four pieces of the same size and shape.



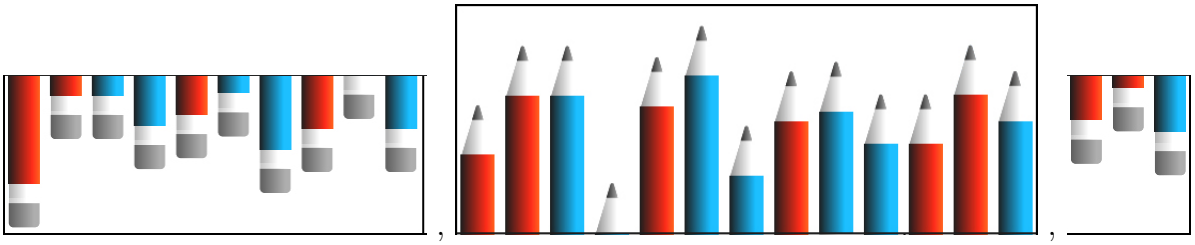
Challenge Problem 5: Dissection

Cut the region into two pieces of the same size and shape.

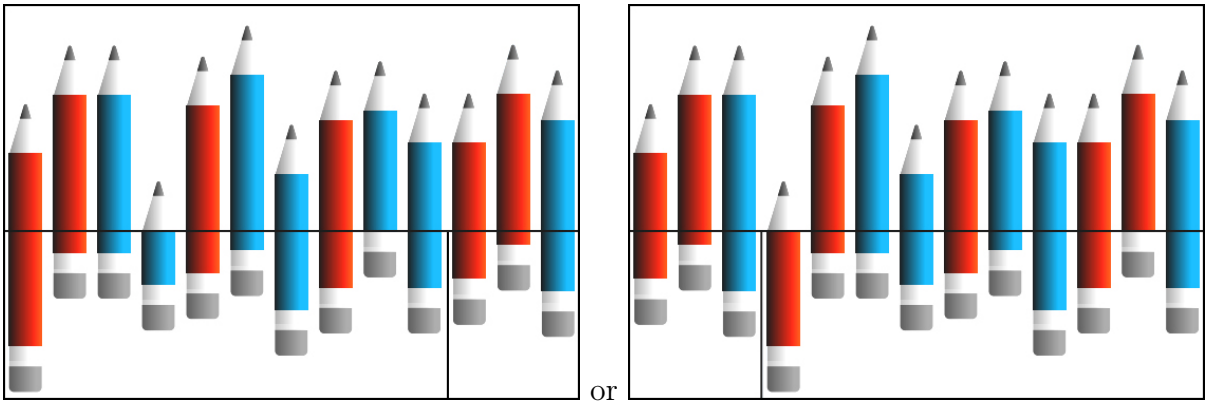


Challenge Problem 6: Red vs Blue

It's possible to arrange the figures



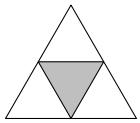
into pencils in two different ways:



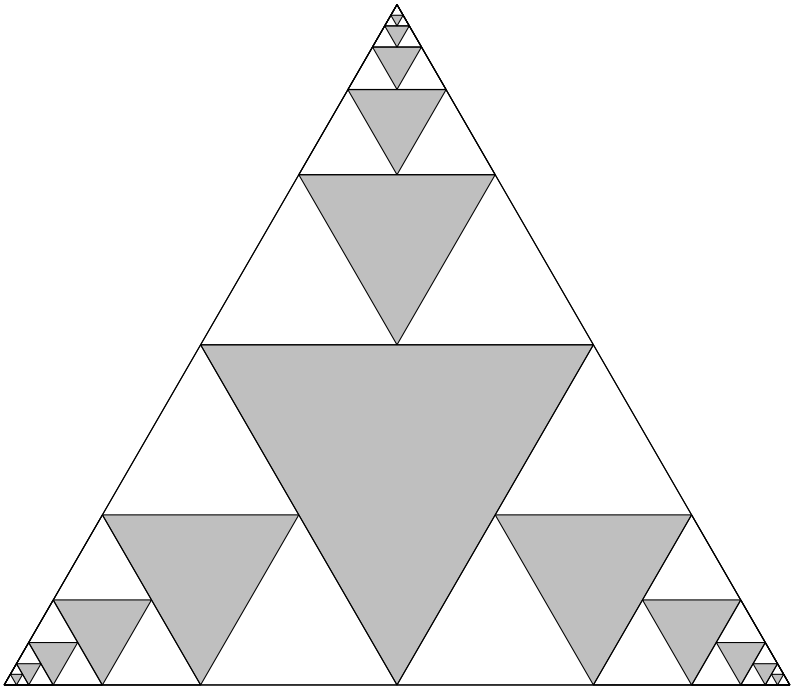
The arrangement on the left has 6 reds and 7 blues, while the arrangement on the right has 7 reds and 6 blues. How is this possible?

Challenge Problem 7: Triangle Shaded Outward

A triangle is divided into fourths, and the center triangle is shaded.

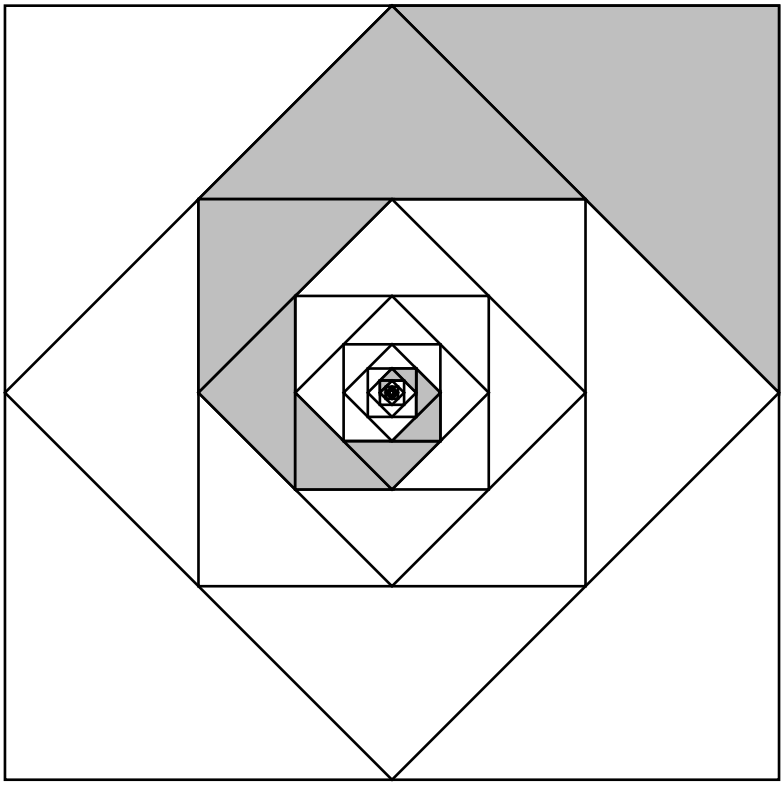


Then the process is repeated over and over with in the triangles closest to the original three corners as pictured below. What portion of the original triangle is shaded?



Challenge Problem 8: A Different Shading of a Square

A sequence of inscribed squares is partially shaded along adjacent triangles as pictured below.



What portion of the big square is shaded?

Challenge Problem 9: A Bee and 2 Trains

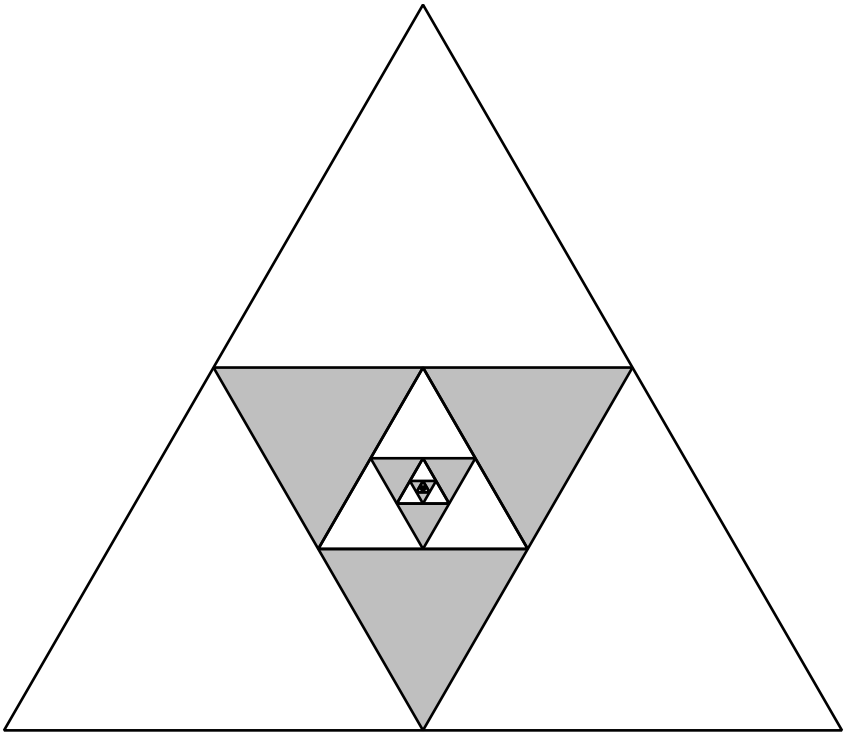
Two towns are 300 miles apart. Trains leave both towns at noon and head towards each other, one at 35 mph and the other at 25 mph. Meanwhile, a bee flies back and forth between the two trains at 100 mph (with no stop time). How far has the bee flown when the trains meet?

Challenge Problem 10: Find the Heavy Ball

Nine balls have the same size and shape. All nine balls are identical, except that one ball is slightly heavier than the other eight. Using a balance scale, how can you identify the heavier ball in only 2 weighings?

Challenge Problem 11: Triangle Shaded Inward

A sequence of triangles is formed by successively inscribing equilateral triangles. Then any point-down triangles are shaded as pictured below. What portion of the original triangle is shaded?



Challenge Problem 12: Four 4s

It's possible to write the numbers 1 through 10 using exactly three 3's along with mathematical operations such as $+, -, \div, \cdot, \sqrt{}, !$, and decimals. For example

- $1 = 3^{3-3}$
- $2 = 3 - \frac{3}{3}$
- $3 = 3 + 3 - 3$
- $4 = 3 + \frac{3}{3}$
- $5 = 3! - \frac{3}{3}$
- $6 = 3! - 3 + 3$
- $7 = 3! + \frac{3}{3}$
- $8 = \left(\frac{3!}{3}\right)^3$
- $9 = 3 + 3 + 3$
- $10 = \frac{\sqrt{3} \cdot \sqrt{3}}{.3}$

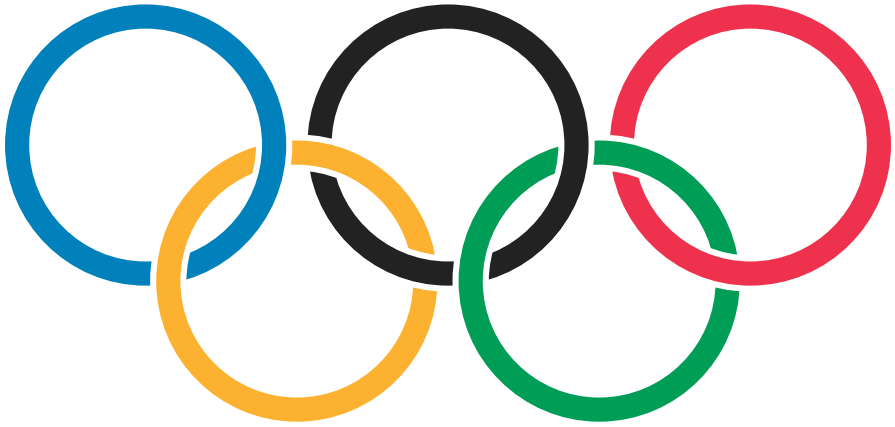
Write each of the numbers using 1 through 10 using exactly four 4's.

Challenge Problem 13: 5 rows of 4

Arrange 10 points into 5 rows of 4. No two lines may share more than one point.

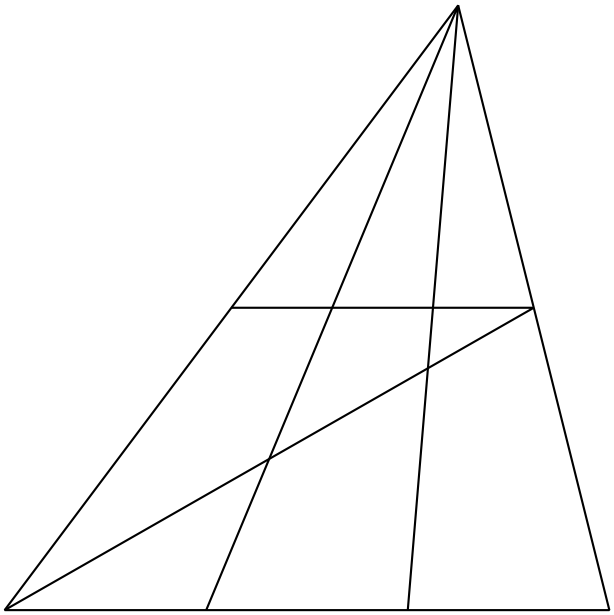
Challenge Problem 14: Olympic Rings Challenge

Place 9 dots in the Venn Diagram formed by the Olympic rings so that each ring contains the same number of dots.



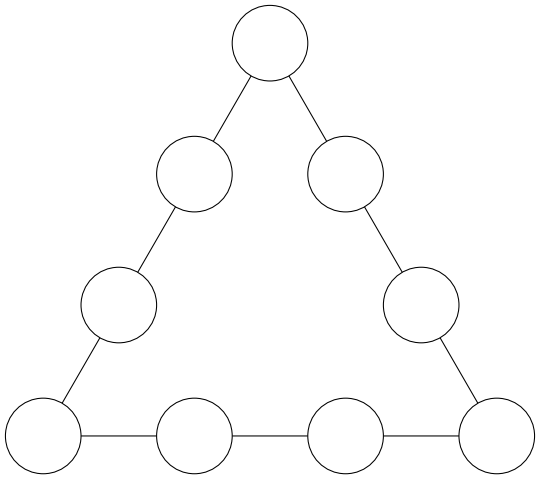
Challenge Problem 15: How Many Triangles

How many triangles are in the figure below?



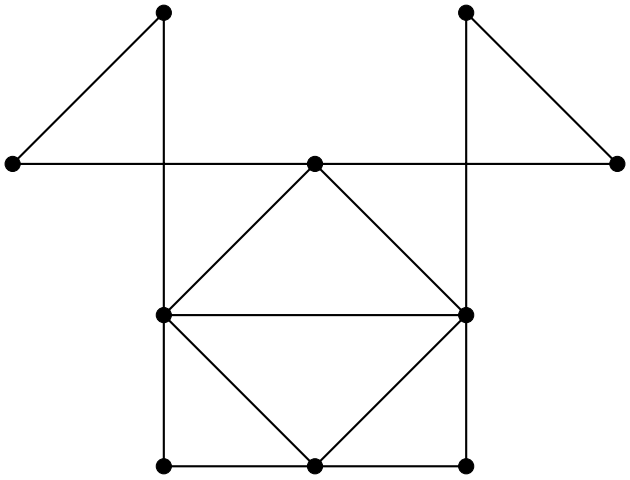
Challenge Problem 16: Magic Triangle

Use the digits 1 through 9 to fill in the circles below so that the sum of the numbers along each side of the triangle is the same.



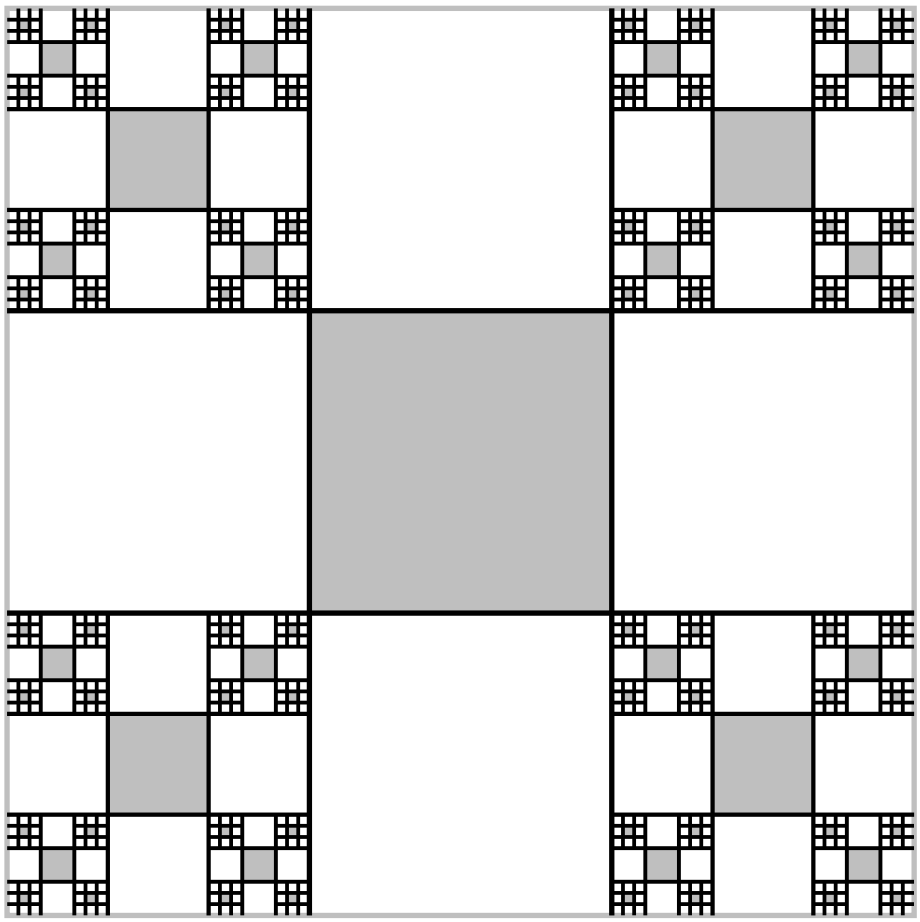
Challenge Problem 17: Untraceable

Start at one vertex and trace the whole figure without retracing any edges.



Challenge Problem 18: Tic-Tac-Fractal

A fractal shape is created by subdividing a square into 9 equal squares and then shading the center square. Then this process is repeated in of the four corner squares resulting in a sequence of smaller shaded squares as pictured below. What portion of the original square is shaded?



Challenge Problem 19: A Chicken and a Half

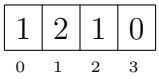
A chicken and a half can lay an egg and a half in a day and a half. How many eggs can one chicken lay in one day?

Challenge Problem 20: Timing 9-minutes

How can you time exactly 9 minutes using only a 7-minute hourglass and a 4-minute hourglass?

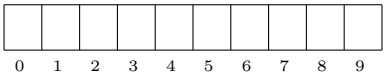
Challenge Problem 21: Self-Counting Sequence

The sequence of numbers in the boxes below is what we might call a self-counting sequence because the i^{th} box contains a number telling how many time the digit i appears in the boxes.



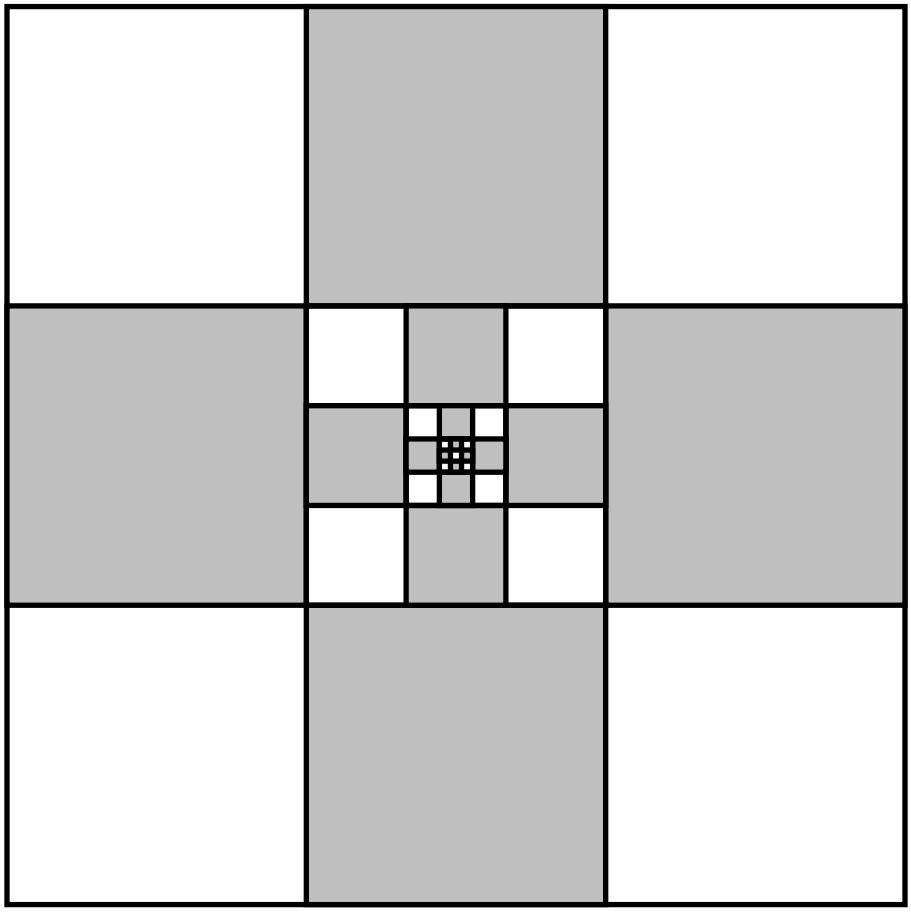
In particular the boxes contain one 0, two 1s, one 2, and zero 3s.

Fill in the boxes below to obtain a self-counting sequence.



Challenge Problem 22: Cross-Inward

A square is subdivided into 9 equal squares, and the middle square of each outer edge is shaded. Then this process is repeated inside the center square. giving a sequence of smaller shaded squares. This process is repeated forever. What portion of the original square is shaded?



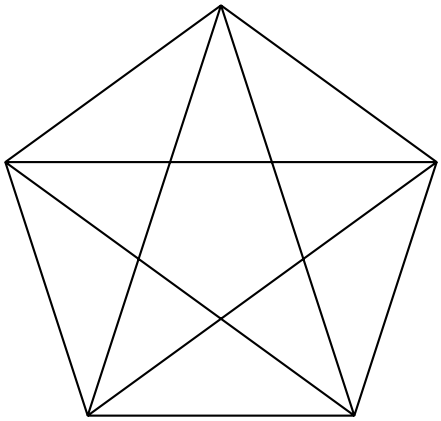
Challenge Problem 23: Sum to 100

From the array below, identify three numbers that sum to 100.

21	43	29	45
25	37	53	41
23	51	19	22
33	61	17	39

Challenge Problem 24: How Many Triangles in K_5

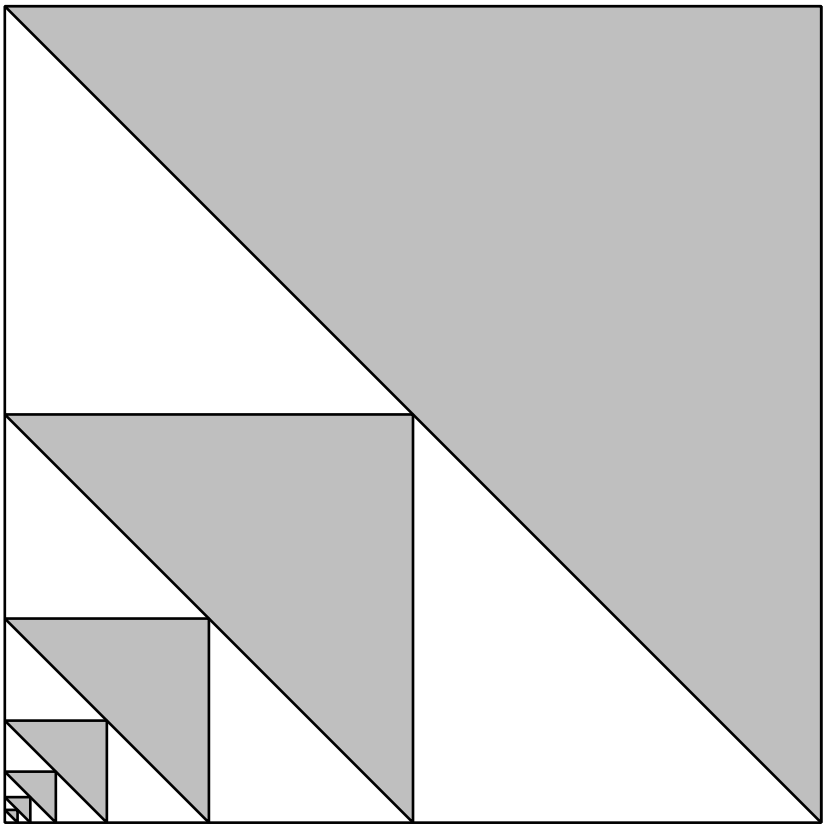
How many triangles are in the figure below?



Challenge Problem 25: Hungry Fish

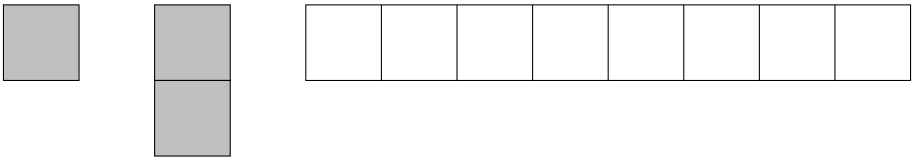
A square is divided in half along a diagonal, and the upper half is shaded. A smaller square is inscribed in the bottom triangle. and the shading process is repeated.

This process is repeated forever. What portion of the original square is shaded?



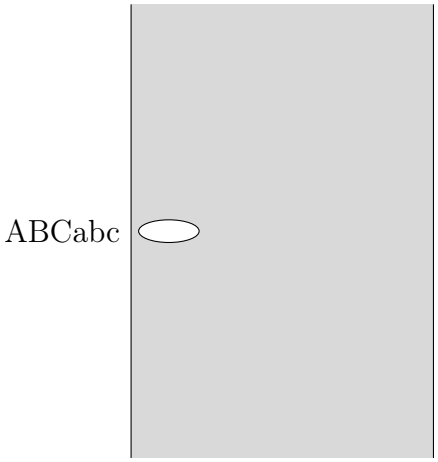
Challenge Problem 26: Counting Domino Tilings

In how many ways can we we tile a 1×8 grid using only 1×1 and 1×2 dominoes?



Challenge Problem 27: 3 Authors and 3 Agents

Three authors (a, b, and c) and their three agents (A, B, and C) need to cross a river in a boat that can only hold two people. The three agents are extremely jealous, and they refuse to let one of their clients be in the presence of another agent unless they, themselves are present. How is it possible to get everyone across the river?

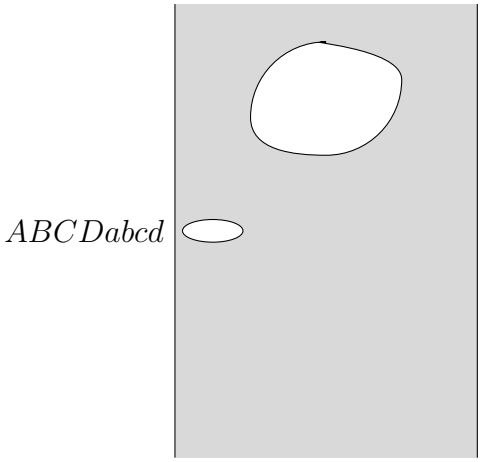


Challenge Problem 28: 4 Authors and 4 Agents

Four authors (a, b, c, and d) and their four agents (A, B, C, and D) need to cross a river in a boat that can only hold two people. The four agents are extremely jealous, and they refuse to let one of their clients be in the presence of another agent unless they, themselves are present.

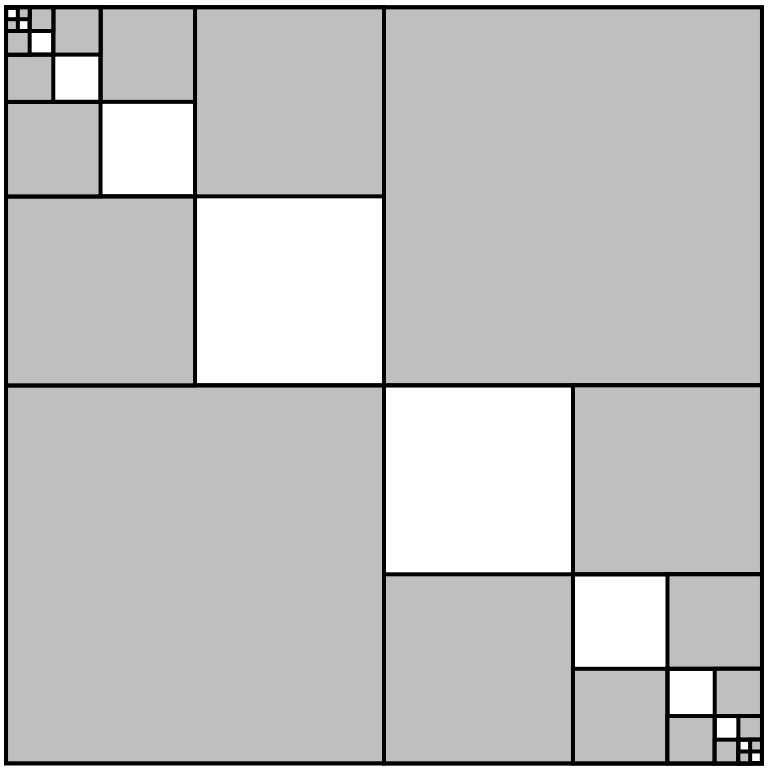
The problem would be impossible, except for the fact that there is an island in the middle of the river that can be used as a layover spot.

How is it possible to get everyone across the river?



Challenge Problem 29: White Diagonal

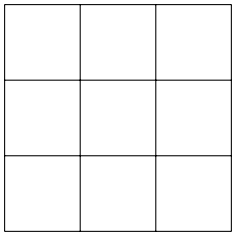
A square is divided into quarters, and the squares off the main diagonal are shaded. The same process is repeated in a sequence heading towards the upper left and lower right corners as pictured below.



What portion of the original square is shaded?

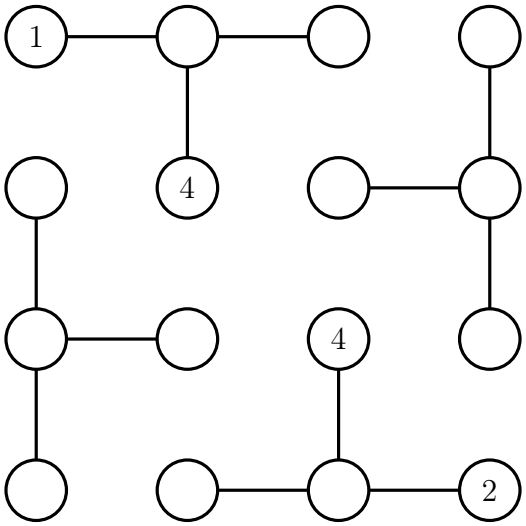
Challenge Problem 30: Rectangle Count

How many rectangles (including squares) are there in an 3×3 grid?



Challenge Problem 31: Strimko

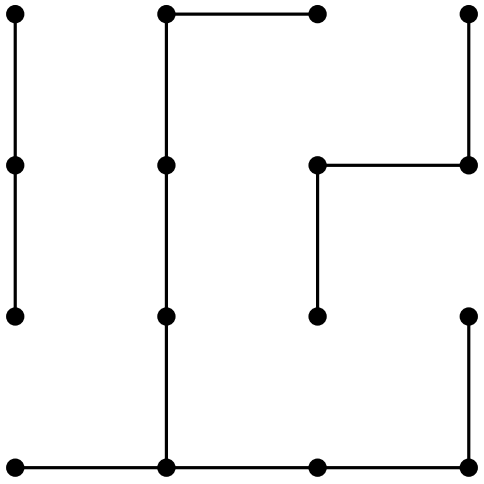
Place the numbers 1, 2, 3, and 4 in the circles so that each number shows up exactly once in each row, column, and stream (connected component).



Challenge Problem 32: Dots and Boxes

In the game of Dots and Boxes, two players take turns drawing horizontal or vertical line segments between adjacent points in a grid. A player scores a point if they complete a 1×1 square, and a player who completes a square gets another turn immediately.

In the current game it is Player 2's turn. Determine which player will win if both players make their best moves.



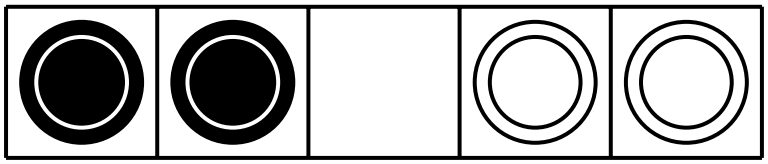
Challenge Problem 33: Handshaking Couples

A couple invites 3 other couples to dinner. Once everyone arrives, each person shakes hands with everyone s/he doesn't know. Then, the host asks everyone how many hands they shook and each person replies with a different number. Assuming that everyone knows his or her own spouse, how many hands did the hostess shake?

Challenge Problem 34: Checker Switcheroo

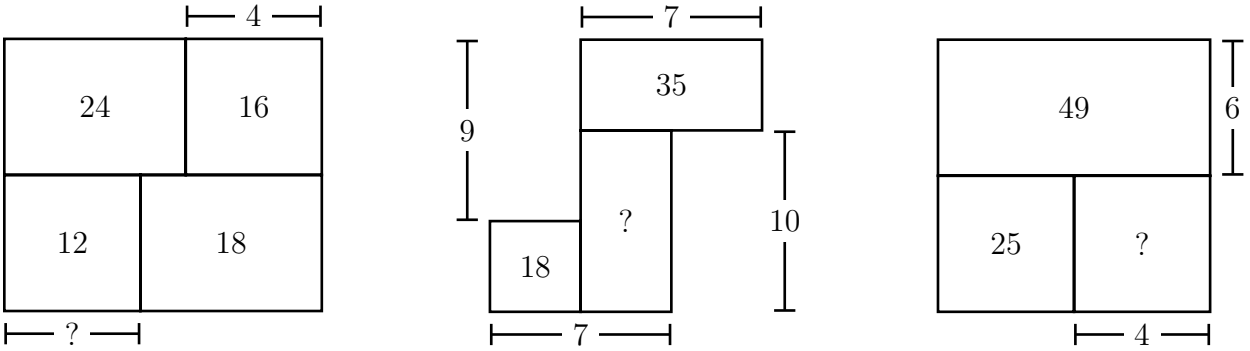
Make the black and white "checkers" switch places in only 8 moves.
There are only two types of moves allowed:

1. You can slide any coin into an empty space.
2. You can jump over any adjacent checker into an empty space.



Challenge Problem 35: Area Mazes

The figures are not drawn to scale. Each number represents either an area or a length. Find the missing value.

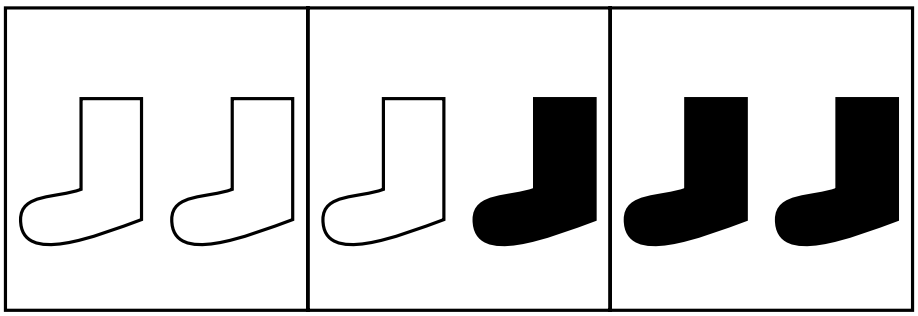


Challenge Problem 36: Black Socks

A dresser has 3 drawers.

- One drawer contains only 2 black socks.
- One drawer contains only 2 white socks.
- One drawer contains 1 black sock and 1 white sock.

You select a drawer at random. Without looking you reach in and pull out a black sock. What is the probability that the other sock in that drawer is black?



Challenge Problem 37: At Least One Boy

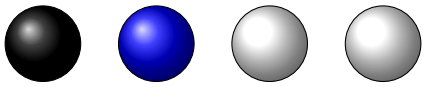
A man truthfully states:

> "I have two children. At least one is a boy."

What is the probability that his other child is a boy?

Challenge Problem 38: Two White Marbles

Four marbles are placed in a bag. One is black, one is blue, and two are white. You draw two marbles in your hand. If there is at least one white marble in your hand, what is the probability that the other marble is also white?

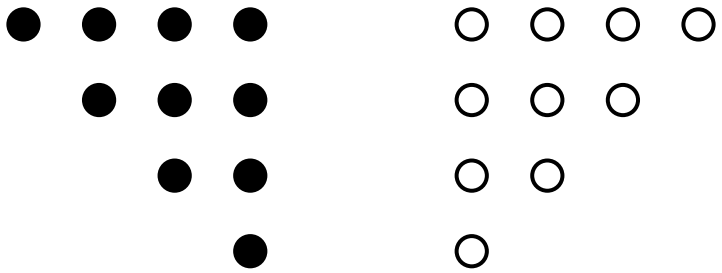


Challenge Problem 39: 6 Rows of 3

Arrange 7 trees so that there are 6 rows of 3 trees?

Challenge Problem 40: Lazy Reflection

Move exactly 3 black points so that the black points are arranged like the white points.



Challenge Problem 41: Shikaku

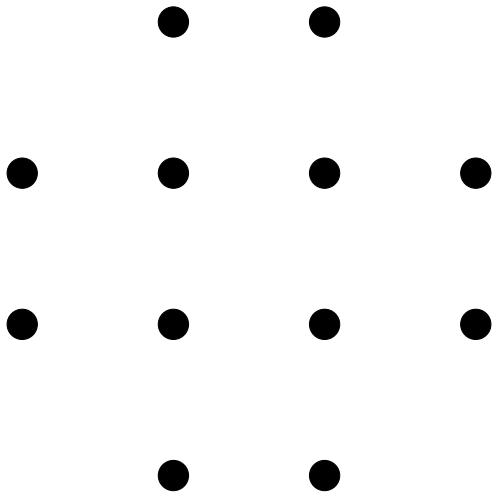
A Shikaku puzzle is completed by dividing up a rectangular grid into smaller rectangles. The objective is to divide the grid into rectangular pieces such that each piece contains exactly one number, and that number represents the area of the rectangle.

Complete the Shikaku puzzle below.

		8					6
6							
		2			4		4
		3					
			6	5		6	
	4		2		4		
			2				2

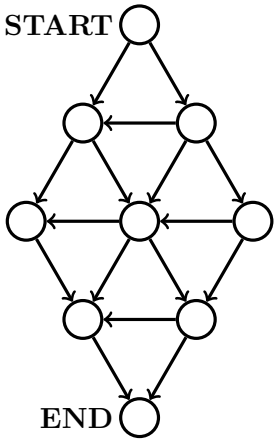
Challenge Problem 42: Squares from a Lattice

How many squares can be made using the points of the figure as the corners of the square?



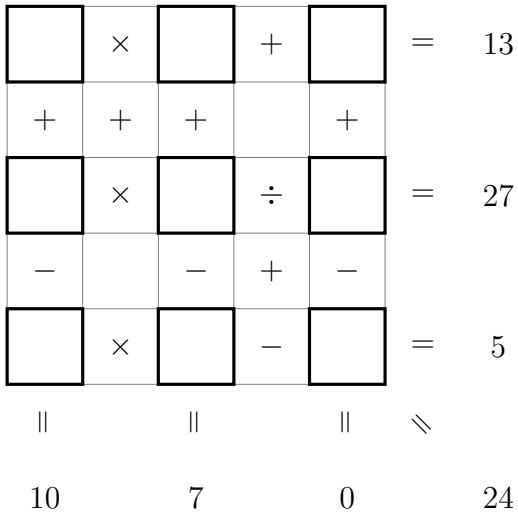
Challenge Problem 43: Path Counter

How many different paths can take you from **START** to **END** following the arrows?



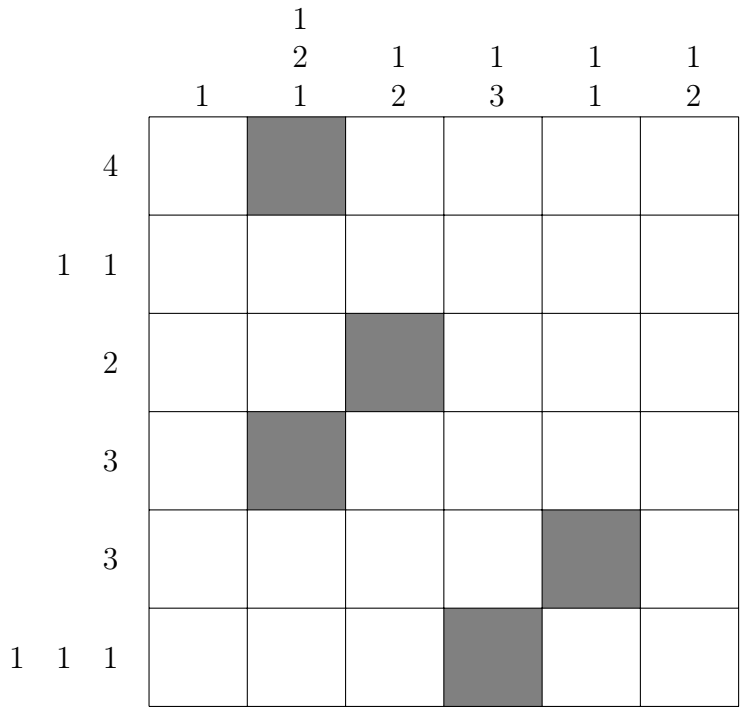
Challenge Problem 44: Mystery Boxes

Use the numbers 1 through 9 to fill in the empty boxes so that the seven equations are satisfied.



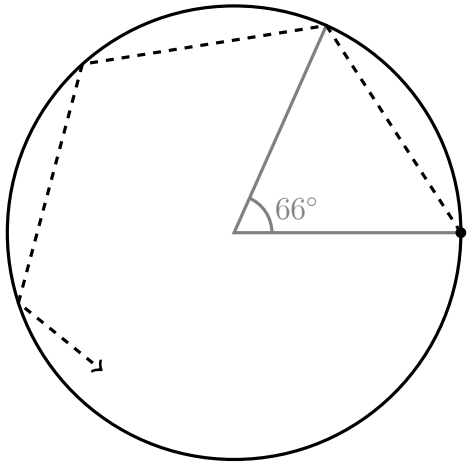
Challenge Problem 45: Grid Shading Puzzle

In this puzzle each square is to be shaded either gray or white. A few of the squares are shaded to get you started. The numbers along the side and top show the sequences of consecutive gray squares to be shaded in each row and column (with at least one white square between them). Shade the remaining squares to complete the puzzle.



Challenge Problem 46: Bouncing Laser

A laser beam is fired inside a circle and bounces around the interior of the circle as pictured.



How many times will the laser bounce before it returns to its starting point?

Challenge Problem 47: Butterfly

Figures 1, 2, 3, and 4 consist of 1, 7, 17, and 31 non-overlapping unit squares, respectively. If the pattern were continued, how many non-overlapping unit squares would there be in Figure 100?

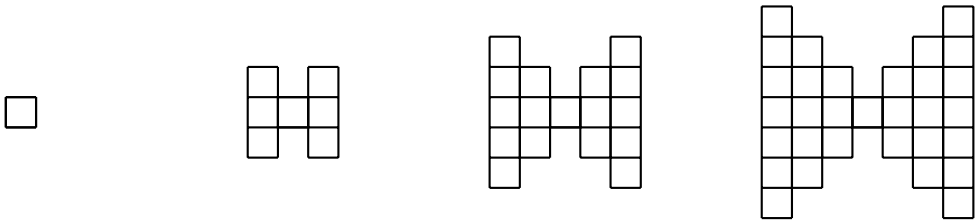
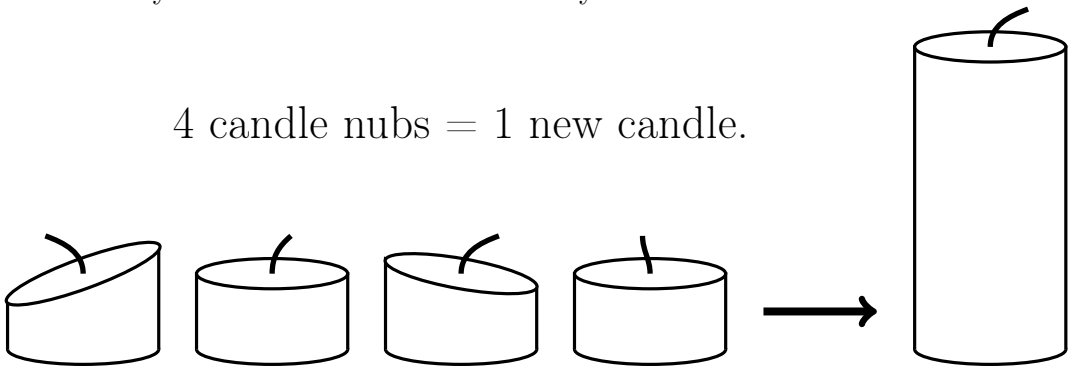


Figure 1 Figure 2 Figure 3 Figure 4

Challenge Problem 48: Candle Recycling

Whenever you have 4 used candle nubs you can combine them to create 1 new candle.

4 candle nubs = 1 new candle.



If you purchase 100 candles, how many total candles can you use?

Challenge Problem 49: Minesweeper

In each of the grids below there are some empty cells. Some of the empty cells contain a hidden mine. The numbers in the grid reveal the total number mines in the eight neighboring cells. Solve all three minesweeper puzzles by identifying the squares that contain mines.

	3			1
2			2	
		3		2
	2			
2				3

1				1
	1		1	
0				2
	1		1	
1				1

	2		3	
	4			1
				2
1		1		
				1

Challenge 50: Negabinary

In base 2, each number can be written using only 0s and 1s. Each digit represents a multiple of a distinct power of 2. For example, we can write the number 23 as

$$23 = 10111_2 \qquad \text{because} \qquad 23 = 1\,(16) + 0\,(8) + 1\,(4) + 1\,(2) + 1\,(1)\,.$$

For example, we can write the numbers 1-10 in base 2 as follows.

$$\begin{aligned} 1 &= 1_2 \\ 2 &= 10_2 \\ 3 &= 11_2 \\ 4 &= 100_2 \\ 5 &= 101_2 \\ 6 &= 110_2 \\ 7 &= 111_2 \\ 8 &= 1000_2 \\ 9 &= 1001_2 \\ 10 &= 1010_2 \end{aligned}$$

In base -2, each number can be written using only 0s and 1s. Each digit represents a multiple of a distinct power of -2. For example, we can write the number 23 as

$$23 = 1101011_2 \qquad \text{because} \qquad 23 = 1\,(64) + 1\,(-32) + 0\,(16) + 1\,(-8) + 0\,(4) + 1\,(-2) + 1\,(1)\,.$$

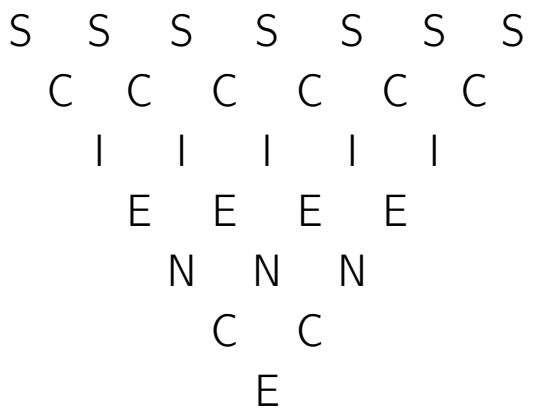
Write the digits 1-10 in base -2.

Challenge 51: Non-attacking Kings

In the game of chess, two kings are said to be attacking each other if they are in squares that are adjacent. What is the maximum number of kings that can be placed on a 100×100 chessboard so that no two kings are attacking each other?

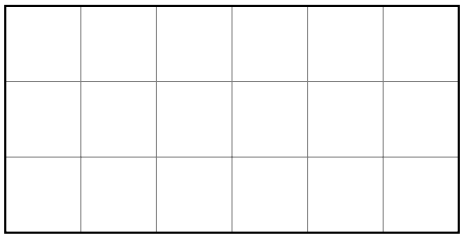
Challenge 52: Science Paths

How many paths in the array below spell out the word "SCIENCE" (traveling only down to the left or down to the right)?



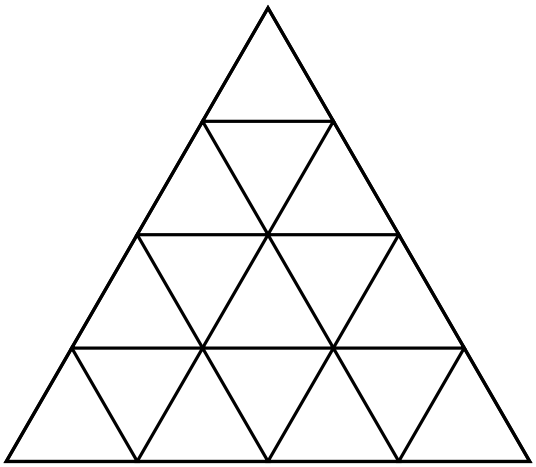
Challenge 53: Rectangle Reassemble

Subdivide the 3×6 rectangle into two pieces that can be reassembled to make a 2×9 rectangle.



Challenge 54: Triangle Counting

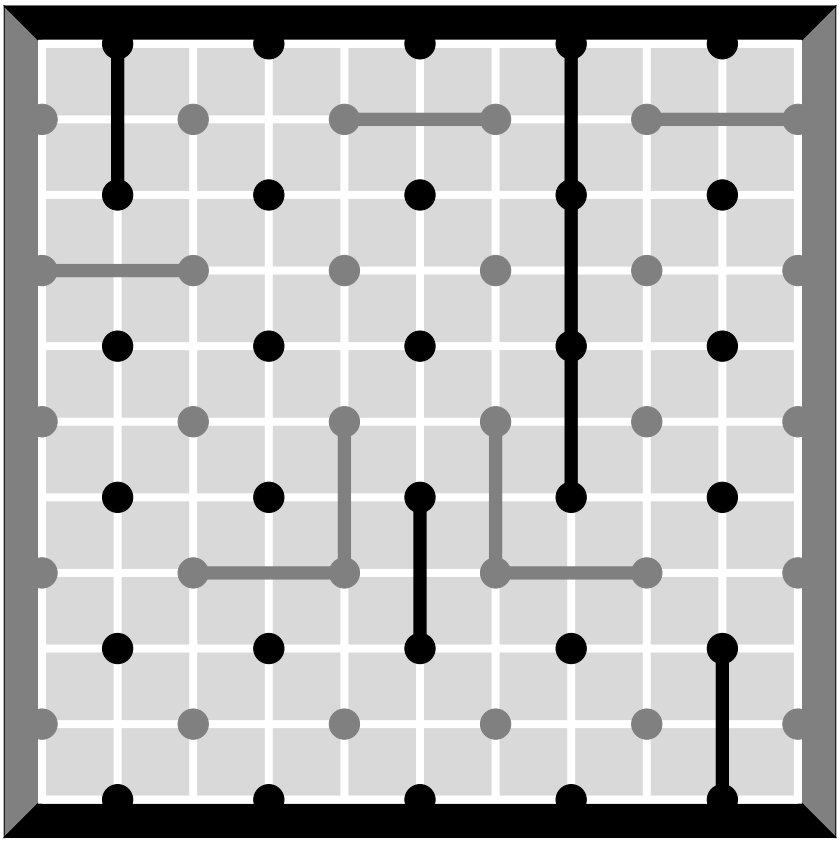
How many triangles are in the figure?



Challenge 55: Side-to-Side

In the game of Side-to-Side, players alternate turns drawing line segments between dots of their own color. The first player to connect their sides with a continuous path is the winner.

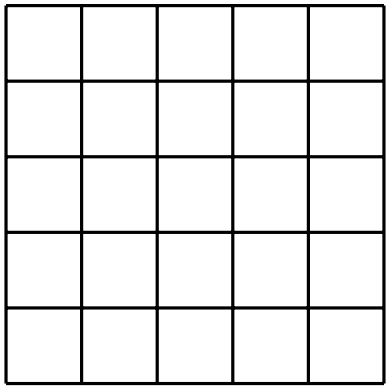
Below is a partially completed game of Side-to-Side. Which player will win if both players play the best possible moves from here on out?



Challenge 56: Obstruction

In the game of Obstruction, players alternate turns claiming a square of rectangular grid. No player may play in an occupied square or in any square that neighbors an occupied square. The last player to claim a square is the winner. (This is equivalent to placing non-attacking chess kings on the board, with the winner being the final player to place a king.)

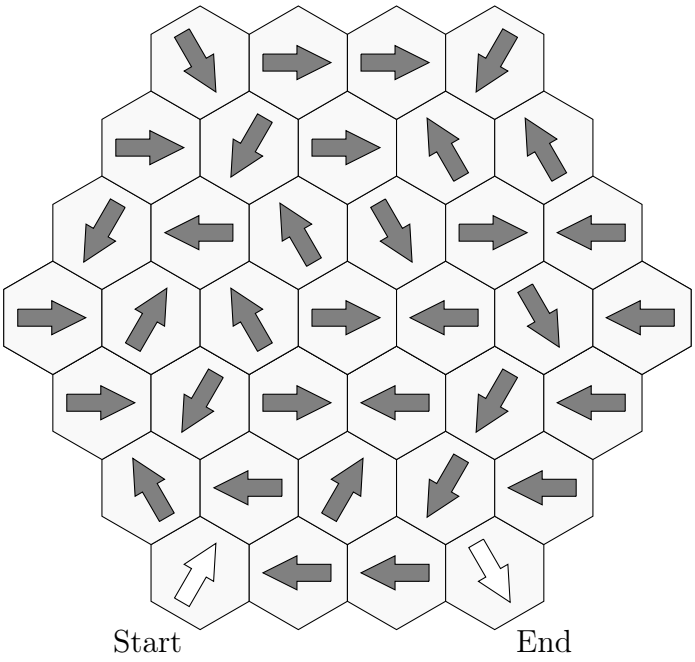
On a 5×5 board, does the first player or second player have a winning strategy?



Challenge 57: Hex Maze

Complete the maze by following the arrows. You must land on a hexagon each move, and you must exit the hexagon in the direction of the arrow. However, you may jump over any number of hexagons each time you move.

Find a path from Start to End!



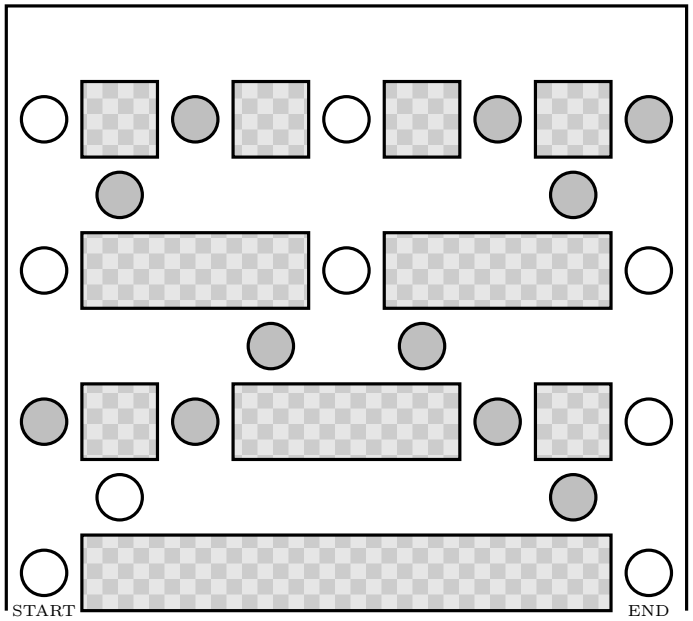
Challenge 58: Counting Maze

In this maze you always travel horizontally or vertically by choosing a direction and taking exactly the number of steps in the cell you have landed in. You are not allowed to exit the 6×6 square. Start in the upper left square and end in the lower left square.

START 3	3	2	2	2
3	2	3	2	3
1	3	2	2	3
3	1	2	3	4
END ★	2	3	2	3

Challenge 59: Alternating Color Maze

Find a path from START to END that never crosses the same color dot twice in a row.



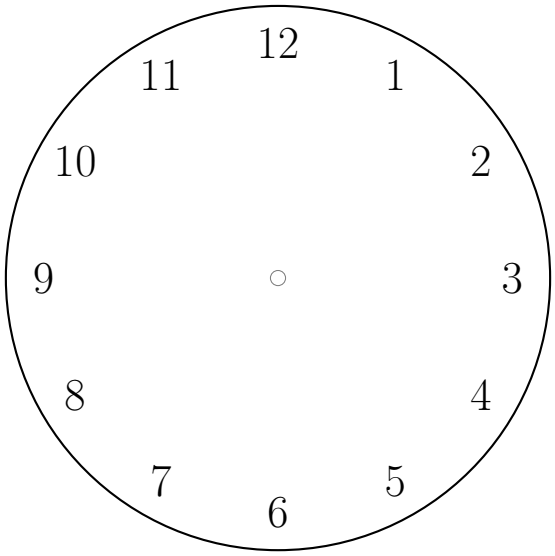
Challenge 60: Fair Division

Three friends (Mike, Ike, and Spike) get together for their annual seafood cookout. Mike contributes 5 fish to dinner, while Ike brings 7 fish. Spike brings no fish, but he gives 20 $1-bills to Mike and Ike to divide among themselves. Then the three friends share the fish equally.

How should Mike and Ike divide up the $20 fairly beween themselves?

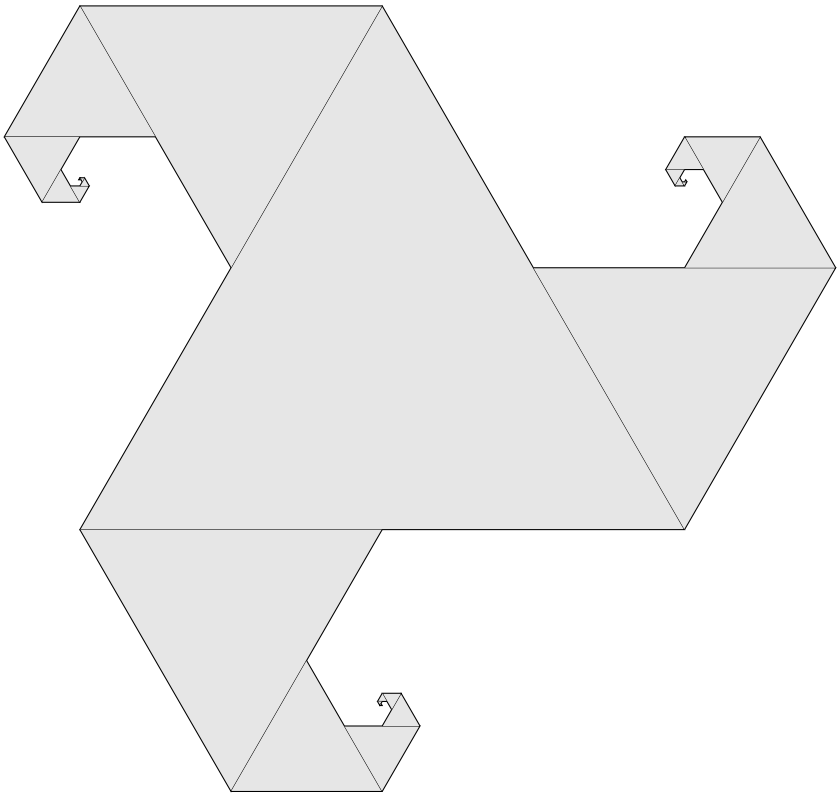
Challenge 61: Dividing a Clock

Slice the clock face into sections by drawing two lines across the clock face so that the sum of the numbers in each section are all equal to one another.



Challenge 62: Spiral Outwards

A figure is made by bisecting the edges of an equilateral triangle and attaching another equilateral triangle as pictured below. If the area of the largest triangle is 64 square units, what is the total shaded area?



Challenge 63: 5 Circles

Draw one straight line that divides the shaded region into two regions of equal area.