

## Lesson Summary

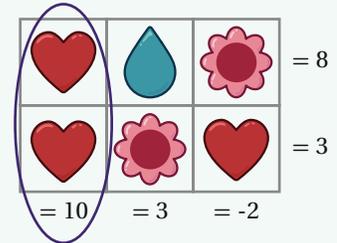
There are many different ways to solve problems and puzzles using math. Let's look at a few strategies for determining the values of shapes in a puzzle.

### Strategy 1: Look for single-shape rows/columns.

If we know the value of two hearts, we can determine the value of one heart and substitute that value in the other parts of the puzzle.

$$2(\heartsuit) = 10$$

$$\heartsuit = 5$$



### Strategy 2: Substitute known shape values to solve for missing values.

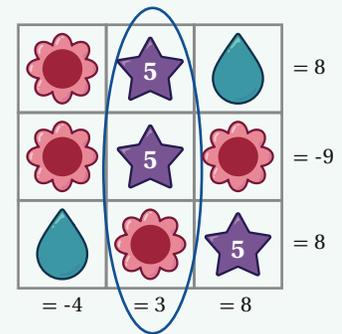
If we know the value of one star, then we can substitute that value in other parts of the puzzle.

$$2(\star) + \text{flower} = 3$$

$$2(5) + \text{flower} = 3$$

$$10 + \text{flower} = 3$$

$$\text{flower} = -7$$



### Strategy 3: Look for repeating shape patterns.

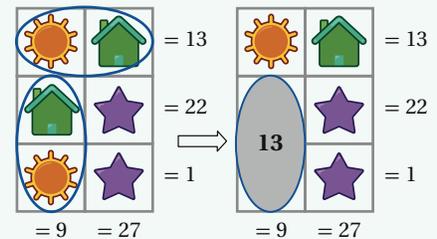
If we know the value of a shape combination and we see it repeat in the puzzle, then we can substitute the value for the shape combination.

$$\text{sun} + \text{house} = 13$$

$$\text{sun} + (\text{house} + \text{sun}) = 9$$

$$\text{sun} + 13 = 9$$

$$\text{sun} = -4$$



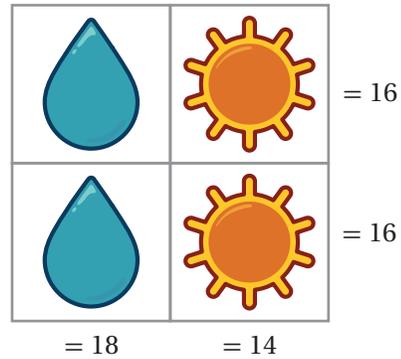
## Things to Remember:

# Lesson Practice

A1.5.01

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

**Problems 1–2:** Here is a shape puzzle. The sum of each row and column is shown.



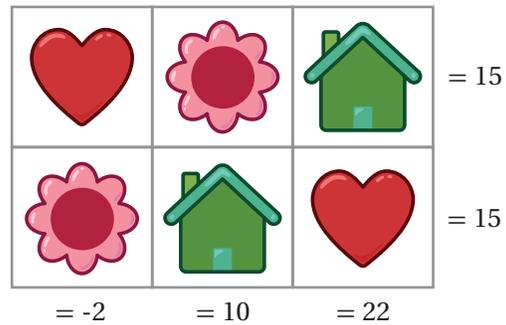
1. Select *all* the true statements.

- A.  +  = 18
- B.  +  = 14
- C.  = 14
- D.  +  = 18
- E.  +  = 16

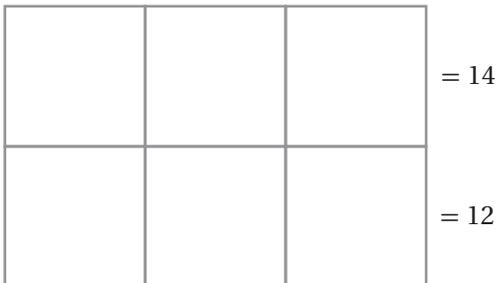
2. Show or explain why this statement is *false*:  = 8.

3. Determine the solution for this puzzle.

Shape	Value
Heart	
Flower	
House	



**Problems 4–5:** Use these two equations:



$$x + x + y = 14$$

$$y + y + y = 12$$

4. Draw a shape puzzle to represent these equations.

5. Determine the values of  $x$  and  $y$ .

$x =$  \_\_\_\_\_  $y =$  \_\_\_\_\_

# Lesson Practice

A1.5.01

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

6. Determine the missing shape in the center of this puzzle. Circle your choice.

Heart      House      Star

Show or explain your thinking.

			= 4
			= 3
			= 2
= 2	= 11	= -4	

## Spiral Review

Problems 7–10: Determine the value of the variable that makes each equation true.

7.  $2.5 + (-3) = a$

8.  $12 + b = -9$

9.  $2c + 3 = 15$

10.  $3d + 2 = 35$

## Reflection

1. Put a heart next to the problem you found most interesting.
2. Use this space to ask a question or share something you're proud of.

## Lesson Summary

A *system of equations* is two or more equations that represent the same constraints using the same variables. The *solution to a system of equations* is the set of all values that makes every equation in the system true. Just like in a shape puzzle where the value of each shape stays the same throughout the puzzle, in a system of equations, the value of the variables stays the same throughout the entire system. There are many solutions to a linear equation in two variables, but there might be only one solution, no solution, or infinitely many solutions to a system of linear equations in two variables.

There are many strategies for determining the solution to a system of equations that makes all equations in a system true. One strategy is called **elimination**, where you add or subtract the equations to produce a new equation with one variable. After finding the value of the first variable, you substitute that value into any original equation to find the value of the second variable. Let's look at some examples.

If the equations in the system share the same *coefficient* with opposite signs on the same variable, you can eliminate a variable by adding. You can solve this system by adding to eliminate the  $y$ -variable.

$$\begin{array}{r} -2x + y = 9 \\ + (8x - y = 3) \\ \hline 6x + 0 = 12 \\ x = 2 \end{array}$$

$$\begin{array}{r} -2(2) + y = 9 \\ y = 13 \end{array}$$

If the equations in the system share the same *coefficient* with the same signs on the same variable, you can eliminate a variable by subtracting. You can solve this system by subtracting to eliminate the  $x$ -variable.

$$\begin{array}{r} x + 2y = 30 \\ - (x + y = 23) \\ \hline y = 7 \end{array}$$

$$\begin{array}{r} x + (7) = 23 \\ x = 16 \end{array}$$

## Things to Remember:

# Lesson Practice

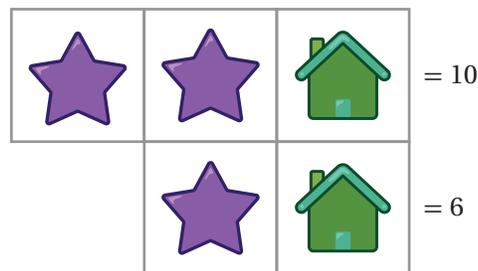
A1.5.02

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

1. Solve this system of equations. Use the shape puzzle if it helps with your thinking.

$$2x + y = 10$$

$$x + y = 6$$



$x =$  \_\_\_\_\_       $y =$  \_\_\_\_\_

**Problems 2–3:** Mateo made a mistake while solving this system of equations.

2. Describe one thing Mateo did *correctly*.

$$\begin{array}{r} 2x + y = 19 \\ -(x - y = 11) \\ \hline x + 0 = 8 \\ x = 8 \end{array}$$

3. Describe one thing Mateo did *incorrectly*.

**Problems 4–5:** Determine the solution for each system of equations.

4.  $3x + 4y = 6$   
 $3x + 2y = 18$

5.  $5x + 6y = 26$   
 $-5x + 2y = -18$

$x =$  \_\_\_\_\_       $y =$  \_\_\_\_\_       $x =$  \_\_\_\_\_       $y =$  \_\_\_\_\_

# Lesson Practice

A1.5.02

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

## Spiral Review

**Problems 6–8:** Write an equivalent expression by combining like terms.

6.  $5a + 3b - 2a$

7.  $3(c - 2) + 2c$

8.  $5d - 2(7d + 3g)$

9. The function  $f(t)$  models a hiker's elevation above or below sea level, in meters,  $t$  hours after noon. Circle the equation that represents this statement:  
*At 7 PM, the hiker was 3 meters below sea level.*

A.  $f(7) = 3$

B.  $f(19) = -3$

C.  $f(7) = -3$

D.  $f(-3) = 7$

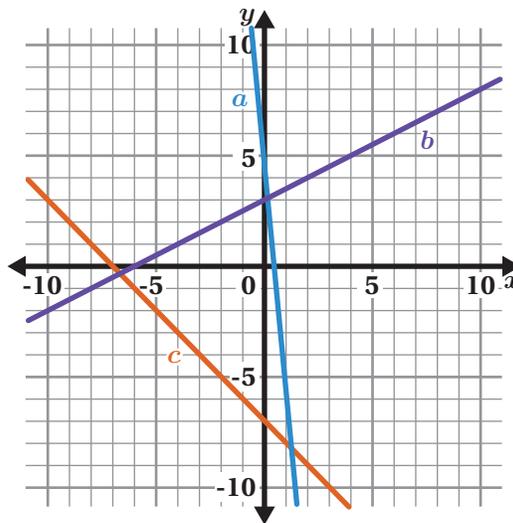
10. Which line represents  $x - 2y = -6$ ?

A. Line  $a$

B. Line  $b$

C. Line  $c$

Explain your thinking.



## Reflection

1. Put a star next to the problem you spent the most time on.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

It can be helpful to write *equivalent equations* when using elimination to solve systems of equations. You can create equivalent equations by multiplying each term of the first or second equation by a number. Your goal is to end up with a system of equations where one variable has the same or *opposite* coefficients so you can add or subtract them to eliminate a variable.

Here is a system of equations:

$$\begin{aligned} 9x - 4y &= 2 \\ 3x + y &= 10 \end{aligned}$$

You can multiply the second equation by -3 to eliminate the  $x$ -variables.

$$\begin{aligned} 9x - 4y &= 2 \\ -3(3x + y = 10) & \\ \hline 9x - 4y &= 2 \\ + -9x - 3y &= -30 \\ \hline 0 - 7y &= -28 \\ \boxed{y = 4} & \\ 3x + (4) &= 10 \\ 3x &= 6 \\ \boxed{x = 2} & \end{aligned}$$

Or you can multiply the second equation by 4 to eliminate the  $y$ -variables.

$$\begin{aligned} 9x - 4y &= 2 \\ 4(3x + y = 10) & \\ \hline 9x - 4y &= 2 \\ + 12x + 4y &= 40 \\ \hline 21x + 0 &= 42 \\ \boxed{x = 2} & \\ 9(2) - 4y &= 2 \\ 18 - 4y &= 2 \\ -4y &= -16 \\ \boxed{y = 4} & \end{aligned}$$

Things to Remember:

# Lesson Practice

## A1.5.03

Name: ..... Date: ..... Period: .....

1. Select *all* expressions that are equivalent to  $6x + 8y = 10$ .

A.  $12x + 16y = 20$

B.  $2x + 6y = 10$

C.  $3x + 4y = 5$

D.  $1.5x + 2y = 2.5$

E.  $4x + 5y = 4$

2. Arnav and Omari are solving this system of equations.

$$4x + 2y = 62$$

They disagree about what the first step should be to eliminate a variable.

$$-8x - y = 59$$

**Arnav's strategy:** Multiply  $4x + 2y = 62$  by 2

**Omari's strategy:** Multiply  $-8x - y = 59$  by 2

Whose strategy will eliminate a variable once the equations are added? Circle your choice.

Arnav's

Omari's

Both

Neither

Explain your thinking.

**Problems 3–4:** Determine the solution to each system of equations.

3.  $2x - 4y = 10$   
 $x + 5y = 40$

4.  $5x + 2y = 20$   
 $2x - 3y = -11$

$x = \dots\dots\dots$       $y = \dots\dots\dots$

$x = \dots\dots\dots$       $y = \dots\dots\dots$

# Lesson Practice

A1.5.03

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

5. Using the digits 0–9, without repeating, fill in each blank to create two equivalent equations.

$$\square x + \square y = \square$$

$$\square x + \square y = \square$$

## Spiral Review

Problems 6–8: Solve each equation.

6.  $x - 2 = 5$

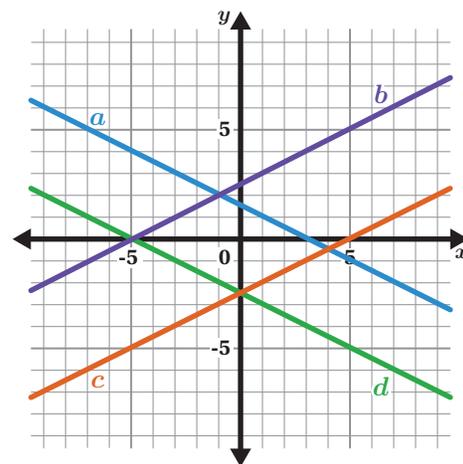
7.  $3(x - 2) = 3 \cdot 5$

8.  $8x - 16 = 40$

9. Solve the equation  $3x - 9y = 72$  for  $y$ .

10. Which line represents  $5x + 10y = 15$ ?

- A. Line  $a$
- B. Line  $b$
- C. Line  $c$
- D. Line  $d$



## Reflection

1. Put a question mark next to a response you'd like to compare with a classmate's.
2. Use this space to ask a question or share something you're proud of.

## Lesson Summary

One strategy you can use to solve a system of equations is **substitution**, where you replace a variable with an equivalent expression. Substitution is a useful strategy when one variable is already isolated in at least one equation in the system.

Here are two examples of systems of equations where substitution may be a useful strategy. Notice that once you substitute, your new equation only has one variable.

In this system, both  $y$ -variables are already isolated. You can substitute the expression  $-4x + 6$  in for  $y$  in the second equation. The new equation sets both expressions equal to each other.

$$\begin{aligned}
 y &= -4x + 6 \\
 y &= 3x - 15 \\
 y &= -4x + 6 \quad \rightarrow \quad y = 3x - 15 \\
 -4x + 6 &= 3x - 15 \\
 -7x &= -21 \\
 \boxed{x = 3} \\
 y &= 3(3) - 15 \\
 \boxed{y = -6}
 \end{aligned}$$

In this system of equations,  $y$  is already isolated, so you can substitute the expression  $2x - 5$  in for  $y$  in the first equation.

$$\begin{aligned}
 -3x - 2y &= 3 \\
 y &= 2x - 5 \\
 y &= 2x - 5 \quad \rightarrow \quad -3x - 2y = 3 \\
 -3x - 2(2x - 5) &= 3 \\
 -3x - 4x + 10 &= 3 \\
 -7x + 10 &= 3 \\
 -7x &= -7 \\
 \boxed{x = 1} \\
 y &= 2(1) - 5 \\
 \boxed{y = -3}
 \end{aligned}$$

## Things to Remember:

# Lesson Practice

## A1.5.04

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

**Problems 1–2:** Show or explain what your first step would be for solving each system of equations.

1.  $4x - y = 20$   
 $x + y = 5$

2.  $6x - 12y = 24$   
 $y = 2x - 1$

3. Determine the solution to this system of equations:

$$7x - y = -3$$
$$y = x - 3$$

**Problems 4–5:** Alma made a mistake as she started to solve this system of equations.

$$y = \frac{1}{2}x - 1$$
$$4x - 2y = 11$$

Alma

$$4x - 2\left(\frac{1}{2}x - 1\right) = 11$$
$$4x - x - 2 = 11$$
$$3x - 2 = 11$$
$$3x = 13$$
$$x = \frac{13}{3}$$

4. What is the error in Alma's work?

5. Determine the solution to this system of equations.

# Lesson Practice

A1.5.04

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

## Spiral Review

**Problems 6–7:** Kadeem made a mistake as he started to solve this system of equations.

$$\begin{aligned} 5x - 4y &= 6 \\ 5(x + y) &= 25 \end{aligned}$$

Kadeem

$$\begin{aligned} 5x - 4y &= 6 \\ 5x + 5y &= 125 \\ \hline -1y &= 131 \\ y &= -131 \end{aligned}$$

6. Show or explain one thing Kadeem did correctly.

7. Show or explain Kadeem's mistake.

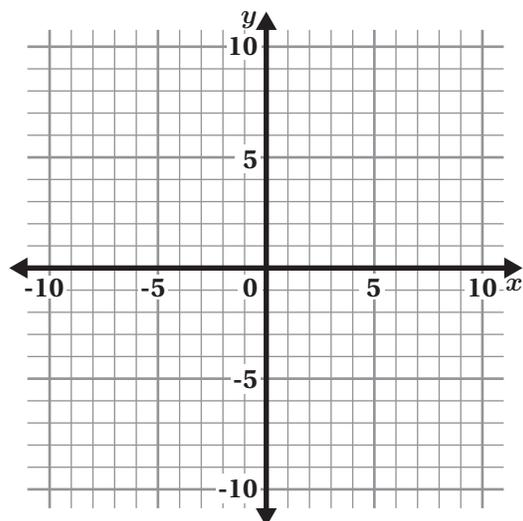
**Problems 8–10:** Solve each equation for the given variable.

8. Solve for  $k$ .  
 $2t + k = 6$

9. Solve for  $x$ .  
 $4x + 3y = 12$

10. Solve for  $y$ .  
 $4x + 3y = 12$

11. Graph the equation  $4x - 6y = 24$ .



## Reflection

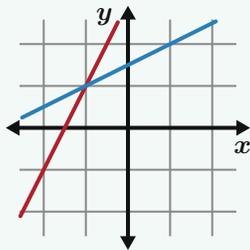
1. Put a heart next to the problem you're most proud of.
2. Use this space to ask a question or share something you're proud of.

## Lesson Summary

On a coordinate plane, you can see the solution of a system of equations at the point(s) where the two lines intersect. A system of linear equations can have:

**One Solution**

The lines intersect at  $(-2, 2)$ .



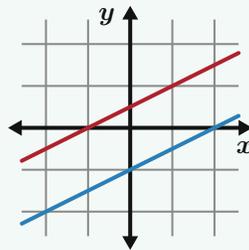
$$y = 2x + 6$$

$$y = \frac{1}{2}x + 3$$

The equations have *different slopes*.

**No Solution**

The lines are *parallel*.



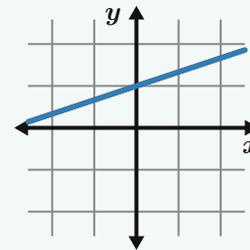
$$y = \frac{1}{2}x + 1$$

$$y = \frac{1}{2}x - 2$$

The equations have the same slope and different *y*-intercepts.

**Infinitely Many Solutions**

The lines are the same.



$$y = \frac{1}{3}x + 2$$

$$3y - x = 6$$

The equations are *equivalent*.

## Things to Remember:

# Lesson Practice

## A1.5.05

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

1. The point  $(-2, 2)$  is on the line  $y = x + 4$ . Explain how you can determine if this point is the solution to this system of equations:

$$y = x + 4$$

$$y = 2x - 1$$

2. Solve this system of equations. Write the solution as a coordinate pair.

$$y = -\frac{1}{2}x - 8$$

$$y = 3x + 6$$

3. Match each system of equations to the number of solutions it has.

a.  $y = -2x + 1$

$$2y = -4x + 2$$

..... No solution

b.  $y = -2x + 1$

$$y = -2x + 4$$

..... One solution

c.  $y = -2x + 1$

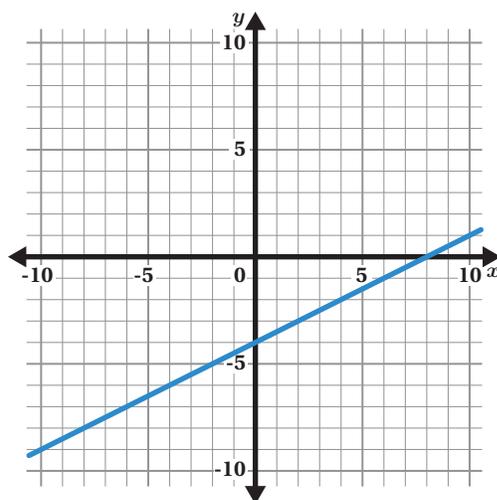
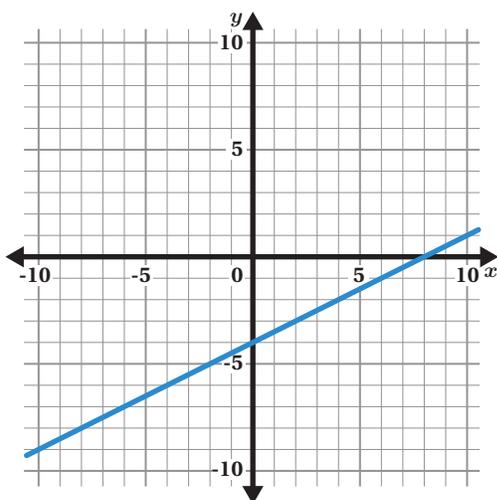
$$y = 2x + 1$$

..... Infinitely many solutions

**Problems 4–5:** Here is a graph of  $y = \frac{1}{2}x - 4$ . Graph a second line to make this system of equations have:

4. No solution.

5. One solution at  $(2, -3)$ .



6. Here is one equation in a system of equations:  $y = 3x - 2$ . Write a different second equation so that the system of equations has infinite solutions.

# Lesson Practice

A1.5.05

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

7. A system of linear equations has no solution. Select *all* of the statements that must be true about the equations in this system.

- A. The equations have different slopes.
- B. The equations have the same slope.
- C. The equations have different  $y$ -intercepts.
- D. The equations have the same  $y$ -intercepts.

## Spiral Review

8. Here is a shape puzzle. What is the value of each shape?

Shape	Value
Star	
House	
Flower	

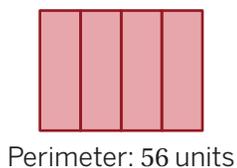
		= -2
		= 13
		= 20
= 23	= 8	

9. Solve this inequality:  $3(x - 3) > 2x - 6$ .

10. Here are two arrangements of identical rectangles. Determine the dimensions  $a$  and  $b$ .

$a =$  \_\_\_\_\_ units

$b =$  \_\_\_\_\_ units



## Reflection

- Put a star next to a problem that looked more difficult than it really was.
- Use this space to ask a question or share something you're proud of.

## Lesson Summary

You can solve systems of equations symbolically using either substitution or elimination. Looking for specific structures in the equations can help you decide which strategy to use.

- It may be helpful to use substitution when at least one of the equations has an isolated variable or at least one is in *slope-intercept form*.
- It may be helpful to use elimination when both equations are in the same form or if the equations have a pair of same or opposite terms.

When solving a system of equations symbolically, sometimes all of the variables are eliminated.

**No Solution**

When the result of eliminating all of the variables is a false statement, there is *no solution* to the system of equations. This means the lines are parallel and will never intersect.

$$\begin{array}{r} y = 3x + 6 \qquad y = 3x - 6 \\ 3x + 6 = 3x - 6 \\ 3x + 12 = 3x \\ 12 = 0 \end{array}$$

**Infinitely Many Solutions**

When the result of eliminating all of the variables is a true statement, there are *infinitely many solutions* to the system of equations. The equations are equivalent and represent the same line.

$$\begin{array}{r} 2 \cdot (2x + 4y = 6) \\ -4x - 8y = -12 \\ \hline 4x + 8y = 12 \\ + -4x - 8y = -12 \\ \hline 0 + 0 = 0 \\ 0 = 0 \end{array}$$

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**Things to Remember:**

# Lesson Practice

A1.5.08

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

**Problems 1–3:** Show or explain what your *first step* would be to solve each system of equations.

1.  $6x + 21y = 103$   
 $-6x + 23y = 51$

2.  $2x + y = 10$   
 $y = 6$

3.  $y = \frac{2}{3}x + 7$   
 $y = \frac{2}{3}x - 3$

**Problems 4–5:** Determine how many solutions each equation has. If there is one solution, what is the solution?

4.  $5x + 2y = 29$   
 $-58 = -10x - 4y$

5.  $2x + 3y = 2$   
 $x = 4y + 12$

No Solution

No Solution

One Solution

One Solution

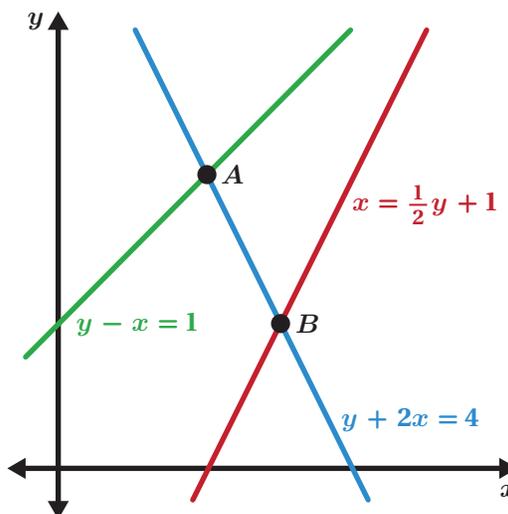
Infinitely many solutions

Infinitely many solutions

6. Determine the coordinates of points *A* and *B*, the intersections of the lines on the graph.

*A*: \_\_\_\_\_

*B*: \_\_\_\_\_



# Lesson Practice

A1.5.08

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

7. Here is a system of equations.

$$\begin{aligned}y &= 6x - 7 \\ 3y &= 6x + 15\end{aligned}$$

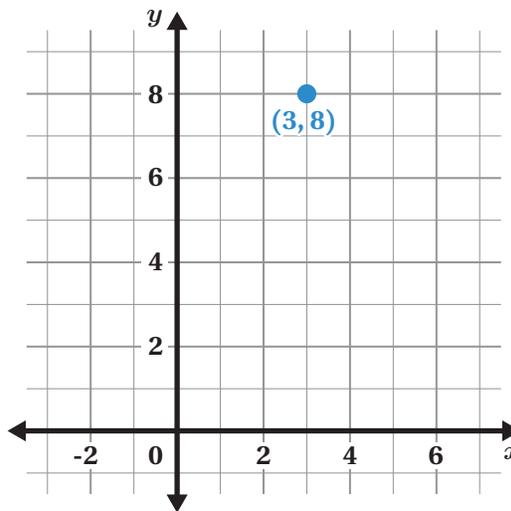
Juliana claims that this system has no solutions. Do you agree with Juliana's claim? Explain your thinking.

8. Write a system of equations where  $(3, 8)$  is the solution.

Use the graph if it helps your thinking.

Equation 1: \_\_\_\_\_

Equation 2: \_\_\_\_\_



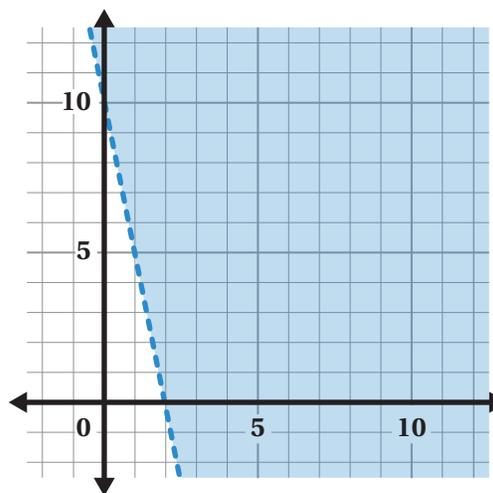
## Spiral Review

9. Select *all* the ordered pairs that are solutions to the inequality  $6y < 30 - 5x$ .

- A.  $(0, 0)$      B.  $(6, 3)$      C.  $(0, 5)$      D.  $(4, 1)$      E.  $(-5, 0)$

10. Which inequality represents this graph?

- A.  $5x + y < 10$   
B.  $5x + y \leq 10$   
C.  $5x + y > 10$   
D.  $5x + y \geq 10$



## Reflection

1. Circle the problem that was the most challenging for you.
2. Use this space to ask a question or share something you're proud of.

## Lesson Summary

A **system of inequalities** is a system of two or more inequalities that represent the *constraints* on a shared set of variables.

You can use different strategies to determine if a point is a solution to a system of inequalities.

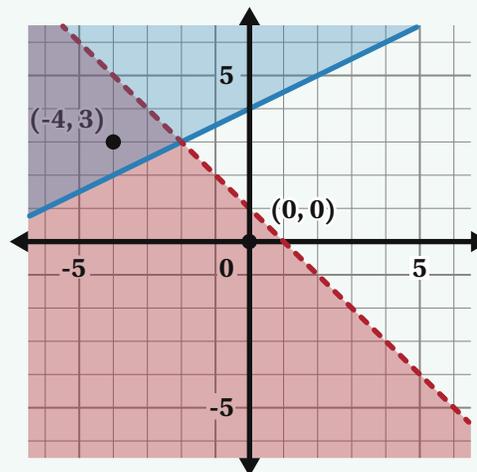
- If the point is in the shaded region for both inequalities, then it is a solution to the system.
- If the  $x$ - and  $y$ -values of the point are substituted into both inequalities and the inequalities are true, then the point is a solution to the system.

Here is a graph for this system of inequalities.

$$\begin{aligned}x + y &< 1 \\ y &\geq \frac{1}{2}x + 4\end{aligned}$$

You can see that the point  $(-4, 3)$  is a solution because it is in the shaded region for both inequalities.

You can also substitute points into both inequalities to determine if they are solutions.  $(0, 0)$  is not a solution and  $(-4, 3)$  is a solution.



## Things to Remember:

# Lesson Practice

A1.5.09

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

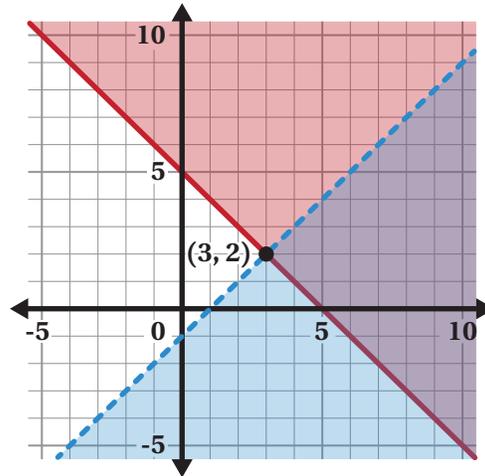
1. The graph shows this system of inequalities.

$$x + y \geq 5$$

$$x - y > 1$$

Is the point  $(3, 2)$  a solution to the system?

Explain your thinking.



**Problems 2–4:** It costs Lukas \$5.00 to mail a package. Lukas has *postcard stamps*,  $p$ , that are worth \$0.34 each and *first-class stamps*,  $f$ , that are worth \$0.49 each.

- Lukas wrote the inequality  $0.34p + 0.49f \geq 5$ . What does this inequality represent?
- Lukas wrote another inequality:  $p + f \leq 12$ . What does this inequality represent?
- If Lukas uses 1 postcard stamp and 9 first-class stamps, will this satisfy both constraints? Explain your thinking.

5. Using the digits 0–9 without repeating, fill in each blank such that each statement is true:

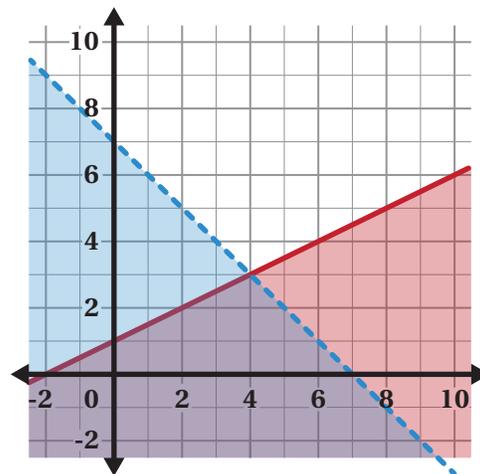
- $A$  is a solution to both inequalities.
- $B$  is a solution to only one inequality.
- $C$  is a solution to only the other inequality.
- $D$  is not a solution to either inequality.

$$A = (\square, \square)$$

$$B = (\square, \square)$$

$$C = (\square, \square)$$

$$D = (\square, \square)$$



# Lesson Practice

A1.5.09

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

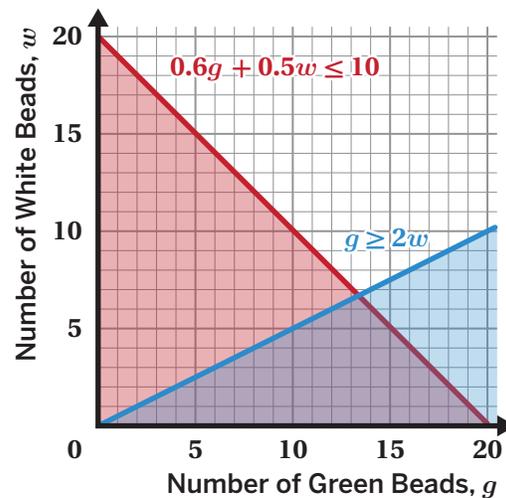
**Problems 6–7:** Arjun is making a bracelet.

He has \$10 to spend on beads.

Green beads cost \$0.60 each and white beads cost \$0.50 each.

His bracelet design needs at least twice as many green beads as white beads.

The graph shows the system of inequalities that represents this situation.



6. What is a combination of green and white beads that meets both constraints?
7. What is a combination of green and white beads that meets *only one* constraint?

## Spiral Review

8. Solve this system of equations. Write your solution as a coordinate pair.

$$2x + y = 8$$

$$y = 2x + 4$$

9. Guiying is at the market with \$14 to buy fruit. Guiying decides to buy apples and grapes. Apples,  $a$ , cost \$1.67 per pound and grapes,  $g$ , cost \$1.87 per pound. Write an inequality to represent this situation.
10. Rio claims that  $(5, 10)$  is *not* a solution to the inequality  $y < 3x - 5$ . Do you agree with Rio's claim? Explain your thinking.

## Reflection

1. Put a star next to the problem you understood best.
2. Use this space to ask a question or share something you're proud of.

## Lesson Summary

The **solutions to a system of inequalities** are all the points that make both inequalities true. The solutions can be seen in the region that is shaded by all inequalities in the system, called the **solution region**.

One strategy for determining the location of the solution region is to test a point. Choose a point that is not on either *boundary line*, substitute the  $x$ - and  $y$ -values into each inequality to see if it makes the statement true, and shade based on the results of the test.

Let's look at this system of inequalities:

$$3x + y \geq 6$$

$$y > x + 2$$

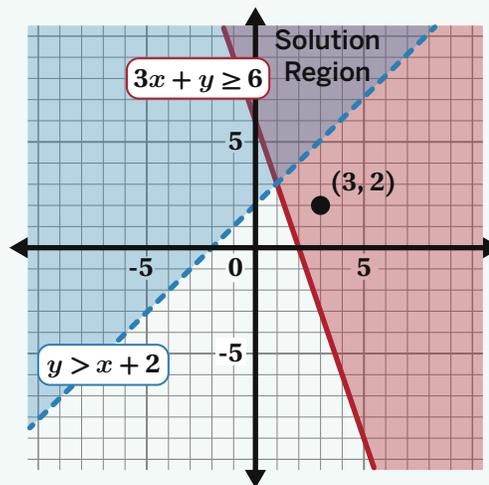
You can test the point  $(3, 2)$  to help determine the solution region.

**Solid Line**  
 $3x + y \geq 6$   
 $3(3) + 2 \geq 6$   
 $11 \geq 6$   
 True ✓

Shade the side of the solid line that includes  $(3, 2)$

**Dashed Line**  
 $y > x + 2$   
 $2 > 3 + 2$   
 $2 > 5$   
 False X

Shade the side of the dashed line that does not include  $(3, 2)$



## Things to Remember:

# Lesson Practice

## A1.5.10

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

**Problems 1–2:** Here is the graph of a system of inequalities.

$$y > -x + 2$$

$$3x + y \geq 1$$

1. Which letter represents the solution region to the system of inequalities?

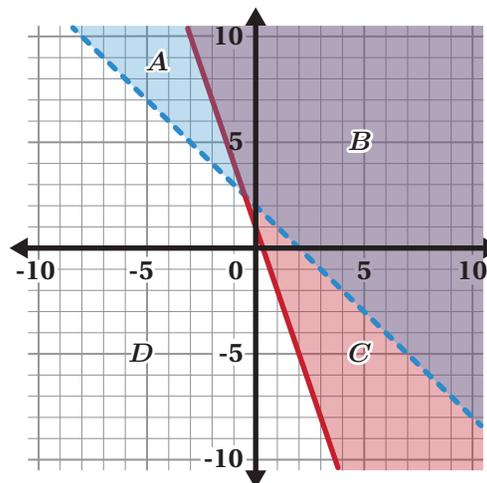
Circle one:

A      B      C      D

2. Is the point (5, -4) a solution to the system?

Circle one:

Yes      No

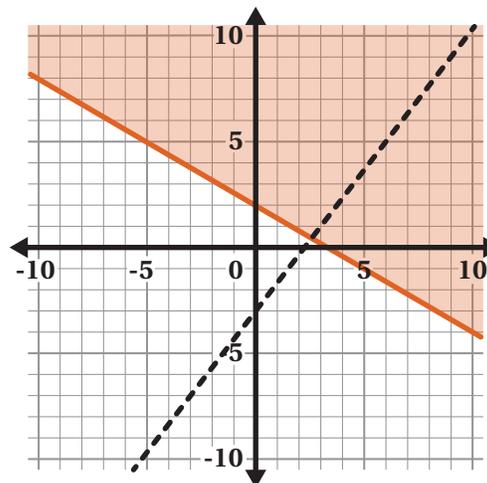


**Problems 3–4:** Javier graphed the first inequality and the boundary line of the second inequality.

$$3x + 5y \geq 10$$

$$4x - 3y < 9$$

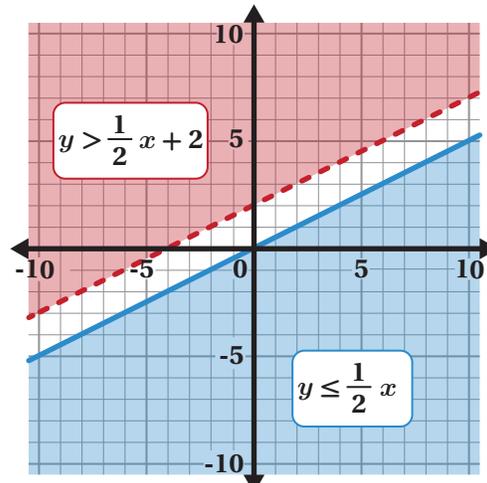
3. Complete the graph of the second inequality.
4. Explain how you knew where to shade the second inequality.



**Problems 5–6:** Here is a system of inequalities that has no solutions.

5. Explain how you know the system has no solutions.

6. Describe one change you could make to the system of inequalities that would have solutions.



# Lesson Practice

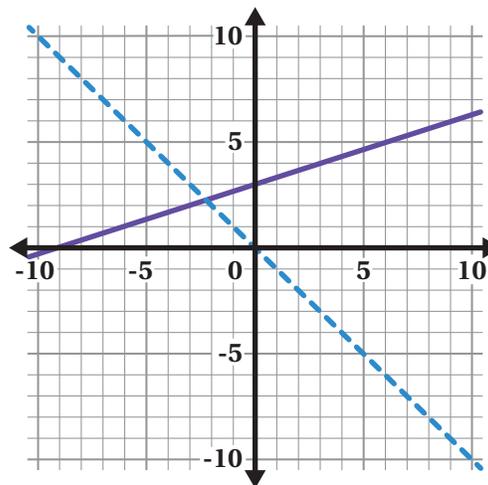
A1.5.10

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

**Problems 7–8:** Nyanna graphed the boundary lines of this system of inequalities:

$$\begin{aligned}x + y &> 0 \\ -x + 3y &\leq 9\end{aligned}$$

7. Complete the graph of the system of inequalities.



8. Identify a coordinate pair that is in the solution region.

**Problems 9–10:** Fill in each blank with an inequality symbol such that:

9. The system has no solutions.

$$x - y \quad \square \quad 0$$

$$x - y \quad \square \quad 0$$

10. Only points with matching  $x$ - and  $y$ -coordinates are a solution to the system.

$$x - y \quad \square \quad 0$$

$$x - y \quad \square \quad 0$$

## Spiral Review

**Problems 11–13:** Determine the value of each expression.

11.  $\frac{4}{6} \cdot 8$

12.  $\frac{3}{5} \cdot 2 \cdot \frac{10}{9}$

13.  $\frac{3}{6} \cdot 3 \cdot \frac{6}{9} \cdot 2$

## Reflection

1. Circle the problem you feel least confident about.
2. Use this space to ask a question or share something you're proud of.

**Lesson Summary**

Situations that include repeated multiplication can be modeled using an equation of the form  $y = a \cdot b^x$ , where  $a$  represents the initial value and  $b$  represents the base. The base determines whether the quantity in the situation will increase or decrease over time. You can use the equation to solve problems about the situation.

For example, Carlos bought a new mega-growing fish. The fish weighed 4 grams when Carlos bought it, and its mass grows 1.5 times greater every hour.

He wrote the equation  $m = 4 \cdot 1.5^t$  to represent the relationship, where:

- $m$  represents the mass of Carlos's fish in grams.
- $t$  represents the time in hours.

You can use the equation to determine the mass of the fish after 4 hours by substituting  $t = 4$  and solving for  $m$ .

$$m = 4 \cdot 1.5^t$$

$$m = 4 \cdot 1.5^4$$

$$m = 4 \cdot 5.0625$$

$$m = 20.25$$

After 4 hours, the fish will have a mass of 20.25 grams.

---

**Things to Remember:**

# Lesson Practice

## A1.6.01

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

**Problems 1–3:** Carlos has a pack of toy fish whose mass doubles every hour when you add water. This table shows the mass of a toy fish over time.

1. What was the mass of the toy fish before it was in water?
2. What will the mass of the toy fish be after 6 hours?

Time (hr)	Mass (g)
0	?
1	30
2	60
3	120
4	240
...	...
6	?

3. Wohali buys another brand of toy fish that claims to grow faster than Carlos's fish. Wohali wrote this equation:  $m = 10 \cdot 3^t$ . He used  $m$  for the mass, in grams, and  $t$  for time, in hours. Explain what the 10 and 3 mean in this situation.

**Problems 4–5:** Jamar had 80 followers on social media. His number of followers tripled every month for 4 months.

4. *Select all* the expressions that represent Jamar's followers after 4 months.  
 A.  $80 \cdot 3 \cdot 3 \cdot 3 \cdot 3$        B.  $80 + 4^3$        C.  $80 \cdot 3^4$   
 D.  $80 + 3 + 3 + 3 + 3$        E.  $80 \cdot 4 \cdot 4 \cdot 4$
5. Complete the table.

Time (months)	0	1	2	3	4
Followers					

# Lesson Practice

A1.6.01

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

**Problems 6–7:** A group of biologists tracked the number of squirrels in a town. They wrote the equation  $n = 40 \cdot 1.5^t$ , where  $n$  is the total number of squirrels and  $t$  is the number of years since the biologists started counting.

6. Explain what the 40 and 1.5 mean in this situation.
7. How many squirrels do the biologists predict there will be 2 years after they started counting?
8. A baby chicken weighs 32 grams when it hatches. The chicken's mass increases by 45% each week for the first 12 weeks of its life. Complete the table of the chicken's mass over time.

Time (weeks)	0	1	2	3	4	...	12
Mass (g)						...	

## Spiral Review

**Problems 9–12:** Determine the value of each expression.

9.  $3^3$

10.  $2(3^3)$

11.  $3^3 + 4$

12.  $2 \cdot 3^3 + 4$

## Reflection

1. Circle the problem you feel least confident about.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

An **exponential function** increases or decreases by a *constant ratio*. The constant ratio will multiply consecutive *y*-values. Another name for the constant ratio is the **growth factor**.

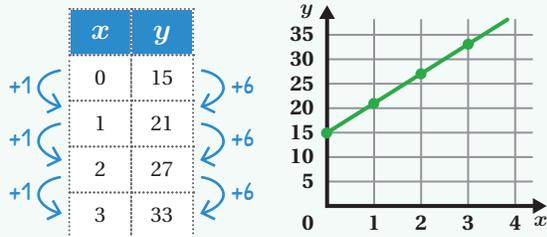
A **linear function** increases or decreases by a *constant difference*. Another name for the constant difference is the *rate of change*.

Here are two examples.

Linear Function

This pattern has a constant difference of 6, so the rate of change is 6.

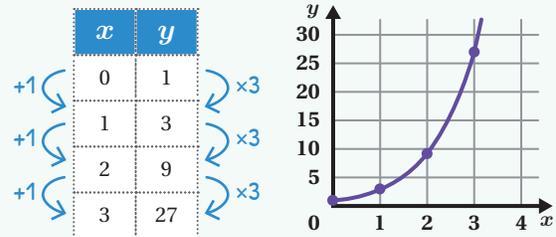
The graph of this linear function is a straight line.



Exponential Function

This pattern has a constant ratio of 3, so the growth factor is 3.

The graph of this exponential function is a curve that gets steeper and steeper.



Things to Remember:

# Lesson Practice

A1.6.02

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

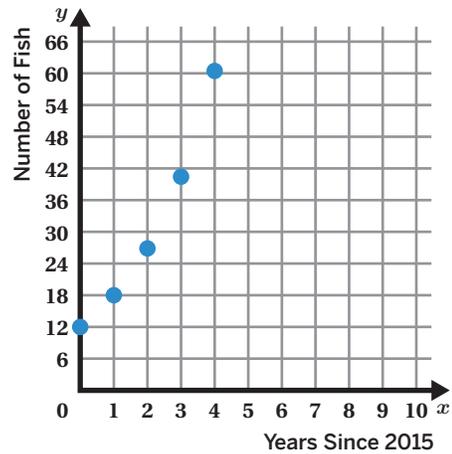
**Problems 1–3:** These tables show the number of red and yellow globs each day.

Day	0	1	2	3	4
Red Globs	50	70	90	110	

Day	0	1	2	3	4
Yellow Globs	5	10	20	40	

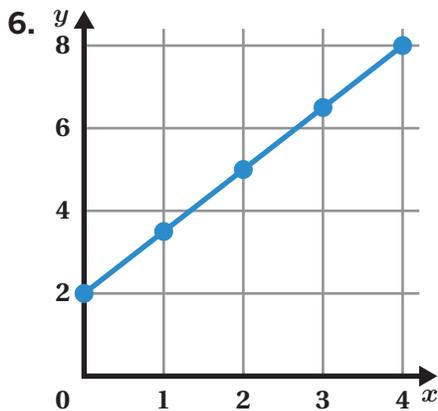
- How many of each type of glob will there be on day 4?
- Will there be more red or yellow globs on day 10? Show or explain your thinking.
- Which group of globs changes by a constant growth factor? Show or explain how you know.

**Problems 4–5:** This graph shows the number of fish in a pond from 2015 to 2019.

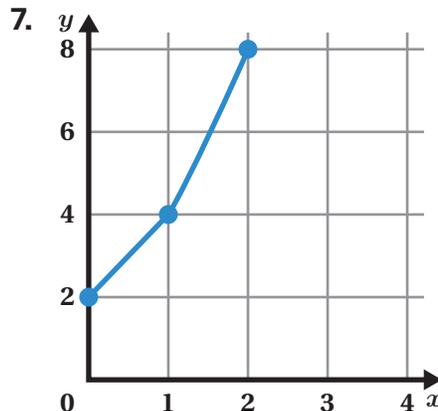


- How many fish are in the pond in 2015?
- Does the number of fish grow by a *constant difference*? Show or explain how you know.

**Problems 6–7:** Determine whether each graph shows a constant rate of change or a constant growth rate. Circle your choice.



Constant rate of change      Constant growth rate



Constant rate of change      Constant growth rate

# Lesson Practice

A1.6.02

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

**Problems 8–9:** Determine whether each table shows a linear or exponential function. Circle your choice.

8.

$x$	$y$
0	4
1	8
2	16
3	32

Linear                  Exponential

9.

$x$	$y$
0	4
1	8
2	12
3	16

Linear                  Exponential

## Spiral Review

10. Determine the value of each expression when  $n = 4$ .

**a**  $n^2 - 5$

**b**  $n(n + 6)$

**c**  $3n^2$

11. Approximate the value of each radical expression.

**a**  $\sqrt{10}$

**b**  $\sqrt{54}$

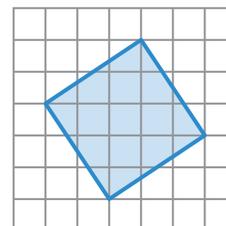
**c**  $\sqrt{41}$

Between \_\_\_\_\_ & \_\_\_\_\_

Between \_\_\_\_\_ & \_\_\_\_\_

Between \_\_\_\_\_ & \_\_\_\_\_

12. Each grid square represents 1 square unit. What is the exact side length of the shaded square?



## Reflection

- Put a question mark next to a response you'd like to compare with a classmate's.
- Use this space to ask a question or share something you're proud of.

## Lesson Summary

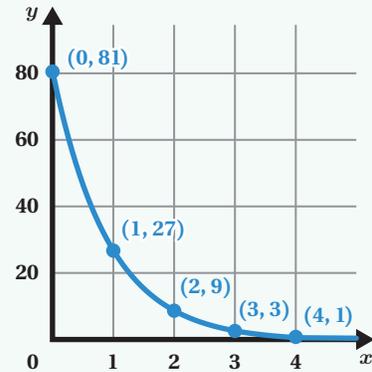
In the exponential function  $f(x) = a \cdot b^x$ ,  $a$  represents the  $y$ -intercept and  $b$  represents the growth factor. Both parts can be seen on a graph.

Let's look at two examples.

Here is the graph of the exponential function

$$g(x) = 81 \cdot \left(\frac{1}{3}\right)^x.$$

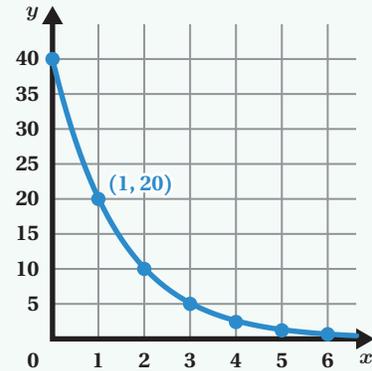
- 81 means that  $(0, 81)$  is the  $y$ -intercept.
- $\frac{1}{3}$  is the growth factor. As  $x$  increases by 1, the  $y$ -values are multiplied by a factor of  $\frac{1}{3}$ .  $81 \cdot \left(\frac{1}{3}\right) = 27$ .



You can also write an exponential equation to represent a graph.

- The  $y$ -intercept is  $(0, 40)$ , so 40 is the  $a$ -value.
- The points  $(1, 20)$ ,  $(2, 10)$ , and  $(3, 5)$  are on the graph of the function. Since 10 is  $\frac{1}{2}$  of 20 and 5 is  $\frac{1}{2}$  of 10, this function has a growth factor of  $\frac{1}{2}$ , which is the  $b$ -value in the equation.

One way to write the equation of this exponential function is  $f(x) = 40 \cdot \left(\frac{1}{2}\right)^x$ .



## Things to Remember:

# Lesson Practice

A1.6.03

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

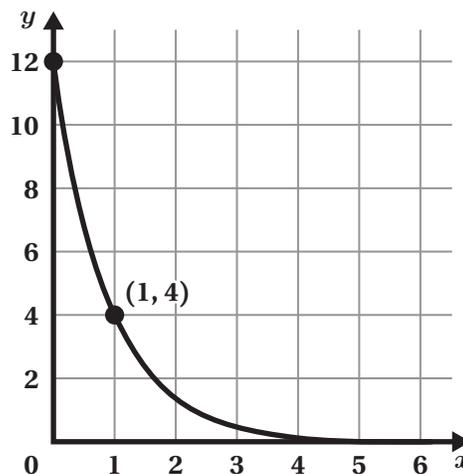
**Problems 1–3:** Determine the value of each expression when  $x = 2$ .

1.  $4^x$

2.  $\left(\frac{1}{3}\right)^x$

3.  $5(6^x)$

4. Here is a graph of  $y = 12 \cdot \left(\frac{1}{3}\right)^x$ . Explain where you can see the 12 and the  $\frac{1}{3}$  in the graph.



**Problems 5–7:** Match each equation to the graph that represents it.

**Equation A**

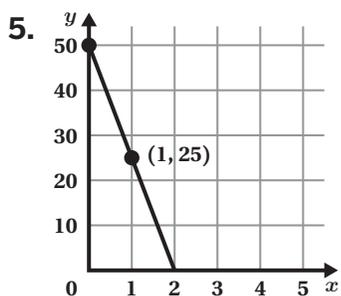
$$y = 50 \cdot \left(\frac{1}{2}\right)^x$$

**Equation B**

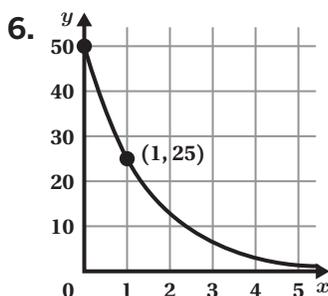
$$y = 50 \cdot 2^x$$

**Equation C**

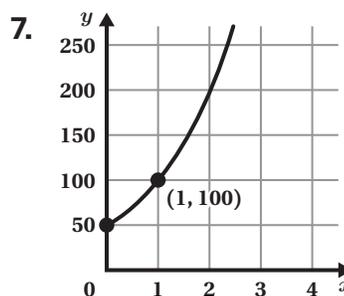
$$y = 50 - 25x$$



Equation \_\_\_\_\_



Equation \_\_\_\_\_



Equation \_\_\_\_\_

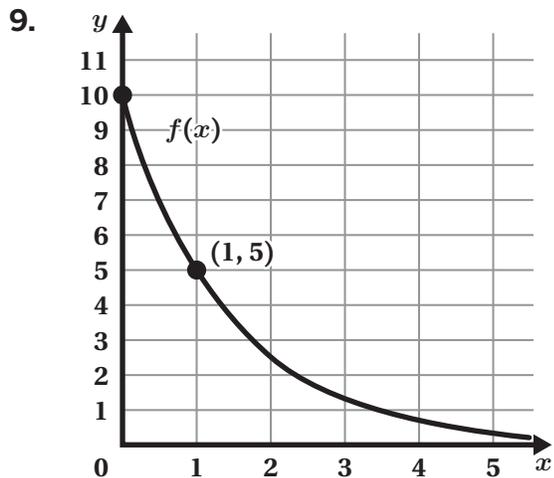
8. Explain how you determined which equation to match with the graph in problem 5.

# Lesson Practice

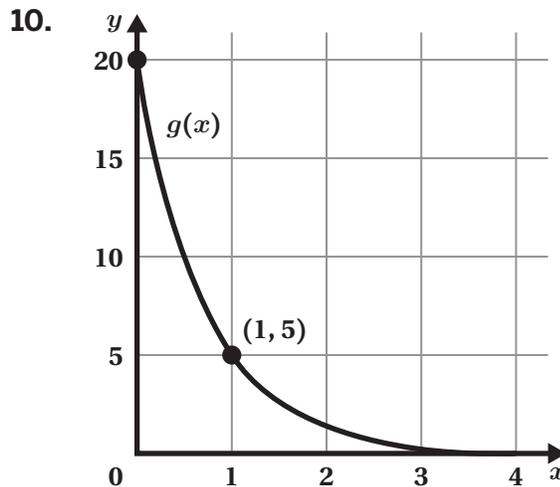
A1.6.03

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

**Problems 9–11:** Here are graphs of two different exponential relationships. Write an equation to represent each graph.



Equation \_\_\_\_\_



Equation \_\_\_\_\_

11. Pick Problem 9 or 10 and explain how you determined the initial value and the growth factor.

## Spiral Review

**Problems 12–15:** Rewrite each expression as a single power.

12.  $4^4 \cdot 4^8$

13.  $\frac{3^5}{3^8}$

14.  $(12^3)^5$

15.  $\frac{7^3 \cdot 7^4}{7^5}$

## Reflection

1. Put a smiley face next to the problem you learned from most.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

You can evaluate exponential functions for inputs that are positive, negative, or zero.

Let's evaluate the function  $f(x) = 9 \cdot 4^x$  for  $f(-3)$  using the equation or a table.

You can substitute a value into the *equation*.

Steps	Explanation
$f(-3) = 9 \cdot 4^{(-3)}$	Substitute $x = (-3)$ into the function.
$f(-3) = 9 \cdot \left(\frac{1}{4}\right)^3$	Apply the property of negative exponents to rewrite the expression with a positive exponent.
$f(-3) = 9 \cdot \left(\frac{1}{64}\right)$	Rewrite the expression without any exponents.
$f(-3) = \frac{9}{64}$	Multiply to determine the value of $f(-3)$ .

You can move backward in a *table*.

$x$	$f(x)$
-3	$\frac{9}{64}$
-2	$\frac{9}{16}$
-1	$\frac{9}{4}$
0	9

Multiply 9 by  $\frac{1}{4}$  three times to determine the value of  $f(-3)$ .

The domain of an exponential function can include values like zero, all positive, or all negative values. It is important to use the information in the situation to decide which values make sense and which do not. For example, a function that models the amount of caffeine remaining in the body after drinking coffee does not include negatives in the domain. However, an exponential model of the population of ants can include negative numbers in the domain.

Things to Remember:

# Lesson Practice

A1.6.05

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

1. Which equation best models the data in the table?

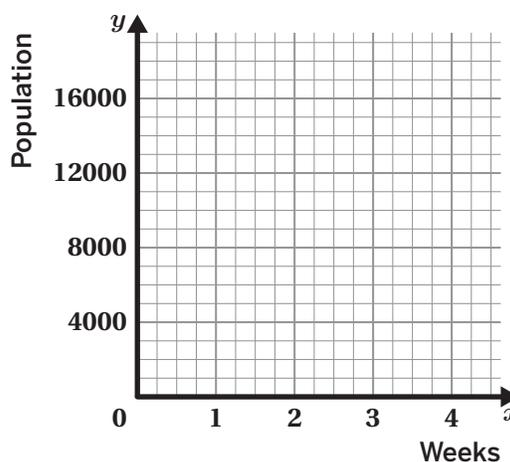
- A.  $f(x) = 80(1.25)^x$
- B.  $f(x) = 64(1.25)^x$
- C.  $f(x) = 64 + 1.25x$
- D.  $f(x) = 60 + 20x$

$x$	$f(x)$
1	80
2	100
3	125
4	156.25

**Problems 2–4:** The equation  $p(w) = 1000 \cdot 2^w$  models a population of mosquitos,  $p(w)$ , where  $w$  is the number of weeks after the population was first measured.

2. Complete the table and plot the values on the graph.

Weeks, $w$	Population, $p(w)$
0	
1	
2	
3	
4	



3. Where on the graph do you see the 1,000 from the equation?

4. Determine the value of  $p(-2)$  and explain what it means in this situation.

**Problems 5–7:** The equation  $f(t) = 800 \cdot \left(\frac{1}{2}\right)^t$  models a fish population,  $f(t)$ , where  $t$  is time in years since the beginning of 2015.

5. What is the population of fish at the beginning of 2015?

6. What is the population of fish at the beginning of 2018?

7. What is the population of fish at the beginning of 2012? Do you think this makes sense in this situation?

# Lesson Practice

A1.6.05

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

## Spiral Review

**Problems 8–9:** Charlie's gaming club wants to make at least \$300 selling boxes of cookies and pies. They make \$9 for each box of cookies and \$15 for each pie.

- Write an inequality to represent the number of boxes of cookies,  $c$ , and pies,  $p$ , the team can sell to make at least \$300.
- If the team sells 5 boxes of cookies, what is the minimum number of pies they need to sell in order to meet their goal?

**Problems 10–13:** Determine the value of each expression.

10.  $2^4$

11.  $-2^4$

12.  $(-2)^4$

13.  $\left(\frac{1}{2}\right)^4$

## Reflection

- Star a problem you're still feeling confused about.
- Use this space to ask a question or share something you're proud of.

**Lesson Summary**

Exponential functions can be written in the form  $f(x) = a \cdot b^x$ , where  $a$  is the *initial value* and  $b$  is the growth factor. Exponential functions represent repeated *percent increase* when the growth factor is larger than 1.

In situations that model repeated percent increase, the growth factor  $b$  can be written as 1 (representing 100%) plus the percent increase in decimal form.

For example, the value of a baseball card collection increases by 4% every year. In 2020, the collection was valued at \$500. Let  $f(x)$  represent the value of the collection and  $x$  represent the years since 2020.

- At first, the collection was valued at \$500. The initial value, or  $a$ , is 500.
- The value increases by 4% every year. Because the value of the cards increases by a repeated percent each year, we can represent the growth by changing the percent to a decimal (4% to 0.04) and adding 1. The growth factor, or  $b$ , will be 1.04.

We can write the exponential function that represents this situation as  $f(x) = 500 \cdot (1.04)^x$ .

---

**Things to Remember:**

# Lesson Practice

## A1.6.07

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

**Problems 1–3:** A group of biologists tracked the number of deer in a forest over several years. There were 600 deer when they first counted. The population has increased by 15% each year.

1. How many deer are in the forest 1 year after the biologists first counted?
2. Write an expression that represents the deer population after 3 years.
3. Write an expression that represents the deer population after  $t$  years.
4. Sothy's family paid \$1,300 in property tax last year. This year, the county will increase the property tax by 2.1%.

Select *all* the expressions that represent Sothy's family's property taxes this year.

- A.  $1300 + (1.021)$
- B.  $1300(1.21)$
- C.  $1300(1.021)$
- D.  $1300(1.0021)$
- E.  $1300 + 1300(0.021)$

**Problems 5–6:** Sai gets a \$500 loan from the bank with an annual interest rate of 6%.

5. Write a function,  $f(t)$ , to represent the amount Sai will owe, in dollars, after  $t$  years.
6. Complete the table to determine how much money Sai will owe over time without making any payments.

Time (yr)	Amount Owed (\$)
0	500
1	
2	
3	
4	

# Lesson Practice

A1.6.07

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

**Problems 7–9:** Three cities have the same initial population and different percent increases each year. Match each function  $p(t)$ , representing the population after  $t$  years, with its correct description.

$$p(t) = 5000 \cdot (1.20)^t$$

$$p(t) = 5000 \cdot (1.02)^t$$

$$p(t) = 5000 \cdot (1.002)^t$$

**7.** City A has a 0.2% annual increase in population.

**8.** City B has a 20% annual increase in population.

**9.** City C has a 2% annual increase in population.

.....

.....

.....

## Spiral Review

**10.** Order these values from least to greatest.

75% of 12

25% of 32

50% of 20

10% of 95

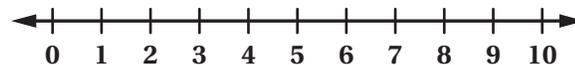
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Least

Greatest

**11.** Create a dot plot that has:

- At least 5 data points.
- A median of 7.
- A mean that is less than the median.



## Reflection

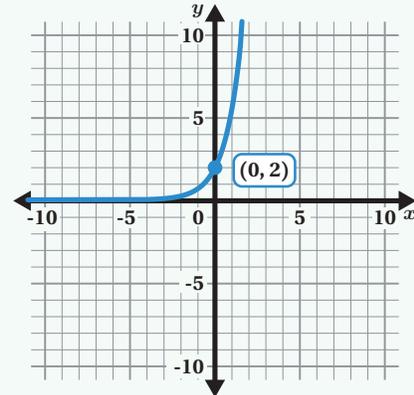
- 1.** Circle the problem you enjoyed doing the most.
- 2.** Use this space to ask a question or share something you're proud of.

Lesson Summary

Here is the graph of  $f(x) = 2 \cdot 3^x$ .

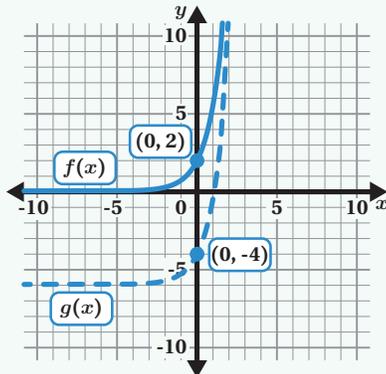
Functions can be translated horizontally and vertically.

Here are two examples of *translations* of  $f(x)$ .



Vertical Translations

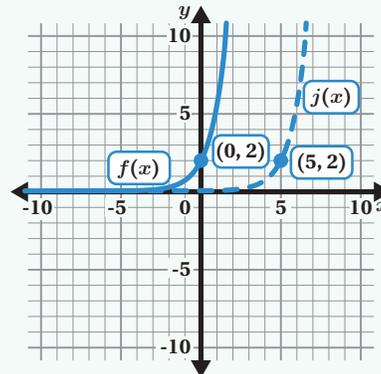
The equation  $f(x) = 2 \cdot 3^x + k$  represents a vertical translation by  $k$  units.



Here  $f(x)$  was translated 6 units down and can be represented with the new equation:  $g(x) = 2 \cdot 3^x - 6$ .

Horizontal Translations

The equation  $f(x) = 2 \cdot 3^{(x-h)}$  represents a horizontal translation by  $h$  units.



Here  $f(x)$  was translated 5 units to the right and can be represented with the new equation:  $j(x) = 2 \cdot 3^{(x-5)}$ .

Things to Remember:

# Lesson Practice

A1.6.10

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

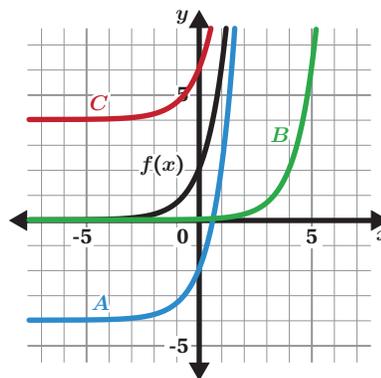
1. Here is the graph of  $f(x) = 2 \cdot 3^x$ .

Match each function with its graph.

$f(x) = 2 \cdot 3^x + 4$  .....

$f(x) = 2 \cdot 3^x - 4$  .....

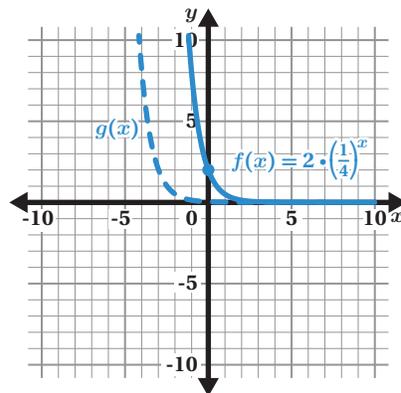
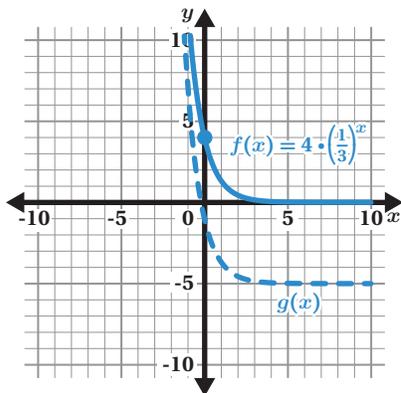
$f(x) = 2 \cdot 3^{(x-4)}$  .....



**Problems 2–3:** For each set of graphs shown, write an equation for the dotted curve,  $g(x)$ .

2.  $f(x) = 4 \cdot \left(\frac{1}{3}\right)^x$

3.  $f(x) = 2 \cdot \left(\frac{1}{4}\right)^x$



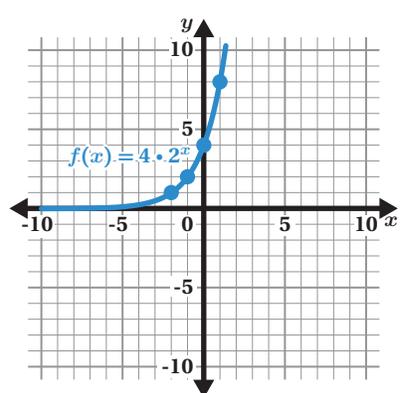
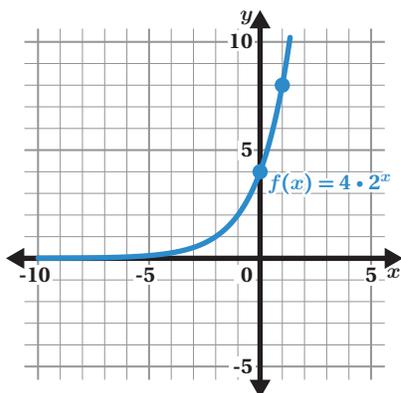
$g(x) =$  .....

$g(x) =$  .....

**Problems 4–5:** The function  $g(x)$  is a transformation of  $f(x) = 4 \cdot 2^x$ .

4. Graph  $g(x) = 4 \cdot 2^x + 1$ .

5. Graph  $g(x) = 4 \cdot 2^{(x-3)}$ .



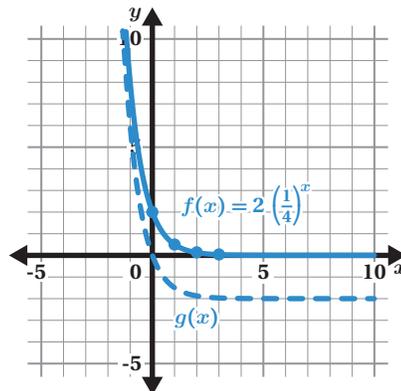
# Lesson Practice

A1.6.10

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

**Problems 6–8:** The function  $g(x)$  is a transformation of  $f(x) = 2 \cdot \left(\frac{1}{4}\right)^x$ .

6. Ariana says  $g(x) = 2 \cdot \left(\frac{1}{4}\right)^x - 4$  because the  $y$ -intercept of  $f(x)$  shifted down 4 units. Explain why Ariana's thinking is incorrect.



7. Kyrie says  $f(x)$  transformed using both vertical and horizontal translations. Explain why Kyrie's thinking is incorrect.
8. Write the correct equation for  $g(x)$ .

## Spiral Review

9. What is the solution to this system of equations?

$$5x + y = 18$$

$$x - 3y = 10$$

- A.  $(-2, -4)$   
B.  $(4, -2)$   
C. There is no solution.  
D. There are an infinite number of solutions.

## Reflection

- Put a star next to a problem you could explain to a classmate.
- Use this space to ask a question or share something you're proud of.

Lesson Summary

You can invest money in accounts that earn simple or compound interest. Accounts that earn **simple interest** can be modeled by linear functions, while accounts that earn **compound interest** can be modeled by exponential functions. Which account earns the most interest depends on how much time there is to invest and other variables.

Let's look at an example. Adah has \$100 to invest in an account. Which account should she choose if she has 12 years to invest?

Simple Interest	Compound Interest
Adah could invest \$100 in an account that earns 10% simple interest annually. The function $a(t) = 100 + 10t$ models the account balance after $t$ years.	Adah could invest \$100 in an account that earns 10% compound interest annually. The function $b(t) = 100 \cdot (1.10)^t$ models the account balance after $t$ years.
To determine the balance of the account after 12 years, substitute $t = 12$ into each function and solve for $a(t)$ .	
$a(12) = 100 + 10(12)$ $a(12) = 220$ After 12 years, the account balance will be \$220.	$b(12) = 100 \cdot (1.10)^{12}$ $b(12) = 313.84$ After 12 years, the account balance will be about \$313.84.

Adah may choose to invest in the account that earns compound interest because it earns more money over 12 years.

Things to Remember:

# Lesson Practice

## A1.6.13

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

**Problems 1–4:** Determine if each equation or table represents simple or compound interest. Circle your choice.

1.  $b(t) = 1000(1.03)^t$

Simple

Compound

2.  $b(t) = 1000 + 30t$

Simple

Compound

3.

Time (yr)	Account Balance (\$)
0	300
1	330
2	360

Simple

Compound

4.

Time (yr)	Account Balance (\$)
0	200
1	230
2	264.50

Simple

Compound

**Problems 5–7:** Jin invests \$4,000 in an account that earns 5% compound interest per year.

5. Complete the table.

6. Which function represents the amount of money in Jin's account after  $x$  years?

A.  $f(x) = 4000 + 1.05x$

B.  $f(x) = 4000(1.05)^x$

C.  $f(x) = 4000(0.05)^x$

D.  $f(x) = 4000 + (1.05)^x$

Time (yr)	Account Balance (\$)
0	
1	4,200
2	4,410
3	
4	

7. What will the balance of the account be after 10 years?

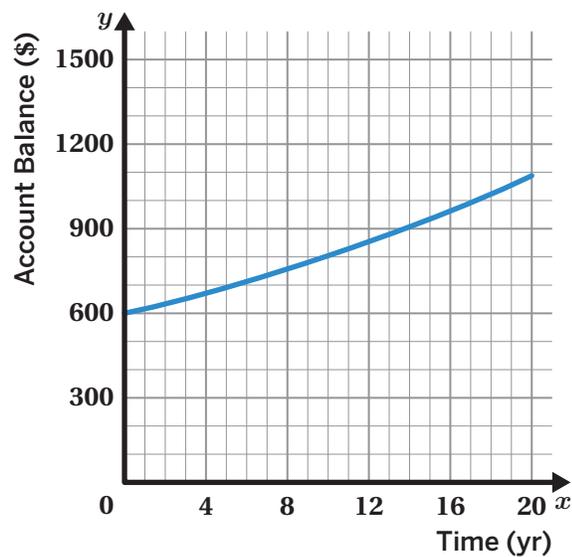
# Lesson Practice

A1.6.13

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

**Problems 8–9:** Keya invests \$600 in an account that earns 3% compound interest per year. The graph shows the function  $f(t) = 600(1.03)^t$ , which gives Keya's account balance after  $t$  years.

- 8. About how many years will it take for her account balance to reach \$1,000?
- 9. Use the graph to determine the value of  $f(14)$ .



What does that tell you about the situation?

- 10. You just won a contest and have two prize options.
  - **Option A:** One payment of \$20 million
  - **Option B:** 2 cents on day one, 4 cents on day two, 8 cents on day three, and so on, for 30 days

Which option would you choose? Explain your choice.

## Spiral Review

**Problems 11–13:** Determine whether each function is linear, exponential, or something else. Circle your choice.

- |                      |        |             |                |
|----------------------|--------|-------------|----------------|
| 11. $f(x) = x^2 + 5$ | Linear | Exponential | Something else |
| 12. $g(x) = 2x + 5$  | Linear | Exponential | Something else |
| 13. $h(x) = 2^x + 5$ | Linear | Exponential | Something else |

## Reflection

- 1. Put a star next to a problem you want to understand better.
- 2. Use this space to ask a question or share something you're proud of.

**Lesson Summary**

You can write exponential expressions representing compound interest in multiple equivalent ways to reveal different information about the account and situation.

Here is an example.

The amount owed on a \$400 loan has a monthly interest rate of 3%. Let  $t$  represent the number of years since taking out the loan if no payments are made.

- The expression  $400 \cdot 1.03^{12t}$  represents a 3% monthly interest rate 12 times each year,  $t$ .
- The expression  $400 \cdot (1.03^{12})^t$  uses the powers of powers property to help us think about the interest rate for every  $t$  year.
- Since  $1.03^{12} = 1.4258$ , the expression  $400 \cdot (1.4258)^t$  shows the annual interest rate of 42.58%.

Each expression reveals different information about the monthly and annual interest rates applied to the account. While a monthly interest rate of 3% may not seem like it impacts the account balance much, the annual interest rate reveals that the loan amount is increasing by 42.58% each year, and that really adds up!

---

**Things to Remember:**

# Lesson Practice

## A1.6.14

Name: ..... Date: ..... Period: .....

**Problems 1–2:** Alina takes out a \$1,000 loan with a monthly interest rate of 5%. She makes no additional payments, deposits, or withdrawals.

1. Select *all* the expressions that can be used to calculate her balance after  $t$  years.

- A.  $1000 \cdot 1.05^t$
- B.  $1000 \cdot 1.05^{12t}$
- C.  $1000(1.05^{12})^t$
- D.  $1000 \cdot 1.7959^t$
- E.  $1000(1.7959)$

2. What is the interest rate per year for this loan?

**Problems 3–7:** Alejandro invests money into a college savings account. He writes the expression  $750(1.006^{12})^3$  to help him calculate what the account balance will be in 3 years.

3. Explain what 750 represents in the expression.

4. Explain what 1.006 represents in the expression.

5. Explain what 12 represents in the expression.

6. Explain what 3 represents in the expression.

7. Write an equivalent expression that could represent Alejandro's account balance in 3 years.

# Lesson Practice

## A1.6.14

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

**Problems 8–9:** Rebecca is considering taking out a payday loan that has a 17% monthly interest rate.

Monthly Interest Rate	17%
Monthly Growth Factor	
Growth Factor per Year	
Interest Rate per Year	

8. Complete the table.

9. If Rebecca takes out a \$300 payday loan, how much would she owe after 2 years if she made no additional payments?

10. Wohali takes out an \$8,000 federal student loan with a monthly interest rate of 0.38% for professional trade school. Write a function,  $g(t)$ , to calculate the amount Wohali owes after making no payments for  $t$  years.

### Spiral Review

**Problems 11–13:** Determine the value of each function when  $n = 2$ .

11.  $f(n) = 4 \cdot 2^n$

12.  $g(n) = 2 \cdot 4^n$

13.  $h(n) = 8 + 2^n$

14. Using the digits 0 to 9, without repeating, fill in each blank to create four equivalent expressions.

$$7^{\square} = 7^{\square} \times 7^{\square} = 7^{\square} \times 7^{\square} \times 7^{\square} = (7^{\square})^{\square}$$

### Reflection

1. Put a star next to a problem that looked more difficult than it really was.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

When you take out a loan on a credit card, the annual interest rate may be compounded at different *intervals*, or different lengths of time.

You can use the formula  $P\left(1 + \frac{r}{n}\right)^{nt}$  to calculate the total amount in an account that accrues compound interest.

- $P$  represents the initial amount of the loan. In finance this is often called the *principal*.
- $r$  represents the interest rate in decimal form.
- $n$  represents the number of compounding intervals in a year.
- $t$  represents the time in years.

Common compounding periods:

Annually	Semi-annually	Quarterly	Monthly	Daily
$n = 1$	$n = 2$	$n = 4$	$n = 12$	$n = 365$

Let's look at the impact of compound interest applied at different intervals on a loan for \$1,000.

Interest	Owed in	Compounded Monthly	Compounded Quarterly	Compounded Annually
15% annually	5 years	$1000\left(1 + \frac{0.15}{12}\right)^{12 \cdot 5}$ $\approx \$2,107.18$	$1000\left(1 + \frac{0.15}{4}\right)^{4 \cdot 5}$ $\approx \$2,088.15$	$1000(1 + 0.15)^5$ $\approx \$2,011.36$

Things to Remember:

# Lesson Practice

## A1.6.15

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

1. Tyrone puts \$2,500 into a savings account with a 1.2% annual interest rate, compounded semi-annually. He makes no additional payments, deposits, or withdrawals.

Select *all* the expressions that can be used to calculate his balance after 3 years.

- A.  $2500\left(1 + \frac{0.012}{2}\right)^{3 \cdot 2}$                        B.  $2500\left(1 + \frac{0.012}{6}\right)^6$
- C.  $2500(1 + 0.012)^3$                        D.  $2500(1 + 0.006)^6$
- E.  $2500\left(1 + \frac{0.012}{3}\right)^{3 \cdot 2}$

**Problems 2–4:** Maneli wants to take out a \$5,000 loan to help pay for a new washing machine and dryer. The bank offers her the loan with an 18% annual interest rate, compounded quarterly.

Maneli wrote this expression to calculate the balance of the loan in 2 years, but she made an error.

$$5000\left(1 + \frac{0.18}{2}\right)^{(4 \cdot 2)}$$

2. Find the error and explain why it is incorrect.
3. Write a correct expression to represent Maneli's balance after 2 years.
4. What will her balance be in 2 years?

**Problems 5–6:** A payday loan company offers a \$1,000 loan with a 25% annual interest rate.

5. If no other charges or payments are made, what will the balance of the loan be after 1 year at each compounding period?

Compounding Period	Balance (\$)
Annually	
Monthly	
Daily	

6. Describe how changing the compounding period affects the balance of the loan.

# Lesson Practice

## A1.6.15

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

**Problems 7–8:** Xavier has \$5,000 to invest and has to choose between three investment options.

- Option A: 2.25% interest applied each quarter
- Option B: 3% interest applied every 4 months
- Option C: 4.5% interest applied twice each year

7. Write an expression for each account to represent Xavier's balance after 5 years.

8. Which option will have the largest balance after 5 years?

### Spiral Review

**Problems 9–10:** Irene needs to make at least 25 dinners for a party, including chicken dinners and vegetarian dinners. She has \$250 to spend. Chicken dinners cost \$8.75 each and vegetarian dinners cost \$5.50 each.

- $c$  represents the number of chicken dinners.
- $v$  represents the number of vegetarian dinners.

9. Write a system of inequalities that represents Irene's constraints.

10. Can Irene make 5 chicken dinners and 20 vegetarian dinners? Explain your thinking.

### Reflection

1. Put a heart next to a problem you understand well.
2. Use this space to ask a question or share something you're proud of.

**Lesson Summary**

Exponential and linear functions are often used to model the population growth of a city or country. *Models* can help us predict unknown data values, including future values. While some models can be useful, they also have limitations.

- Exponential functions increase toward infinity, but populations are limited by space, time, and available land.
- Linear models may generate a negative  $y$ -intercept, but populations do not have less than 0 people.
- Models may only be useful for predicting unknown values within a specific *domain*.

Let's look at an example. Here is data about the population of a city since 1880.

An exponential model is a better fit than a linear model. The data fits the shape of a curve more closely than it follows a line.

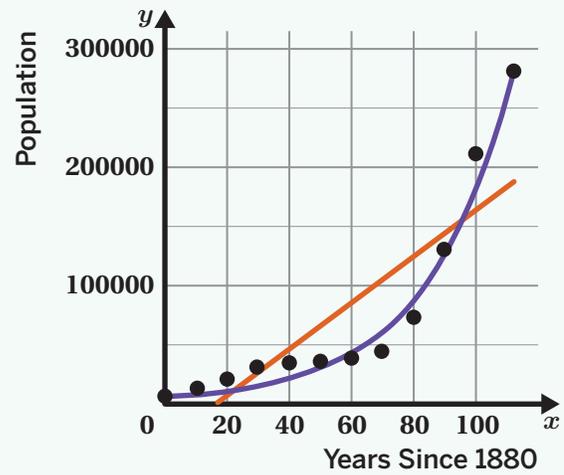
You can use the exponential function  $p(t) = 3992(1.0397)^x$  to model the number of people in 1945.

Since 1945 is 65 years after 1880, substitute 65 into  $p(t)$  to estimate the unknown value.

$$p(65) = 3992(1.0397)^{65}$$

$$p(65) = 50143$$

The model estimates that there were 50,143 people in 1945.



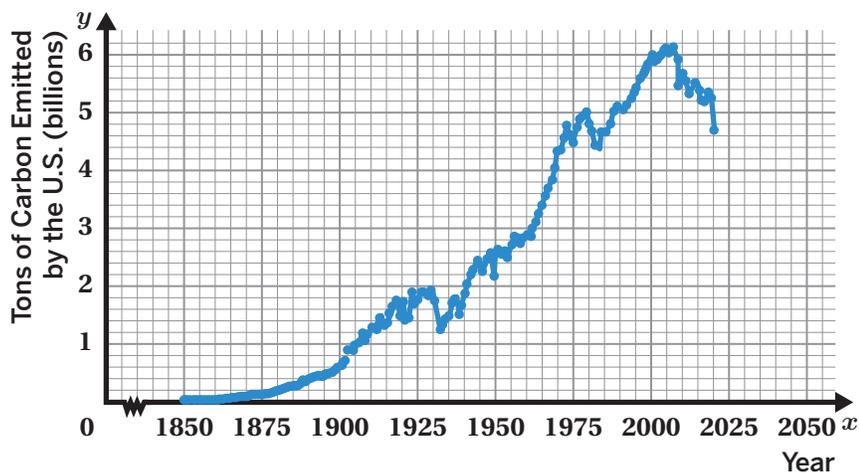
**Things to Remember:**

# Lesson Practice

A1.6.16

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

**Problems 1–5:** The graph shows the number of tons of carbon, in billions, that the United States emitted from 1850 to 2020.



Source: Our World in Data

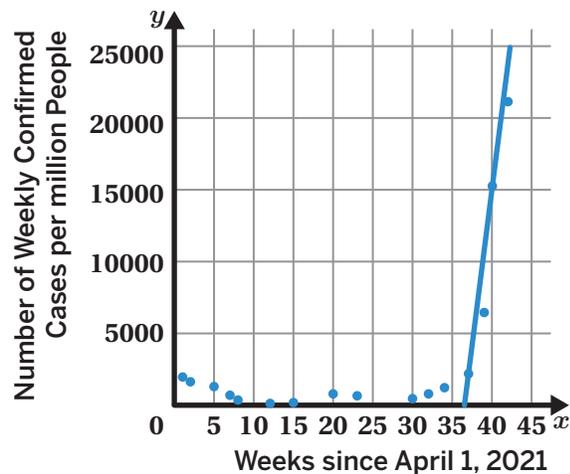
1. How would you describe the data in this graph?
2. What change occurs in the data around 2005? What do you think may have caused that change?
3. Sketch a line or exponential curve of best fit to model the data from 1850 to 2005.
4. Sketch a line or exponential curve of best fit to model the data after 2005.
5. Use your model to predict how many tons, in billions, the United States will emit in 2050.

# Lesson Practice

A1.6.16

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

**Problems 6–8:** The graph shows the number of weekly confirmed cases of COVID-19 per million people in Italy. Emma modeled the given data with a linear function.



Source: Our World in Data

- Describe an advantage of using Emma's model.
- Describe a disadvantage of using Emma's model.
- Provide a suggestion for Emma to improve her model.

## Spiral Review

- The line of best fit  $y = 8.23x - 1.84$  was calculated for a data set. Which value could be the  $r$ -value of the data?
  - $r = 0.72$
  - $r = -0.72$
  - Both are possible.
  - Neither is possible.

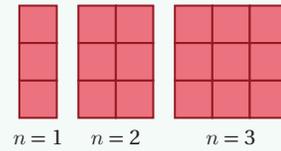
## Reflection

- Put a star next to the problem you understood best.
- Use this space to ask a question or share something you're proud of.

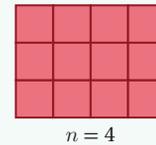
## Lesson Summary

You can investigate visual patterns to determine how to build a particular figure or write an expression to represent the pattern.

Here is an example of the first three steps of a visual pattern.



You can describe what changes and what stays the same to help you draw the next figure, where  $n = 4$ .



Describing how you see a visual pattern changing can help you write a rule or expression to determine the other values in the pattern.

Here are two ways you may determine value of the tenth step of this pattern, or how many tiles are in  $n = 10$ .

- If you see the pattern increasing by a column of 3 each step, you could add 3 more until you reach  $n = 10$ .
- If you see each step in the pattern as a rectangle that has the dimensions 3-by- $n$ , you could multiply 3 by 10 to find the total number of tiles.
- In both cases, you would determine that there will be 30 tiles when  $n = 10$ .

$n$	Number of Tiles
1	3
2	6
3	9
...	...
10	30

Arrows on the right side of the table indicate an increase of +3 tiles from one step to the next. A curved arrow at the bottom points from the value 10 in the 'n' column to the value 30 in the 'Number of Tiles' column, labeled 'x3'.

There are many ways to see and describe visual patterns accurately.

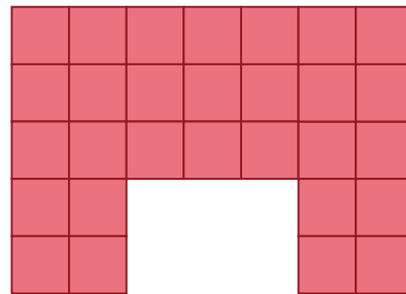
## Things to Remember:

# Lesson Practice

A1.7.01

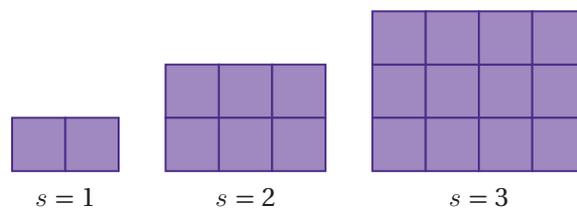
Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

1. Select *all* of the expressions that could represent the number of tiles in this diagram.



- A.  $5 \cdot 7$
- B.  $5 \cdot 7 - 6$
- C.  $3 \cdot 7 + 2 \cdot 2$
- D.  $5 \cdot 7 - 2 \cdot 3$
- E.  $2 \cdot 5 + 3 \cdot 3 + 2 \cdot 5$

2. Here are the first three steps in a pattern. How many tiles will there be when  $s = 10$ ?



Explain your thinking.

3. What type of relationship does the pattern in the table represent? Circle one.

Linear      Exponential      Neither

$s$	Number of Tiles
1	3
2	9
3	27
4	81

Explain your thinking.

# Lesson Practice

A1.7.01

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

**Problems 4–6:** A teacher gives her class a table with only the first two rows in the pattern.

$s$	Number of Tiles
1	5
2	25

4. Rishi says the pattern is an exponential relationship.

Ichiro says there is not enough information to be sure.

Whose thinking is correct? Explain your thinking.

5. How many tiles would be in the next step if the relationship were *linear*?

6. How many tiles would be in the next step if the relationship were *exponential*?

## Spiral Review

7. Juana began hiking at 6:00 AM. At noon, she had hiked 12 miles. At 4:00 PM, Juana finished her hike with a total distance of 26 miles.

On average, during which time interval was Juana hiking faster? Circle one.

6:00 AM to noon

Noon to 4:00 PM

Explain your thinking.

**Problems 8–10:** Use the distributive property to write an equivalent expression for each.

8.  $3(n + 5)$

9.  $16x + 20$

10.  $2(4m - 6)$

## Reflection

1. Circle the problem you feel least confident about.

2. Use this space to ask a question or share something you're proud of.

**Lesson Summary**

You can determine if a visual pattern represents a linear, exponential, or **quadratic relationship**, or something else.

If you can observe a square that is changing throughout a pattern, the relationship might be quadratic and you can use a squared term to write a quadratic expression.

Here are some strategies you might use to write an expression to represent a visual pattern:

You can look for what is changing and staying the same.

- The 3 outside tiles stay the same.
- The interior square is growing from 1 to 4 to 9.

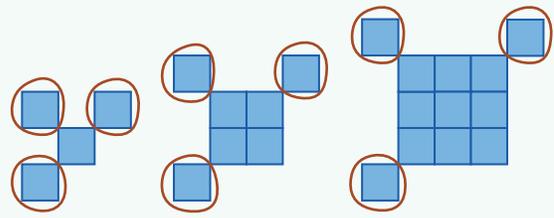


Figure 1      Figure 2      Figure 3

You can look for where you see the figure number,  $n$ , in each diagram.

- You can see the figure number as the side length of the growing square.
- You can write each number of tiles as an expression.

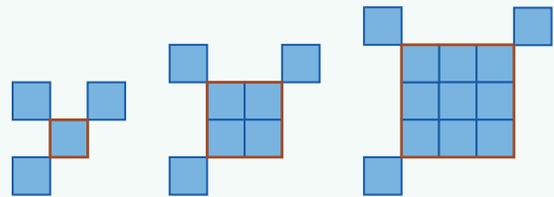


Figure 1      Figure 2      Figure 3  
 $1^2 + 3$        $2^2 + 3$        $3^2 + 3$

You can write a quadratic expression to represent the number of tiles in Figure  $n$  as  $n^2 + 3$ .

**Things to Remember:**

# Lesson Practice

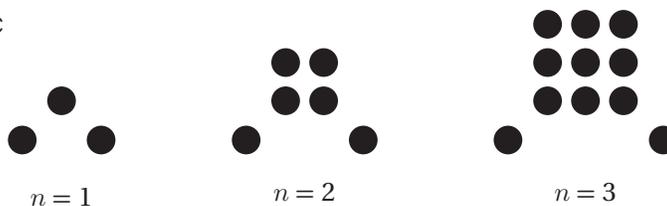
A1.7.02

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

**Problems 1–4:** Circle whether each expression represents a quadratic relationship, linear relationship, or neither.

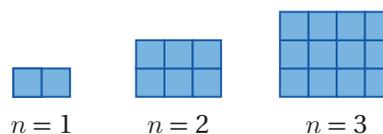
- |                |           |        |         |
|----------------|-----------|--------|---------|
| 1. $2^n + n$   | Quadratic | Linear | Neither |
| 2. $2n^2 + 2$  | Quadratic | Linear | Neither |
| 3. $2(n - 5)$  | Quadratic | Linear | Neither |
| 4. $2n(n + 3)$ | Quadratic | Linear | Neither |

5. Does this pattern show a quadratic relationship?



Explain your thinking.

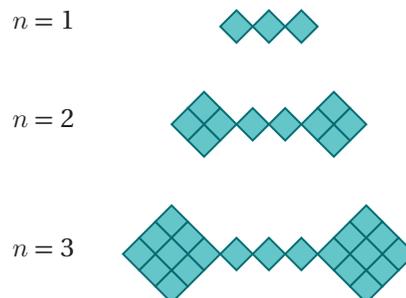
6. Karima says that she sees a square plus one more row in each pattern.



Circle *all* the expressions Karima could use to represent the number of tiles in this pattern.

$n^2 + 1$                        $n^2 + n$                        $n(n + 1)$

7. Write an expression to represent the relationship between the figure number,  $n$ , and the total number of tiles.



# Lesson Practice

A1.7.02

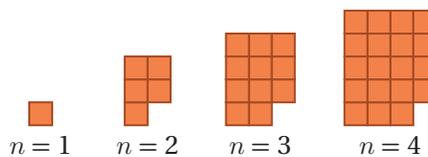
Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

8. Three students each wrote different, but correct, expressions to represent this pattern.

**Ethan**  
 $n^2 + (n - 1)$

**Ama**  
 $n(n + 1) - 1$

**Nasir**  
 $n^2 + n - 1$



Choose a student and describe how they see the expression in the pattern.

## Spiral Review

9. Select all the expressions that are equivalent to  $6m + 3q$ .

A.  $4m + 2m + 5q - 2q$

B.  $3(2m + q)$

C.  $(6 + 3)(m + q)$

D.  $3q + 2m + 3q + m$

E.  $q + 15m + 2q - 9m$

10. Complete the table for the function  $h(x) = 5(2)^x$ .

$x$	-2	-1	0	1	2
$h(x)$					

## Reflection

- Put a smiley face next to a problem you were stuck on and then figured out.
- Use this space to ask a question or share something you're proud of.

Lesson Summary

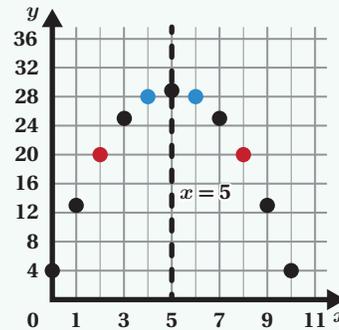
The graph of a quadratic function is called a **parabola**. Parabolas have a **line of symmetry** that goes through the maximum or minimum point. If you fold a parabola along this line, you get two identical halves. Here is a table and graph that represent a quadratic relationship.

Table

$x$	$y$
2	20
4	28
5	29
6	28
8	20

You can see the points are symmetrical across the line of symmetry at  $x = 5$ .

Graph



You can see the points create the shape of a parabola and are symmetrical across the line of symmetry at  $x = 5$ .

Things to Remember:

# Lesson Practice

A1.7.04

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

**Problems 1–3:** For each pair of symmetrical points on a parabola, determine the equation for the line of symmetry.

1. (0, 0) and (13, 0)

$x = \dots\dots\dots$

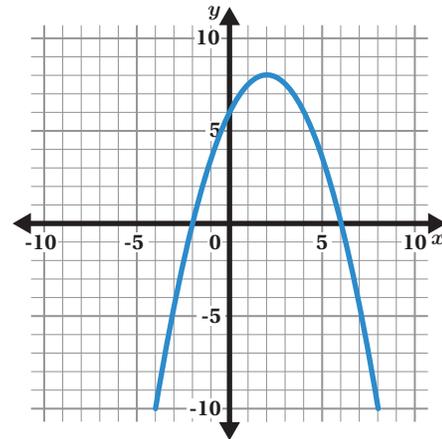
2. (4, 2) and (20, 2)

$x = \dots\dots\dots$

3. (7, 10) and (28, 10)

$x = \dots\dots\dots$

**Problems 4–5:** Here is a graph of a parabola.



4. Draw the *line of symmetry* where you think it is located on this parabola.

5. Write the equation for the line of symmetry.

$x = \dots\dots\dots$

**Problems 6–8:** Here are a few points that belong to a function  $g(x)$ .

6. Does  $g(x)$  represent a quadratic relationship? Circle your response and explain your thinking.

Yes

No

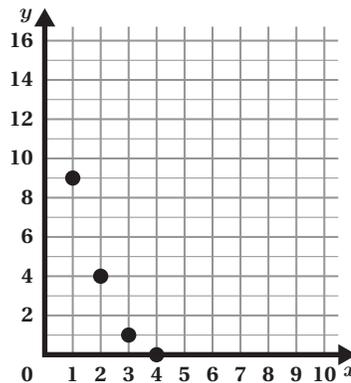
Not enough information

$x$	$g(x)$
0	
1	9
2	4
3	1
4	0
5	
6	
7	
8	

7. Complete the table for  $g(x)$  and plot the new points on the graph.

8. Write the equation for the line of symmetry.

$x = \dots\dots\dots$



# Lesson Practice

A1.7.04

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

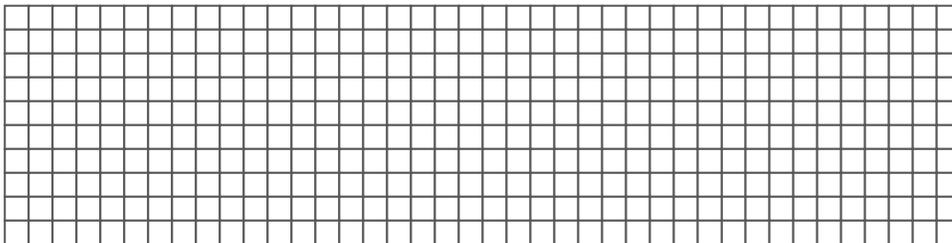
**Problems 9–10:** Here is an incomplete table that could represent several types of functions.

Figure	Number of Tiles
1	1
2	
3	9

9. Select a function type and determine the number of tiles that would be in Figure 2. Circle one.

Linear                  Quadratic                  Exponential

10. Draw three figures to match the pattern in the table.

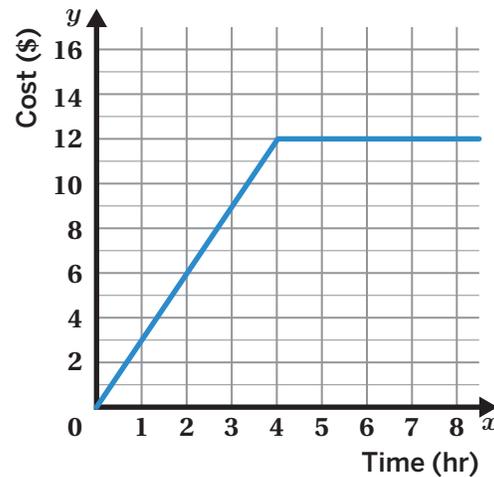


## Spiral Review

**Problems 11–12:** The graph represents the relationship between the amount of time a car is parked, in hours, and the cost of parking, in dollars.

11. Is the relationship a function?

12. Describe the relationship between the amount of time a car is parked and the cost of parking.



## Reflection

- Put a star next to a problem you're still wondering about.
- Use this space to ask a question or share something you're proud of.

Lesson Summary

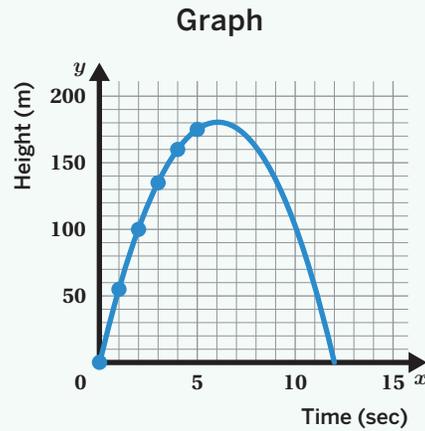
You can use tables and graphs to make predictions about quadratic relationships in context. This table and graph show the height of a stomp rocket at various times.

**Table**

Time (sec)	Height (m)
0	0
1	55
2	100
3	135
4	160
5	175

$+55$   
 $+45$   
 $+35$   
 $+25$   
 $+15$

$-10$   
 $-10$   
 $-10$   
 $-10$



You can extend the pattern of the table using the second difference.

From the table, you can see how the rocket is 160 meters high after 4 seconds and 175 meters high after 5 seconds.

You can use the graph to determine the maximum height of the rocket by looking for the highest point on the parabola.

The maximum height of the rocket is 180 meters at 6 seconds.

You can also see from the graph that it takes 12 seconds for the rocket to land.

Things to Remember:

# Lesson Practice

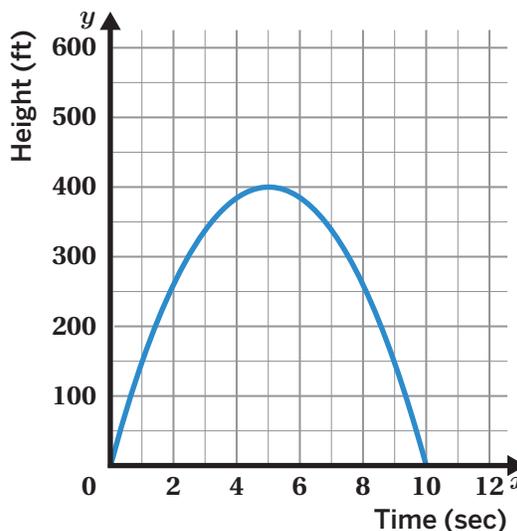
A1.7.05

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

**Problems 1–3:** This table and graph show the height of a stomp rocket at various times.

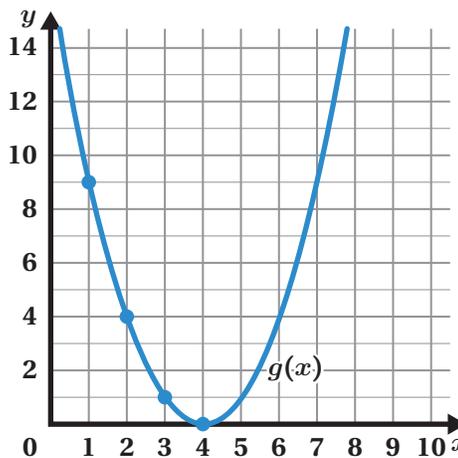
Time (sec)	0	1	2	3
Height (ft)	0	144	256	336

1. What is the maximum height that the stomp rocket reached?
2. How long did it take for the stomp rocket to land?
3. How high was the stomp rocket after 4 seconds?



4. Here is the graph of the function  $g(x)$ . Write the equation for the line of symmetry of  $g(x)$ .

$x = \dots\dots\dots$



**Problems 5–6:** Oliver jumps off a diving board into a swimming pool. The relationship between his height and time can be modeled by a quadratic relationship.

Time (sec)	0	0.2	0.4	0.6	0.8	1	1.2	1.4
Height (m)	3	4.8	6.2	7.2				

5. Complete the table.
6. After how many seconds will Oliver reach his maximum height?

# Lesson Practice

A1.7.05

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

7. The table shows the height of a stomp rocket from the time it is launched. How many seconds will it take for the rocket to land? Circle one.

- 8 seconds    9 seconds    Between 8 and 9 seconds    Between 9 and 10 seconds

Time (sec)	Height (m)
0	0
1	42
2	74
3	96

Explain your thinking.

## Spiral Review

**Problems 8–10:** Mohamed can run a mile in 9 minutes. The number of miles Mohamed runs is a function of the amount of time he has been running.

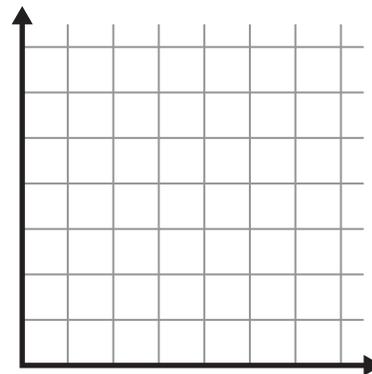
8. Graph this function. Be sure to label the axes.

9. Describe the domain of this function.

Domain:

10. Describe the range of this function.

Range:



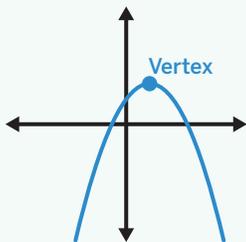
## Reflection

1. Put a star next to a problem you could explain to a classmate.
2. Use this space to ask a question or share something you're proud of.

## Lesson Summary

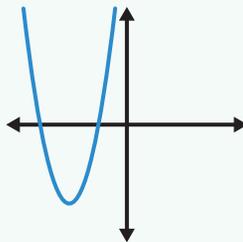
You can describe key features of *parabolas* with terms like: vertex, concave up, or concave down.

Vertex



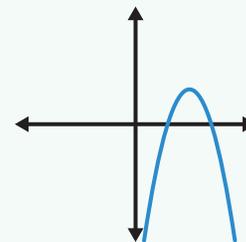
The vertex is the *maximum* or *minimum* point on a parabola (where a parabola changes from increasing to decreasing, or vice versa).

Concave Up



A parabola that opens upward is concave up.

Concave Down



A parabola that opens downward is concave down.

## Things to Remember:

# Lesson Practice

A1.7.06

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

1. The key features of this parabola are labeled *A*, *B*, *C*, and *D*.

Match each key feature with a term from the word bank.

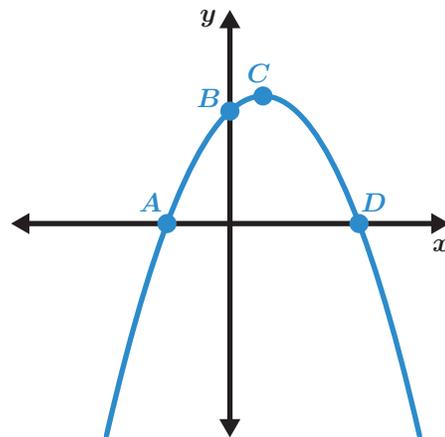
vertex    *x*-intercept    *y*-intercept

*A*: .....

*B*: .....

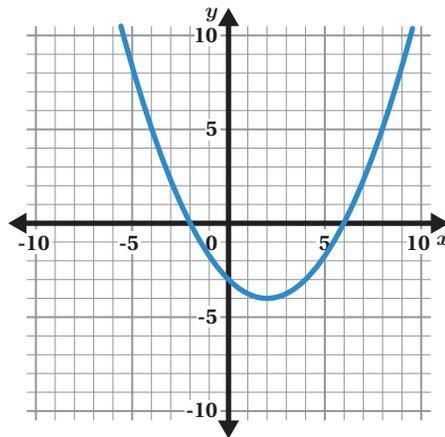
*C*: .....

*D*: .....



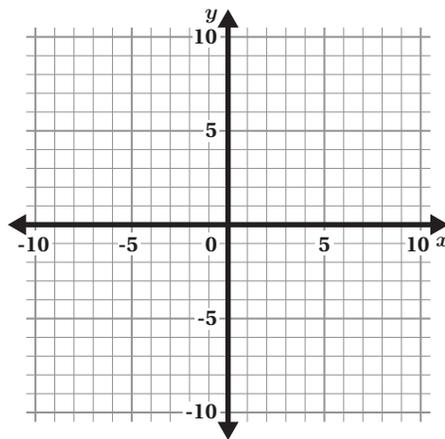
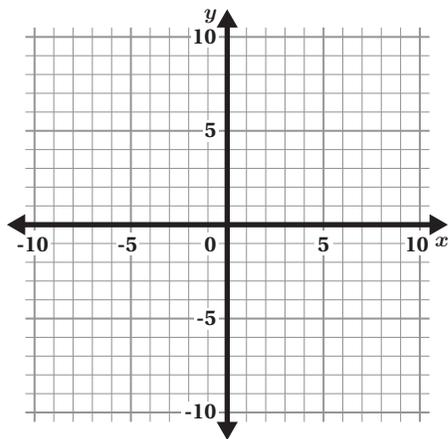
**Problems 2–4:** Use the graph to determine the coordinates of each key feature.

2. vertex:
3. *x*-intercept(s):
4. *y*-intercept:



**Problems 5–6:** Graph a parabola that fits each description.

5. Concave down with a positive *y*-intercept
6. Concave up and vertex at (5, -5)



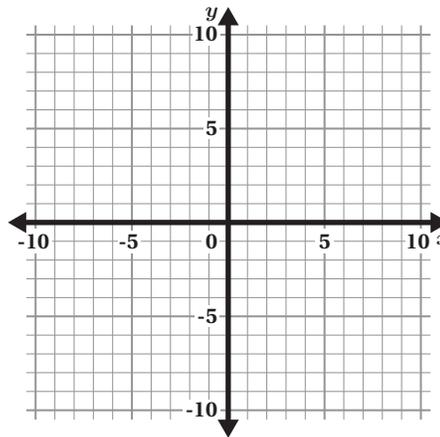
# Lesson Practice

A1.7.06

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

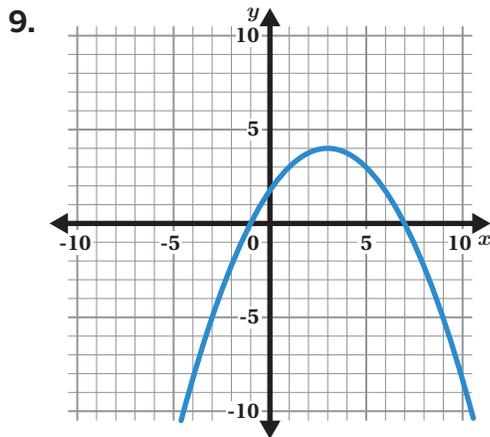
7. A parabola has a vertex at  $(4, 2)$ . Give two possible coordinates for its  $x$ -intercepts.

8. Using only parabolas, make a design that:
- Looks the same when reflected over the  $x$ -axis.
  - Looks the same when reflected over the  $y$ -axis.

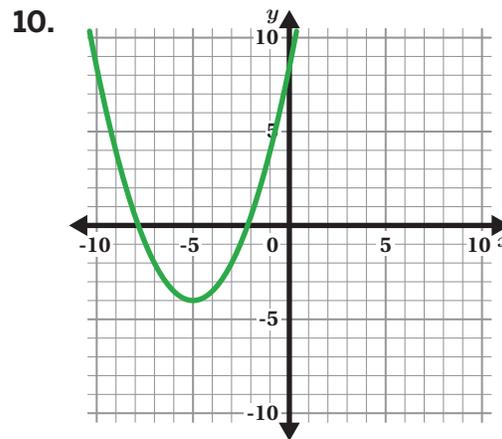


## Spiral Review

Problems 9–10: Write an equation for the line of symmetry of each parabola.



Equation: .....



Equation: .....

## Reflection

1. Put a question mark next to a response you'd like to compare with a classmate's.
2. Use this space to ask a question or share something you're proud of.

**Lesson Summary**

Different forms of quadratic equations help us see different key features of a parabola.

In a standard form quadratic equation, the *y-intercept* is the constant term in the equation.

In a factored form quadratic equation, the *x-intercepts* are the values that make either factor equal to 0.

Here is an example of the same function written in standard and factored form. We can use the different forms to determine key features and graph the parabola.

**Standard Form**

$$f(x) = 2x^2 - 4x - 6$$

$$f(x) = 2x^2 - 4x - 6$$

$$f(0) = 2(0)^2 - 4(0) - 6$$

The *y*-intercept is (0, -6).

**Factored Form**

$$f(x) = (2x + 2)(x - 3)$$

<i>x</i>	$(2x + 2)$	$(x - 3)$	$(2x + 2)(x - 3)$
-1	$2(-1) + 2 = 0$	$(-1) - 3 = -4$	$(0)(-4) = 0$
3	$2(3) + 2 = 8$	$(3) - 3 = 0$	$(8)(0) = 0$

The *x*-intercepts are (-1, 0) and (3, 0).

**Things to Remember:**

# Lesson Practice

A1.7.10

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

**Problems 1–2:** Determine an  $x$ -intercept(s) from each equation.

1.  $a(x) = (x - 2)(x + 1)$

2.  $b(x) = (x + 3)^2$

3. Here is a graph of a quadratic function.

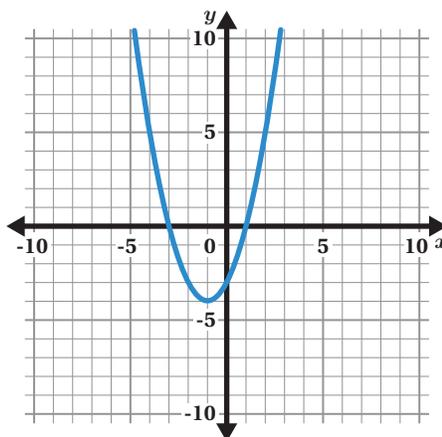
Which function could this be?

A.  $y = (x - 3)(x + 1)$

B.  $y = (x + 3)(x - 1)$

C.  $y = (x - 3)(x - 1)$

D.  $y = (x + 3)(x + 1)$



4. Here is the same function written in two forms.

**Factored form:**  $g(x) = (x + 5)(x - 2)$       **Standard form:**  $g(x) = x^2 + 3x - 10$

What are the  $x$ - and  $y$ -intercepts of the function?

$x$ -intercept: .....

$x$ -intercept: .....

$y$ -intercept: .....

5. Crow and Ariel were working on a problem together.

Crow said: *The  $y$ -intercept of  $y = (x - 3)^2$  is  $(0, 3)$ .*

Ariel said: *The  $x$ -intercept of  $y = (x - 3)^2$  is  $(3, 0)$ .*

Whose thinking is correct? Circle one.

Crow's

Ariel's

Explain your thinking

# Lesson Practice

A1.7.10

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

**Problems 6–7:** Here is a function:  $h(x) = x(x + 6)$ .

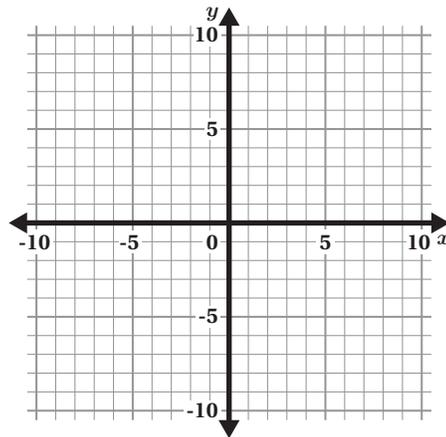
6. Determine the intercepts of  $h(x)$ .

$x$ -intercept: .....

$x$ -intercept: .....

$y$ -intercept: .....

7. Draw the graph of  $h(x)$ .



**Problems 8–10:** Use a graphing calculator to graph  $f(x) = (x + 3)(x + 1)(x - 2)$ . Change the numbers in the equation to determine an equation whose graph . . .

8. Has two  $x$ -intercepts. ....

9. Has one  $x$ -intercept. ....

10. Has an  $x$ -intercept at  $(7, 0)$ . ....

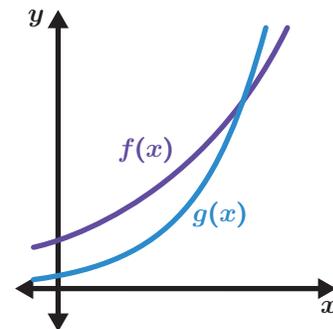
## Spiral Review

11. Here are the graphs of two functions,  $f(x)$  and  $g(x)$ .

$$f(x) = 100 \cdot 2^x$$

Which equation could represent  $g(x)$ ?

- A.  $g(x) = 25 \cdot 4^x$
- B.  $g(x) = 50 \cdot 1.5^x$
- C.  $g(x) = 100 \cdot 4^x$
- D.  $g(x) = 200 \cdot 1.5^x$



**Problems 12–13:** Complete each equation with a number that makes the equation true.

12.  $2 \cdot \dots + 4 = 0$

13.  $3(3 \cdot \dots + 4) = 0$

## Reflection

1. Star a problem you're still feeling confused about.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

We can use key features of a quadratic function to create graphs.

Here is an example. Determine the  $x$ -intercepts and the vertex of the function  $f(x) = (x + 5)(x - 1)$ .

The  $x$ -intercepts of the graph are the  $x$ -values that make each factor equal to 0.

$x$	$(x + 5)$	$(x - 1)$	$(x + 5)(x - 1)$
-5	$(-5) + 5 = 0$	$(-5) - 1 = -6$	$(0)(-6) = 0$
1	$(1) + 5 = 6$	$(1) - 1 = 0$	$(6)(0) = 0$

The  $x$ -intercepts are  $(-5, 0)$  and  $(1, 0)$ .

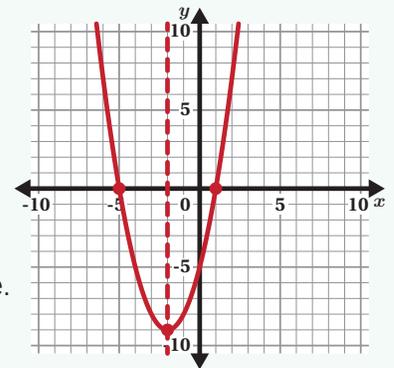
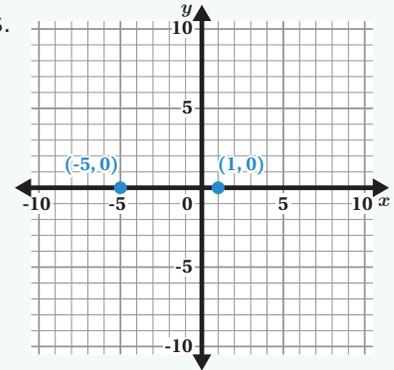
The vertex is always directly between the two  $x$ -intercepts.

The  $x$ -value of the vertex is  $-2$  because  $-2$  is exactly in the middle of the two  $x$ -intercepts.

We substitute  $x = -2$  into the equation to determine the  $y$ -value.

$x$	$(x + 5)$	$(x - 1)$	$(x + 5)(x - 1)$
-2	$(-2) + 5 = 3$	$(-2) - 1 = -3$	$(3)(-3) = -9$

The vertex is at the point  $(-2, 9)$ .



Things to Remember:

# Lesson Practice

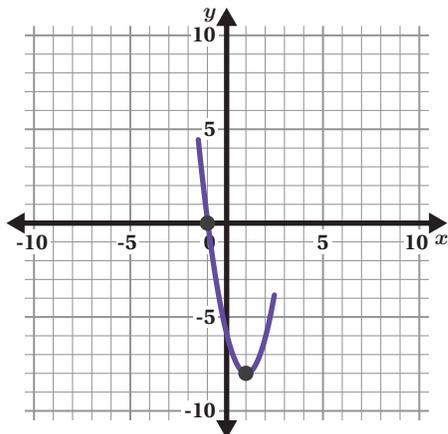
A1.7.11

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

**Problems 1–2:** Determine one point to zap that will light up the entire parabola.

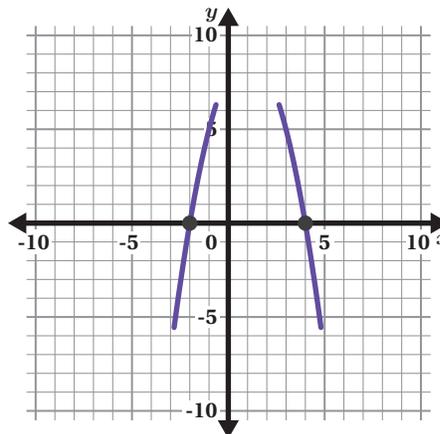
1. Equation:  $y = 2(x - 3)(x + 1)$

Point: \_\_\_\_\_



2. Equation:  $y = -1(x - 4)(x + 2)$

Point: \_\_\_\_\_



**Problems 3–5:** A parabola has  $x$ -intercepts at  $(3, 0)$  and  $(7, 0)$ . Determine whether each statement is true or false, or if there is not enough information.

- |   |      |       |                        |
|---|------|-------|------------------------|
| 3. The vertex of the parabola is at $(5, -4)$ . | True | False | Not enough information |
| 4. The line of symmetry is at $x = 5$ .         | True | False | Not enough information |
| 5. The parabola is concave up.                  | True | False | Not enough information |

**Problems 6–7:** Here is a function:  $g(x) = (-2x + 4)(x - 6)$ .

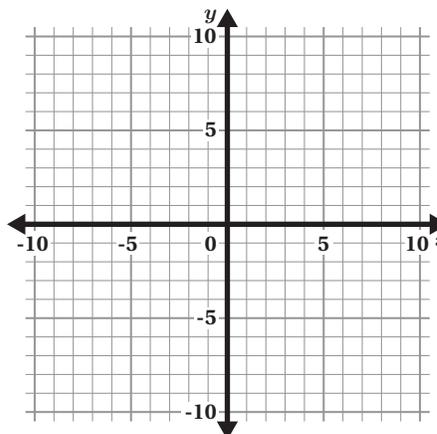
6. Determine the  $x$ -intercepts and vertex of  $g(x)$ .

$x$ -intercept: \_\_\_\_\_

$x$ -intercept: \_\_\_\_\_

Vertex: \_\_\_\_\_

7. Draw the graph of the function  $g(x)$ .



# Lesson Practice

A1.7.11

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

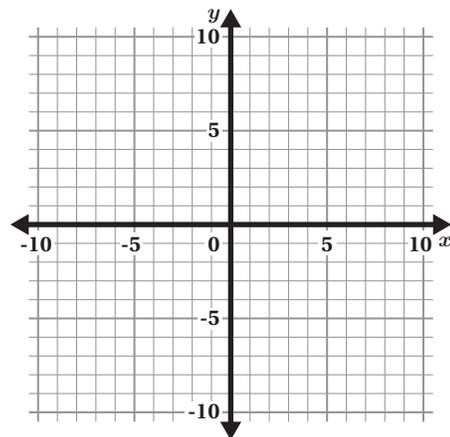
8. Zahra and Santino were graphing  $p(x) = (x + 3)^2$ .  
Zahra says: *This graph doesn't have a vertex because there's only one  $x$ -intercept.*  
Santino says: *The vertex is the same as the  $x$ -intercept.*

Whose thinking is correct? Circle one.

Zahra's                  Santino's                  Both                  Neither

Explain your thinking.

9. Using only parabolas, write the equations of your initials. Use the graph and/or graphing technology to help with your thinking.



## Spiral Review

**Problems 10–11:** Determine the  $x$ - and  $y$ -intercepts for each equation.

10.  $y = 4x + 8$

$x$ -intercept: .....

$y$ -intercept: .....

11.  $2x - 3y = 9$

$x$ -intercept: .....

$y$ -intercept: .....

## Reflection

1. Put a smiley face next to the problem you learned from most.
2. Use this space to ask a question or share something you're proud of.

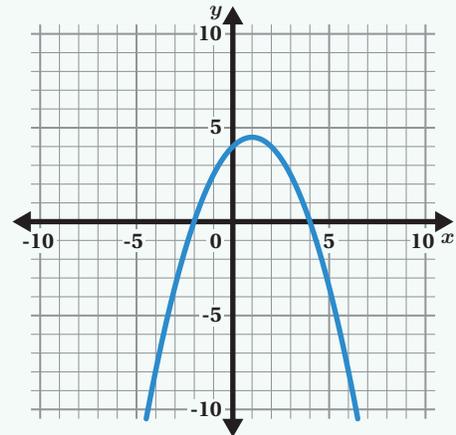
## Lesson Summary

We can write quadratic equations to match graphs and key features. One way to do this is in the form  $y = a(x - m)(x - n)$ .

- Use the  $x$ -intercepts to write the *factors* of the equation.
- If the parabola is concave down, make the  $a$ -value negative.
- Adjust the  $a$ -value to match the vertical position of the vertex and the  $y$ -intercept.

Here is an example strategy for writing an equation to match this graph.

- The  $x$ -intercepts are  $(-2, 0)$  and  $(4, 0)$ . You can write the factors as  $y = (x + 2)(x - 4)$ .
- Since the parabola is concave down, multiply by a negative number, like  $y = -(x + 2)(x - 4)$ .
- Adjust the  $a$ -value to make the vertical position of the vertex match the graph, like  $y = -\frac{1}{2}(x + 2)(x - 4)$ .



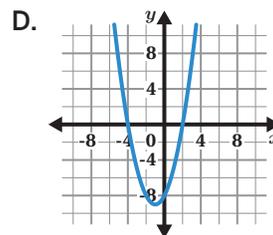
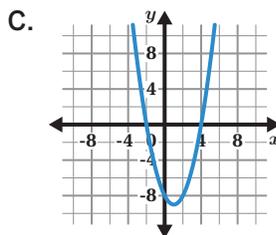
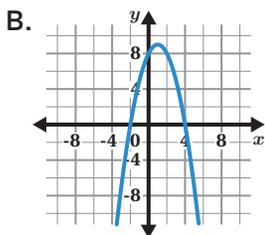
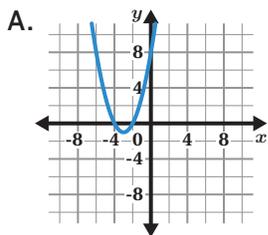
## Things to Remember:

# Lesson Practice

A1.7.12

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

1. Which graph shows the function  $y = (x - 4)(x + 2)$ ?



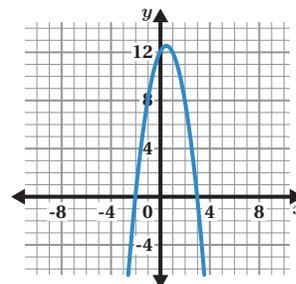
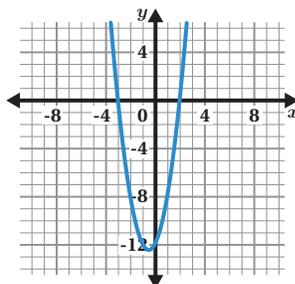
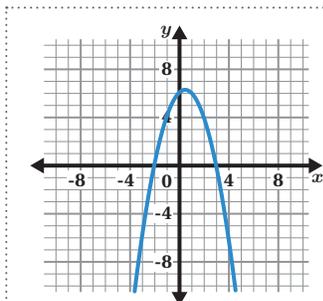
2. Match each equation to the graph it represents. One equation will have no match.

$$y = -(x + 2)(x - 3)$$

$$y = 2(x + 3)(x - 2)$$

$$y = -2(x + 2)(x - 3)$$

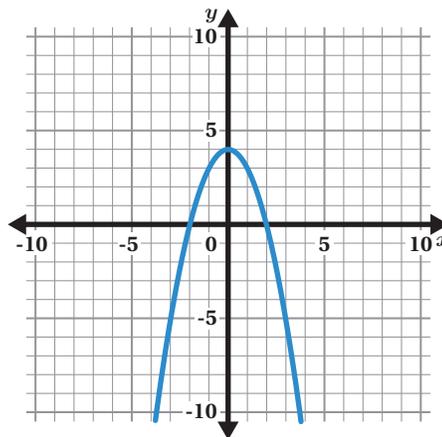
$$y = (x + 2)(x - 3)$$



**Problems 3–4:** Here is the graph of  $y = -1(x + 2)(x - 2)$ . Change the equation so the vertex goes through:

3.  $(0, 8)$

4.  $(0, -2)$



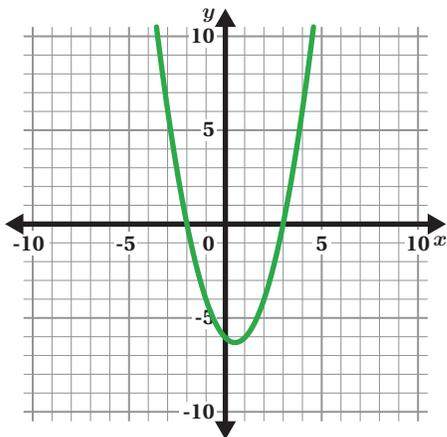
# Lesson Practice

A1.7.12

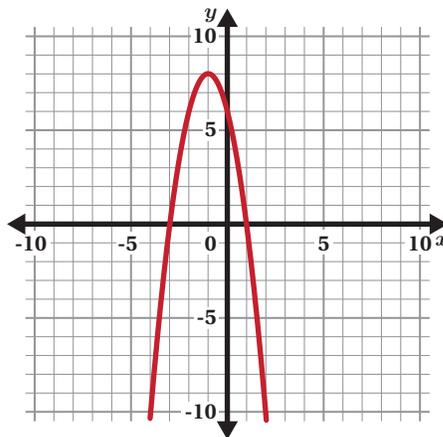
Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

**Problems 5–6:** Write an equation to match each graph.

5. Equation: \_\_\_\_\_



6. Equation: \_\_\_\_\_



7. Write the equations of three different parabolas that have the same vertex but different  $x$ -intercepts.

Equation 1	Equation 2	Equation 3

## Spiral Review

**Problems 8–9:** Here are two linear functions:  $f(x)$  and  $h(x)$ .

$$f(x) = 3x + 2$$

8. Which function has a greater  $y$ -intercept? Circle one.

$f(x)$

$h(x)$

9. Which function has a greater slope? Circle one.

$f(x)$

$h(x)$

$x$	$h(x)$
1	5
2	9
3	13

## Reflection

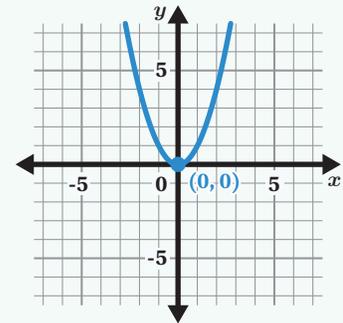
- Put a heart next to a problem you understand well.
- Use this space to ask a question or share something you're proud of.

Lesson Summary

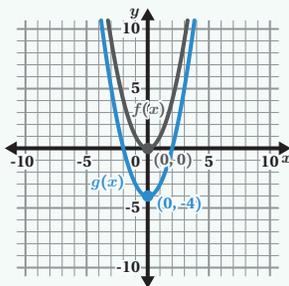
Here is the graph of  $f(x) = x^2$ .

Functions can be translated horizontally and vertically.

Here are three examples of *translations* of  $f(x)$ . The equations for these translations are written in **vertex form**, which highlights the coordinates of the vertex in the equation.



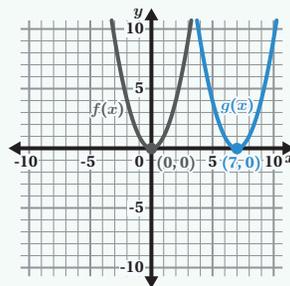
Vertical Translations



$f(x)$  is translated 4 units down.

Its equation is  $g(x) = x^2 - 4$ .

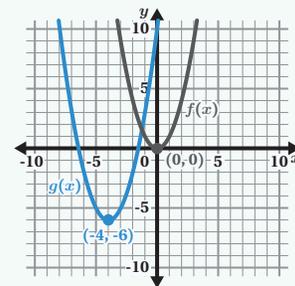
Horizontal Translations



$f(x)$  is translated 7 units right.

Its equation is  $h(x) = (x - 7)^2$ .

Vertical and Horizontal Translations



$f(x)$  is translated 4 units left and 6 units down.

Its equation is  $j(x) = (x + 4)^2 - 6$ .

Things to Remember:

# Lesson Practice

A1.7.14

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

**Problems 1–2:** Circle if the transformation of  $y = x^2$  is a *horizontal translation*, *vertical translation*, or *neither*.

1.  $y = (x - 3)^2$

Horizontal Translation    Vertical Translation    Neither

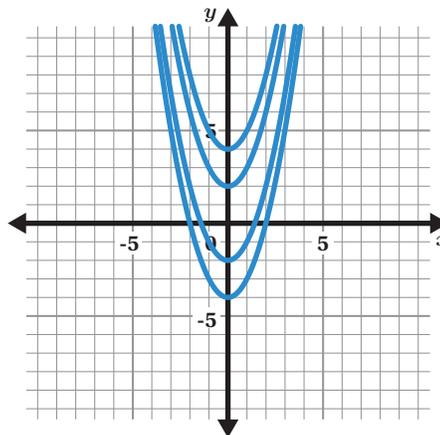
2.  $y = x^2 - 5$

Horizontal Translation    Vertical Translation    Neither

3. These parabolas are translations of  $y = x^2$ .

Select *all* of the equations shown in the graph.

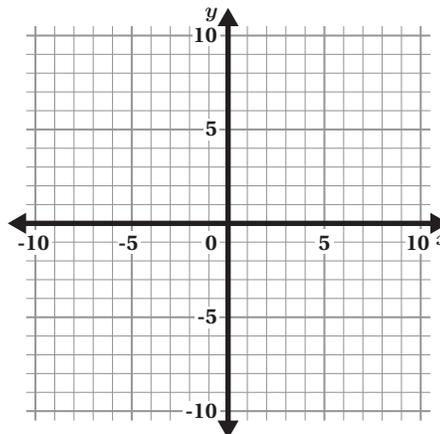
- A.  $y = x^2 + 2$
- B.  $y = (x - 2)^2$
- C.  $y = x^2 - 2$
- D.  $y = x^2 - 4$
- E.  $y = (x + 4)^2$



4. Match each vertex of a parabola to its equation.

- a.  $y = (x - 3)^2 + 5$  ..... (0, -4)
- b.  $y = (x + 7)^2 + 3$  ..... (-7, 3)
- c.  $y = (x - 4)^2$  ..... (-3, 5)
- d.  $y = (x + 3)^2 + 5$  ..... (3, 5)
- e.  $y = x^2 - 4$  ..... (4, 0)

5. Draw the graph of  $y = (x + 3)^2 + 5$ .



# Lesson Practice

A1.7.14

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

6. Here are four equations in vertex form. Which function has a graph with a vertex at (1, 3)?

A.  $y = (x + 1)^2 + 3$

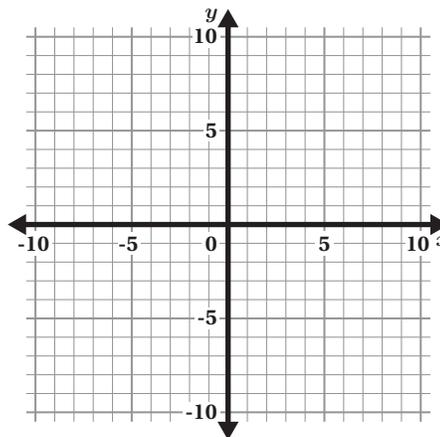
B.  $y = (x - 1)^2 + 3$

C.  $y = (x - 3)^2 + 1$

D.  $y = (x + 3)^2 + 1$

7. Write and graph the equations of five parabolas so that when you connect their vertices, they form a sixth parabola.

Equations



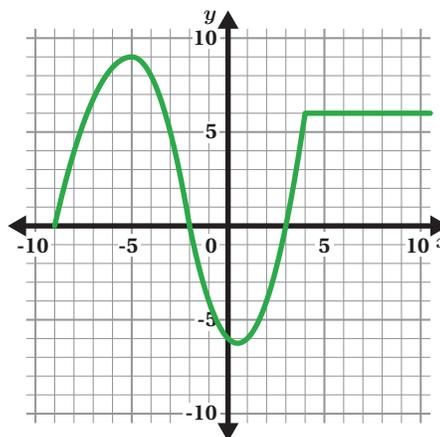
## Spiral Review

Problems 8–10: Here is a graph of  $g(x)$ .

8. At what point does the maximum occur?

9. At what points does  $g(x) = 0$ ?

10. What is the  $y$ -intercept?



## Reflection

- Put a star next to a problem you want to understand better.
- Use this space to ask a question or share something you're proud of.

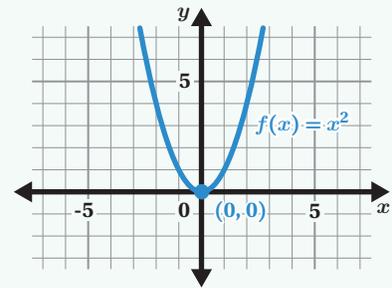
Lesson Summary

Quadratic functions like  $f(x) = a(x - h)^2 + k$  are written in vertex form, where  $(h, k)$  is the vertex of the parabola, and  $a$  shows the vertical scale.

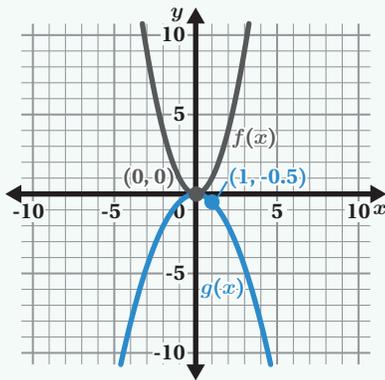
The  $a$ -value:

- Multiplies each output of the function by a constant value.
- Shows by what factor the graph is scaled in the  $y$ -direction.
- Tells whether a parabola is concave up or concave down.

Let's look at some examples of quadratic functions that have been scaled vertically:

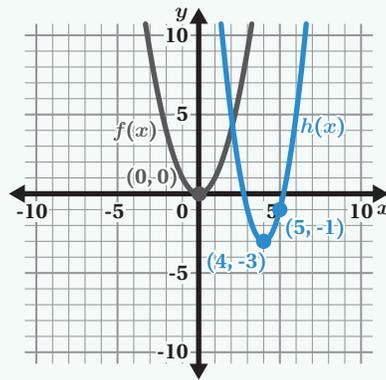


Vertical Scale



Here  $f(x)$  was scaled vertically by a factor of  $-\frac{1}{2}$ . Its equation is  $g(x) = -\frac{1}{2}x^2$ . This made the parabola wider and concave down.

Vertical Scale and Translations



Here  $f(x)$  was scaled vertically by a factor of 2 and translated right 4 units and down 3 units. Its equation is  $h(x) = 2(x - 4)^2 - 3$ .

Things to Remember:

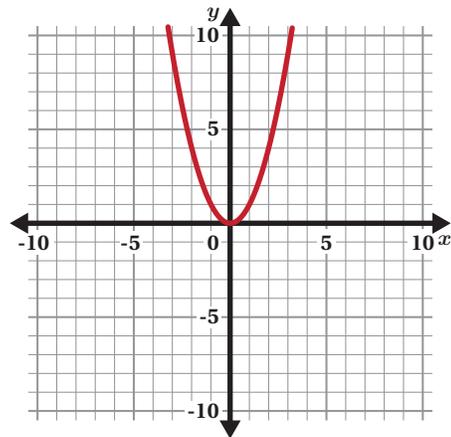
# Lesson Practice

A1.7.15

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

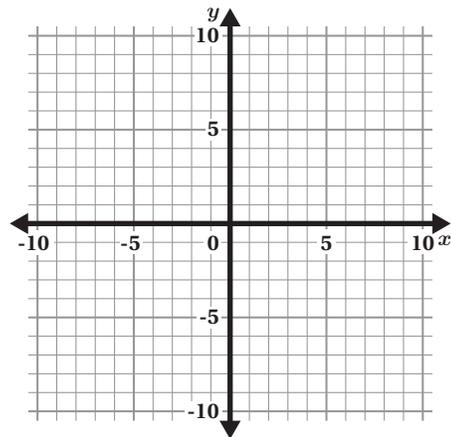
**Problems 1–3:** Here's the graph of  $y = x^2$ . Change one number to make the graph:

1. Wider:
2. Narrower:
3. Open down:



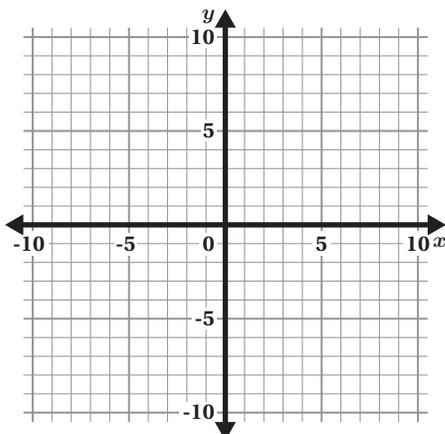
4. Describe how the graph of  $y = x^2$  compares to  $y = -4(x - 2)^2 + 7$ .

5. Draw a graph of a parabola that has a vertex at  $(3, -1)$  and is scaled vertically by a factor of 2.

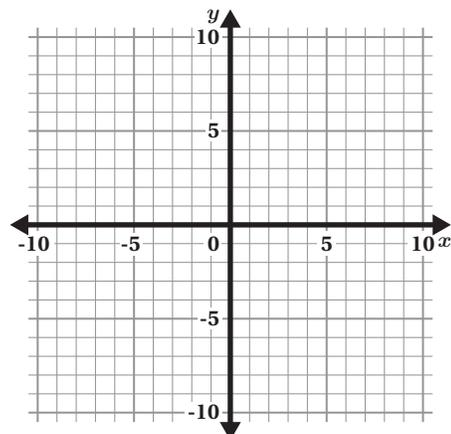


**Problems 6–7:** Draw a graph for each equation.

6.  $a(x) = -2(x - 1)^2 + 4$



7.  $c(x) = 0.5(x + 4)^2 - 2$



# Lesson Practice

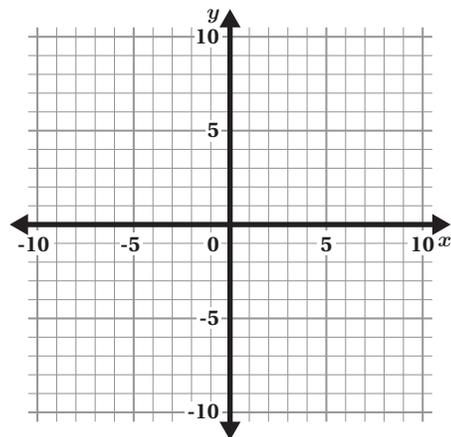
A1.7.15

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

8. Using at least 4 parabolas, draw a graph that reminds you of a rainbow. The space between the  $y$ -intercept of each parabola should remain the same.

Record the functions you use.

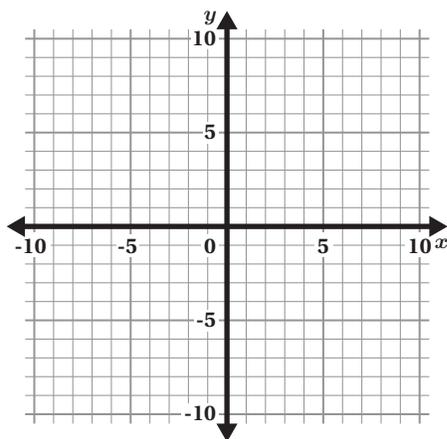
Functions



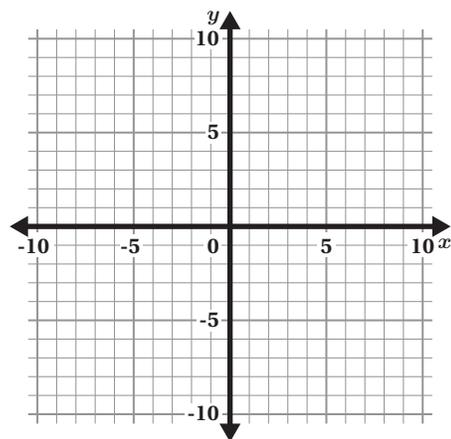
## Spiral Review

Problems 9–10: Draw a graph of each equation.

9.  $-3x + 3y = 9$



10.  $2x + 4y = 8$



## Reflection

1. Circle the problem you enjoyed doing the most.
2. Use this space to ask a question or share something you're proud of.

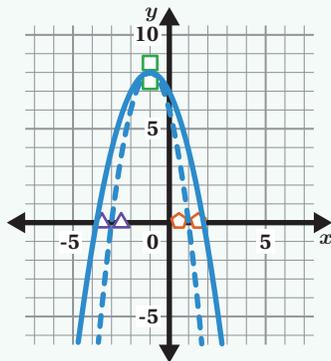
Lesson Summary

You can use key features to write quadratic functions in standard, factored, or vertex form. Here are two strategies for using key features to write an equation of a parabola that will pass through these gates:

**Vertex Form:**  $f(x) = a(x - h)^2 + k$

Identify a possible vertex:  $(-1, 8)$

$$f(x) = a(x + 1)^2 + 8$$



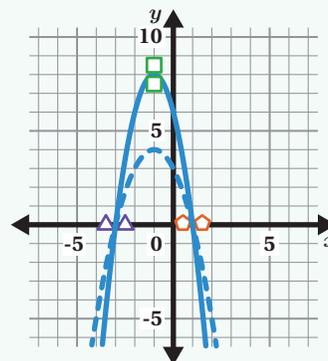
Since the parabola is concave down, the  $a$ -value needs to be negative. Then I can adjust the value to scale the parabola vertically to match the  $x$ -intercepts.

$$f(x) = -2(x + 1)^2 + 8$$

**Factored Form:**  $f(x) = a(x - b)(x - c)$

Identify possible  $x$ -intercepts:  $(-3, 0)$  and  $(1, 0)$  Substitute the  $x$ -values of the  $x$ -intercepts into  $b$  and  $c$ :

$$f(x) = a(x + 3)(x - 1)$$



Since the parabola is concave down, the  $a$ -value needs to be negative. Then I can adjust the value to scale the parabola vertically to adjust the vertical position of the vertex.

$$f(x) = -2(x + 3)(x - 1)$$

Things to Remember:

# Lesson Practice

A1.7.16

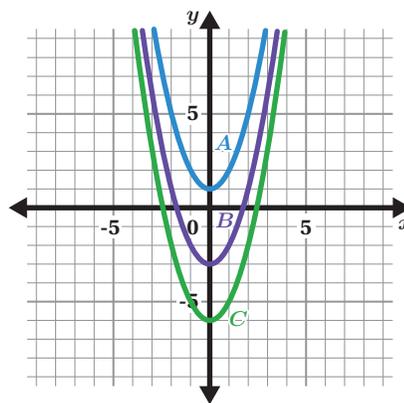
Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

1. Parabolas  $A$ ,  $B$ , and  $C$  are translations of  $y = x^2$ . Determine the equation of each parabola.

A: \_\_\_\_\_

B: \_\_\_\_\_

C: \_\_\_\_\_



2. Which equation has a graph with a vertex at  $(1, 3)$ ?

A.  $f(x) = (x - 1)^2 + 3$

B.  $f(x) = (x - 3)^2 + 1$

C.  $f(x) = (x + 1)^2 + 3$

D.  $f(x) = (x + 3)^2 + 1$

Explain your thinking.

3. Select *all* the equations that match this graph.

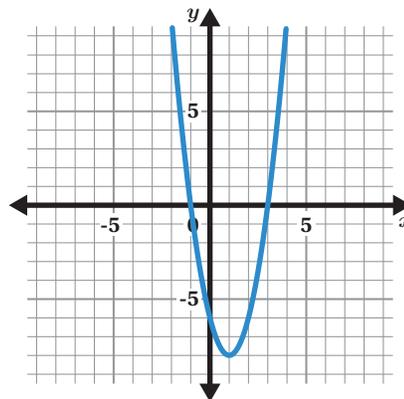
A.  $y = 2(x - 1)^2 - 8$

B.  $y = -(x - 1)^2 - 8$

C.  $y = (2x - 2)(x + 3)$

D.  $y = (2x + 2)(x - 3)$

E.  $y = (x - 1)^2 - 3$



4. Write an equation for a quadratic function with  $x$ -intercepts at  $(-2, 0)$  and  $(6, 0)$ .

**Problems 5–6:** Write an equation for a quadratic function that matches each description. Use graphing technology to check your work.

5. Concave down with a vertex at  $(-2, 6)$ .

6. Concave up with a vertex at  $(0, -8)$ .

# Lesson Practice

A1.7.16

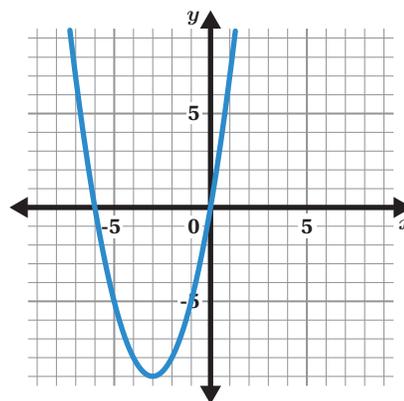
Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

7. Here are two equations:

$$m(x) = x(x + 6)$$

$$p(x) = (x + 3)^2 - 9$$

Show or explain how you know that both equations describe this graph.



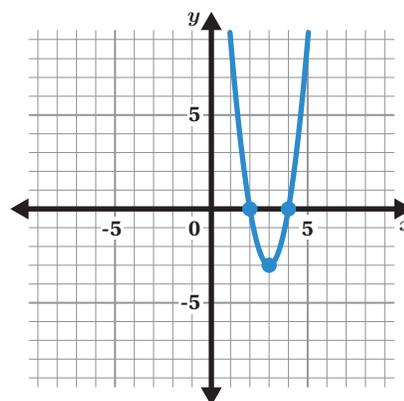
8. Which equation describes this graph?

Circle your choice.

$$g(x) = 3(x + 3)^2 + 3$$

$$h(x) = 3(x - 3)^2 - 3$$

Explain your thinking.



## Spiral Review

**Problems 9–10:** Write an equation that matches each description.

9. Description of  $f(x)$ :

- linear function
- $y$ -intercept of 12
- negative slope

10. Description of  $g(x)$ :

- quadratic function
- $x$ -intercepts of (4, 0) and (7, 0)
- concave up

## Reflection

1. Put a star next to a problem that looked more difficult than it really was.
2. Use this space to ask a question or share something you're proud of.

**Lesson Summary**

We can use quadratic equations, tables, and graphs to make sense of situations in society. These mathematical models can help inform the decisions we make about real-world issues. In real-world situations, it is important to be able to create and compare models of different relationships.

For example, in this lesson we explored two models that represent the cost of housing. We used a linear model to explore the relationship between the price of rent and the number of units rented. We used a quadratic model to explore the relationship between rent and revenue generated. A community organization could use these models to determine a fairer price of rent.

---

**Things to Remember:**

# Lesson Practice

A1.7.17

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

1. Which function has a graph with a vertex at  $(-1, 4)$ ?

A.  $y = (x - 1)^2 + 4$

B.  $y = (x - 4)^2 - 1$

C.  $y = (x + 1)^2 + 4$

D.  $y = (x + 4)^2 - 1$

2. Select *all* the functions whose graphs have an  $x$ -intercept at  $(3, 0)$ .

A.  $a(x) = (x + 2)(x - 3)$

B.  $b(x) = (x + 3)(x - 2)$

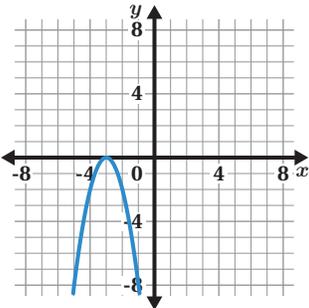
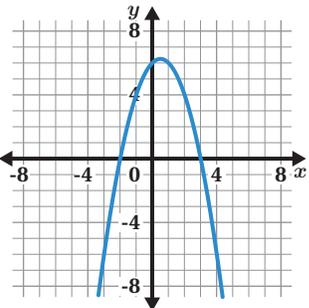
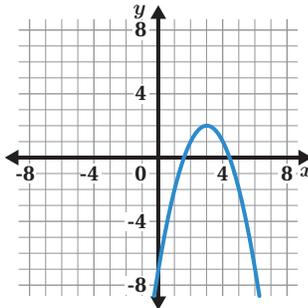
C.  $c(x) = 3x(x - 2)$

D.  $d(x) = (2x - 6)(x + 2)$

E.  $e(x) = (2x - 3)(x + 3)$

3. Match each graph to the quadratic equation it represents. One equation has no match.

a. $y = -(x - 3)^2 + 2$	b. $y = -3x^2 + 2$
c. $y = -2(x + 3)^2$	d. $y = -(x + 2)(x - 3)$

		
Equation: _____	Equation: _____	Equation: _____

4. Write an equation of a parabola that has a vertex at  $(-2, 1)$ .

# Lesson Practice

A1.7.17

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

5. Azul and Latifa try to write an equation to describe this graph. Each equation is incorrect in some way.

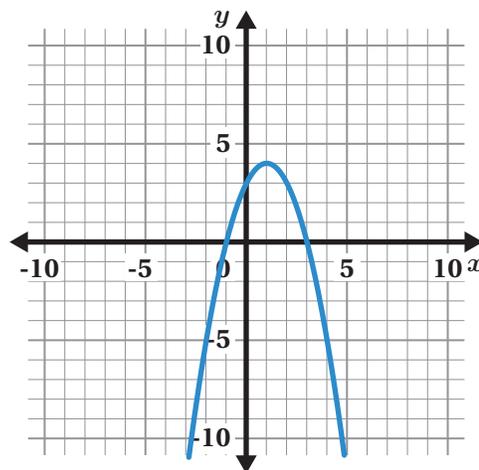
**Azul**

$$y = (x - 1)^2 + 4$$

**Latifa**

$$y = -(x + 3)(x - 1)$$

Choose one equation. Explain how you would change the equation so that it creates this graph.



**Problems 6–7:** Here is a function:  $m(x) = 2x(x - 3)$ .

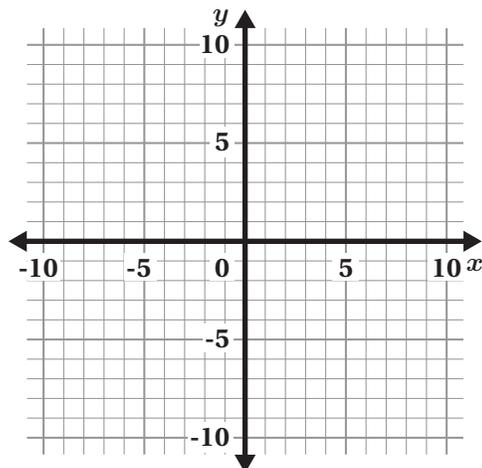
6. Determine the  $x$ -intercepts and vertex of  $m(x)$ .

$x$ -intercept: .....

$x$ -intercept: .....

vertex: .....

7. Draw the graph of the function  $m(x)$ .



## Spiral Review

**Problems 8–10:** Determine the value of each function when  $x = -1$ .

8.  $f(x) = 2(x - 1)^2 + 3$

9.  $g(x) = 3x^2 - 2x - 1$

10.  $h(x) = (2x + 3)(x - 1)$

## Reflection

- Put a heart next to the problem you're most proud of.
- Use this space to ask a question or share something you're proud of.

## Lesson Summary

Quadratic expressions can be written in *factored form* or *standard form*.

You can use an area model to help you rewrite a factored-form quadratic expression into an equivalent expression in standard form.

Here are two examples.

Factored form:  $(3x - 4)(x + 2)$

	$3x$	$-4$
$x$	$3x^2$	$-4x$
$2$	$6x$	$-8$

$$3x^2 + 6x - 4x - 8$$

Standard form:  $3x^2 + 2x - 8$

Factored form:  $x(3x + 1)$

	$3x$	$1$
$x$	$3x^2$	$1x$

Standard form:  $3x^2 + x$

Things to Remember:



# Lesson Practice

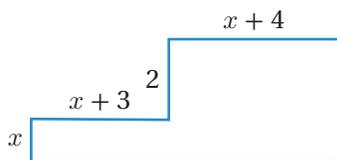
A1.8.02

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

5. Complete the table by writing each expression in standard form.

Factored Form	Standard Form
$(x + 7)(x + 3)$	
$(x - 2)(x - 12)$	

6. Write an expression that represents the area and an expression for the perimeter of this figure:



Area: \_\_\_\_\_

Perimeter: \_\_\_\_\_

## Spiral Review

Problems 7–9: Complete each equivalent expression:

7.  $14a + 21 = \dots (\dots a + \dots)$

8.  $3b + 2 + 3b + 10 = \dots (\dots b + \dots)$

9.  $-3(3c + 3) = \dots c + \dots$

## Reflection

1. Put a star next to the problem you spent the most time on.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

You can analyze the structure of a quadratic expression written in *factored form* to make predictions about what the equivalent expression in *standard form*,  $ax^2 + bx + c$ , will look like.

Here are two strategies for multiplying a factored-form expression to rewrite quadratic expressions in standard form.

Strategy 1

$$(2x - 5)(x + 3)$$

	$2x$	$-5$
$x$	$2x^2$	$-5x$
$3$	$6x$	$-15$

Standard form:  $2x^2 + x - 15$

$a = 2$   $b = 1$   $c = -15$

Strategy 2

$$(3x - 4)(3x + 4)$$

$$(3x - 4)(3x + 4)$$

$$9x^2 + 12x - 12x - 16$$

Standard form:  $9x^2 - 16$

$a = 9$   $b = 0$   $c = -16$

Here are some patterns demonstrated in these two examples:

- If the constants in factored form have opposite signs, the  $c$ -value in standard form will be negative.
- If the factors have the same *coefficients* but opposite constants, then  $b = 0$  in standard form.

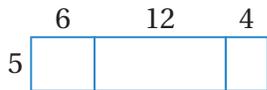
Things to Remember:

# Lesson Practice

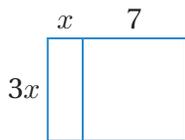
A1.8.03

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

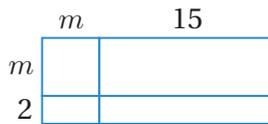
1. Write an expression that represents each area model.



.....

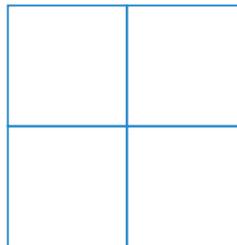


.....



.....

2. Complete the diagram to show that  $(x - 10)(x - 3)$  is equivalent to  $x^2 - 13x + 30$ .



**Problems 3–5:** For each expression in factored form, write an equivalent expression in standard form.

3.  $(x - 2)^2$

4.  $(x + 1)(x - 1)$

5.  $(2x + 4)(x - 3)$

**Problems 6–7:** Write an expression in factored form that has:

6. A  $b$ -value less than 4 and a  $c$ -value greater than 1 when written in standard form.

7. A negative  $a$ -value, negative  $b$ -value, and a positive  $c$ -value when written in standard form.

# Lesson Practice

A1.8.03

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

8. Select any number from the inner square.

- Multiply the number to the left of your selection by the number to the right.
- Multiply the number above your selection by the number below.
- Here's an example with the number 12 selected:

$$\begin{array}{c} 2 \\ 11 \times 13 = 143 \\ 22 \\ = 44 \end{array}$$

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

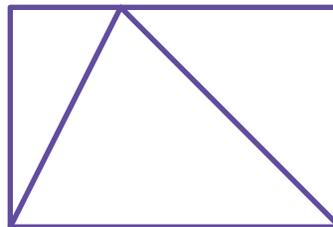
The difference between the numbers will be 99 no matter your selection.  
Explain why.

## Spiral Review

9. Draw a line to show that this rectangle is made only from 2 congruent triangles.



10. Draw a line to show that this rectangle is made only from 2 pairs of congruent triangles.



## Reflection

1. Put a star next to the problem you spent the most time on.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

A diagram can be a helpful tool for rewriting *standard-form* quadratic expressions in *factored form*. Here is an example: Rewrite the expression  $3x^2 + 14x + 15$  in factored form.

Work

$3x^2$	
	15

Explanation

Place the  $ax^2$  term in top-left corner of the diagram and the  $c$ -term in the bottom right.

	$x$	15
$3x$	$3x^2$	$45x$
1	$x$	15

Try different *factors* on the outside of the diagram that multiply to get  $ax^2$  and  $c$  until the inside of the diagram matches standard form.

This attempt didn't work because  $x + 45x \neq 14x$ . The first attempt might not work and that's okay! Try different factors or switching the positions of the current factors.

	$3x$	5
$x$	$3x^2$	$5x$
3	$9x$	15

This attempt works because the linear terms in the diagram combine to match  $bx$  in standard form:  $5x + 9x = 14x$ .

The factored form is  $(3x + 5)(x + 3)$ .

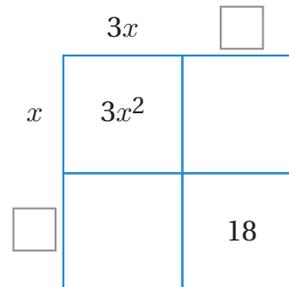
Things to Remember:

# Lesson Practice

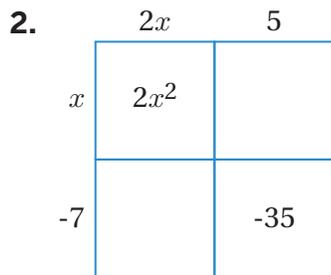
A1.8.04

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

1. Write two possible constants that could complete the outside of the diagram.

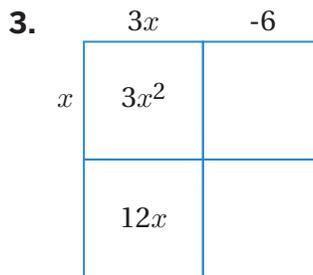


**Problems 2–3:** Complete the diagram puzzles and expressions.



Factored form:  $(2x + 5)(x - 7)$

Standard form: \_\_\_\_\_



Factored form: \_\_\_\_\_

Standard form: \_\_\_\_\_

4. Rewrite each expression in factored or standard form.

Factored Form	Standard Form
	$x^2 + 9x + 18$
$(2x - 3)(2x - 7)$	
	$3x^2 + 10x - 8$

5. Determine values that make the equation true.

$$7x^2 - 10x - \dots = (\dots x - 2)(7x + \dots)$$

# Lesson Practice

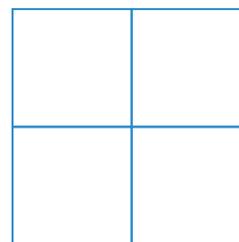
A1.8.04

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

6. This quadratic expression in standard form has an unknown  $b$ -value. If we know the expression can be factored, list two possibilities for the unknown value.

$$3x^2 + \boxed{?}x - 4$$

7. Create and complete a diagram puzzle that only uses factors of 12 as coefficients and constants.



## Spiral Review

Problems 8–10: Fill in the missing values to make each equation true.

8.  $3(\text{.....} + \text{.....}) = 18$

9.  $\text{.....}(\text{.....} + \text{.....}) = 20$

10.  $4(\text{.....} + \text{.....}) = 3(\text{.....}) + 2(\text{.....})$

## Reflection

1. Put a question mark next to a response you'd like to compare with a classmate's.
2. Use the space below to ask one question you have or to share something you're proud of.

Lesson Summary

You can use the structure of a quadratic expression written in *standard form* to predict what the *factored form* will look like. Here are some strategies:

- Try to factor out a common factor first.
- If the standard form expression only has two terms, write the missing term with a coefficient of 0.
- If the *c*-value is negative, the signs of the constants in factored form will be different.

Here are some examples.

$$3x^2 - 9x - 30$$

3 is a common factor.

$$3(x^2 - 3x - 10)$$

Then factor  $x^2 - 3x - 10$ .

	$x$	$-5$
$x$	$x^2$	$-5x$
$2$	$2x$	$-10$

When  $a = 1$ , the constants in factored form multiply to  $c$  and add to  $b$ .

**Factored form:**

$$3(x - 5)(x + 2)$$

$$x^2 - 81$$

Expressions with this structure are called a *difference of squares*.

Rewriting the expression with three terms might be helpful.

$$\text{Factor } x^2 + 0x - 81.$$

	$x$	$9$
$x$	$x^2$	$9x$
$-9$	$-9x$	$-81$

**Factored form:**

$$(x - 9)(x + 9)$$

$$2x^2 + 3x - 27$$

There is not a common factor in this quadratic expression.

Test pairs of expressions that multiply to  $2x^2$  and  $-27$ .

The  $c$ -value is negative so the signs of the constants in factored form will be different.

	$2x$	$9$
$x$	$2x^2$	$9x$
$-3$	$-6x$	$-27$

**Factored form:**

$$(2x + 9)(x - 3)$$

Things to Remember:

# Lesson Practice

A1.8.05

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

1. Write a + or – sign in each box to make true equations.

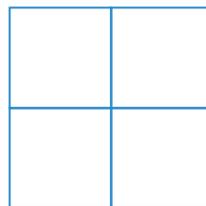
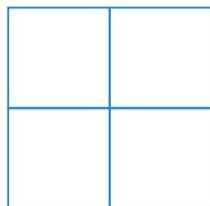
$$(x \square 18)(x \square 3) = x^2 + 15x - 54$$

$$(x \square 18)(x \square 3) = x^2 - 21x + 54$$

**Problems 2–5:** Fill in the blanks to make each equation true. Use the diagrams if they help your thinking.

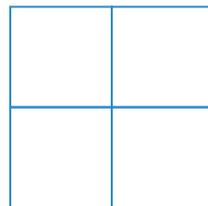
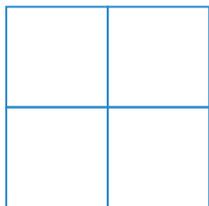
2.  $x^2$  \_\_\_\_\_  $x$  \_\_\_\_\_ =  $(x - 9)(x - 3)$

3.  $x^2 + 12x$  \_\_\_\_\_ =  $(x + 4)(x$  \_\_\_\_\_)



4.  $2x^2 + 11x + 15 = (2x$  \_\_\_\_\_) $(x$  \_\_\_\_\_)

5.  $3x^2 - 11x$  \_\_\_\_\_ =  $(x - 6)(3x$  \_\_\_\_\_)



**Problems 6–8:** Factor each expression.

6.  $x^2 - x - 30$

7.  $4x^2 + 20x + 25$

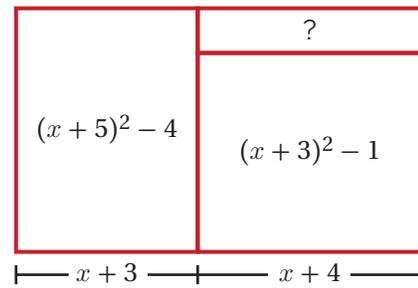
8.  $4x^2 - 81$

# Lesson Practice

A1.8.05

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

9. The diagram shows the expressions for areas and lengths. Determine an expression for the unknown area.



## Spiral Review

Problems 10–12: Solve each equation.

10.  $6 + 2x = 0$

11.  $2x - 5 = 0$

12.  $\frac{1}{2}(x - 87) = 0$

## Reflection

1. Circle the problem that was the most challenging for you.
2. Use this space to ask a question or share something you're proud of.

## Lesson Summary

An  $x$ -intercept is the coordinate point where the graph crosses the  $x$ -axis, and a **zero** is the  $x$ -value of the  $x$ -intercept.

To determine the zeros or  $x$ -intercepts, you determine the  $x$ -values that make the function equal to 0. Rewriting the function in factored form is a helpful step for determining those values.

Here are two examples of determining the  $x$ -intercepts of functions in different forms.

Factored Form:  $h(x) = (x - 1)(x + 6)$

When  $x = 1$ , the factor  $x - 1 = 0$ ,  
so  $h(1) = 0$ .

When  $x = -6$ , the factor  $x + 6 = 0$ ,  
so  $h(-6) = 0$ .

The zeros are  $x = 1$  and  $x = -6$ .

The  $x$ -intercepts are  $(1, 0)$  and  $(-6, 0)$ .

Standard Form:  $g(x) = x^2 - 6x - 40$

First, I can factor the function.

	$x$	$-10$
$x$	$x^2$	$-10x$
$4$	$4x$	$-40$

$$g(x) = (x - 10)(x + 4)$$

When  $x = 10$ , the factor  $x - 10 = 0$ ,  
so  $g(10) = 0$ .

When  $x = -4$ , the factor  $x + 4 = 0$ ,  
so  $g(-4) = 0$ .

The zeros are  $x = 10$  and  $x = -4$ .

The  $x$ -intercepts are  $(10, 0)$  and  $(-4, 0)$ .

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 Things to Remember:

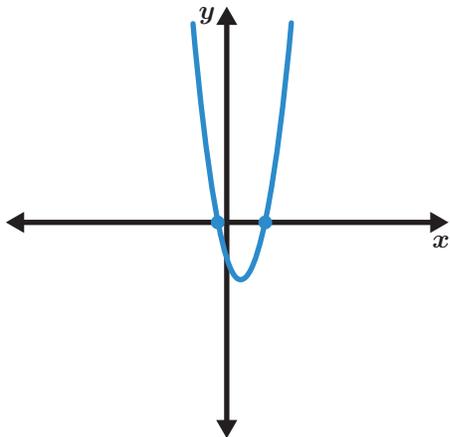
# Lesson Practice

A1.8.06

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

**Problems 1–3:** What are the  $x$ -intercepts of the function?

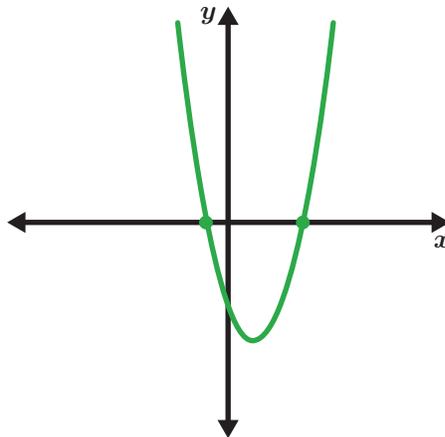
1.  $a(x) = (x - 2)(2x + 1)$



$x$ -intercept #1: \_\_\_\_\_

$x$ -intercept #2: \_\_\_\_\_

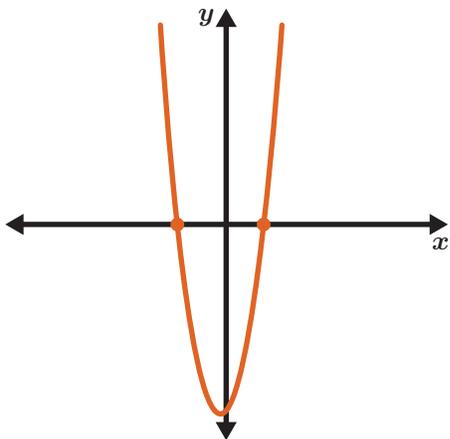
2.  $b(x) = x^2 - 3x - 4$



$x$ -intercept #1: \_\_\_\_\_

$x$ -intercept #2: \_\_\_\_\_

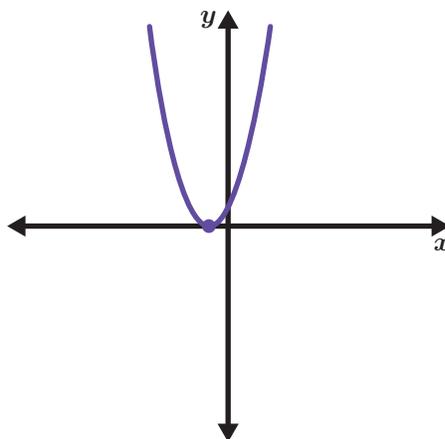
3.  $c(x) = 2x^2 + x - 10$



$x$ -intercept #1: \_\_\_\_\_

$x$ -intercept #2: \_\_\_\_\_

4. Write an equation that could represent this graph.



$f(m) =$  \_\_\_\_\_

5. Select *all* the functions that have 5 and -1 as their  $x$ -intercepts.

A.  $f(x) = (x + 5)(x - 1)$

B.  $g(x) = (x + 1)(x - 5)$

C.  $h(x) = x^2 + 4x - 5$

D.  $j(x) = 2x^2 - 8x - 10$

E.  $k(x) = (15 - 3x)(4x + 4)$

# Lesson Practice

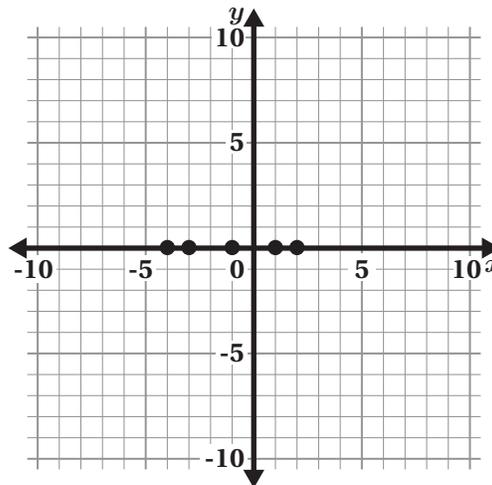
A1.8.06

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

6. Change one number so that the function has  $x$ -intercepts at -1 and 3.

$$g(x) = x^2 - 1x - 3$$

7. Write the equations of 3 parabolas that go through all of the points.



## Spiral Review

Problems 8–9: Solve for  $x$ .

8.  $2x - 3 \geq 4x + 7$

9.  $3(x - 2) < 2x - 12$

## Reflection

1. Circle the problem that was the most challenging for you.
2. Use this space to ask a question or share something you're proud of.

## Lesson Summary

The **zero-product property** states that if the product of two or more *factors* is 0, then at least one of the factors is 0. You can use this property to determine the  $x$ -intercepts of a function or the *solutions* to quadratic equations using the following steps.

- Set the quadratic equation equal to 0.
- Factor the equation.
- Set each factor equal to 0.
- Solve for  $x$ .

Here are two examples of solving quadratic equations.

$$(5x - 3)(2x + 3) = 0$$

Set each factor equal to 0 and solve for  $x$ .

$$\begin{array}{l} (5x - 3) = 0 \\ 5x = 3 \\ x = \frac{3}{5} \end{array} \quad \begin{array}{l} (2x + 3) = 0 \\ 2x = -3 \\ x = -\frac{3}{2} \end{array}$$

$$2x^2 - x = 21$$

First, rewrite the equation so that it is equal to 0.

$$2x^2 - x - 21 = 0$$

Then factor the equation.

$$(2x - 7)(x + 3) = 0$$

Set each factor equal to 0 and solve for  $x$ .

$$\begin{array}{l} (2x - 7) = 0 \\ 2x = 7 \\ x = \frac{7}{2} \end{array} \quad \begin{array}{l} (x + 3) = 0 \\ x = -3 \end{array}$$

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Things to Remember:

# Lesson Practice

A1.8.07

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

1. Rewrite each standard-form quadratic equation in factored form.

**a**  $x^2 + 7x + 6 = 0$

**b**  $x^2 - 5x + 6 = 0$

**c**  $x^2 + 5x - 6 = 0$

2. Rewrite the equation  $x^2 - x = 6$  so that one side is equal to 0. Then solve the equation.

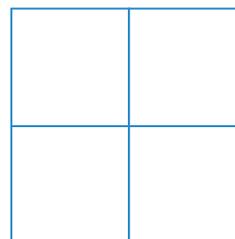
**Problems 3–5:** Solve each equation.

3.  $(4 - 5x)(x + 4) = 0$

4.  $x^2 + 14x + 33 = 0$

5.  $5x + 12 = x^2 - 5x - 12$

6. Solve the equation  $3x^2 - 18x + 24 = 0$ .  
Use the diagram if it helps with your thinking.



**Problems 7–8:** Solve each equation.

7.  $x^2 - 7x = 0$

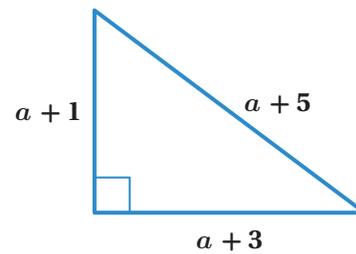
8.  $(x + 2)(x + 4) = 3$

# Lesson Practice

A1.8.07

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

9. Determine the value of  $a$  in the right triangle.  
Explain your thinking.



10. Write two equations with  $x = \frac{2}{3}$  and  $x = -9$  as solutions.

## Spiral Review

11. Decide whether this equation has *one solution*, *no solution*, or *infinitely many solutions*:  $-3(x + 2) = -x + 6 - 2x$ .

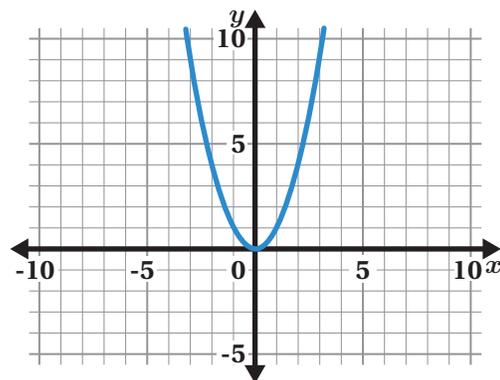
One solution

No solution

Infinitely many solutions

12. Select *all* the true statements about the function  $f(x) = x^2$ .

- A. The domain has no negative values.
- B. The range has no negative values.
- C. The function has no minimum.
- D. The function has no maximum.



## Reflection

- Put a star next to a problem you could explain to a classmate.
- Use this space to ask a question or share something you're proud of.

## Lesson Summary

To determine the number of solutions to a quadratic equation, you can use reasoning or use the structure of the equation. Here are some examples:

No Solutions	One Solution	Two Solutions
$(x + \dots)^2 = \text{a negative number}$  No value squared will result in a negative number.	$(x + \dots)^2 = 0$  Only one value squared will equal 0.	$(x + \dots)^2 = \text{a positive number}$  There are two values that when squared will equal a positive number.
$(x + 10)^2 = -25$  $(x - 3)^2 + 1 = 0$  $x^2 + 4 = 0$	$(x + 4)^2 = 0$  $x^2 + 9 = 9$  $(x - 3)(x - 3) = 0$	$(x + 4)^2 = 1$  $x^2 - 12 = -3$  $(x - 3)(x - 3) = 1$

Things to Remember:

# Lesson Practice

A1.8.08

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

1. Write a quadratic equation that has . . .

**a** Two solutions \_\_\_\_\_

**b** One solution \_\_\_\_\_

**c** No solutions \_\_\_\_\_

2. For each equation, determine the number of solutions.

Equation	Number of Solutions
$x(x + 3) = 0$	
$(x + 3)(x + 1) = 0$	
$(x + 1)(x + 1) = 0$	
$x^2 - 10x = -9$	
$(x - 5)(x - 5) = -14$	
$x^2 - 3 = -3$	
$x^2 + 6 = 2$	

**Problems 3–4:** Determine the solution for each equation.

3.  $149 + (x - 2)^2 = 149$

4.  $x^2 + 3x = x - 1$

**Problems 5–6:** Determine the two solutions for each equation.

5.  $100 + (x - 2)^2 = 149$

6.  $x^2 + 4x = x + 18$

7. Which value for  $x$  is a solution to the equation  $x^2 + 10 = 9$ ?

A.  $x = 1$

B.  $x = -1$

C.  $x = \sqrt{19}$

D. There is no solution to the equation.

# Lesson Practice

A1.8.08

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

## Spiral Review

8. Order the expressions by value from *least* to *greatest*:  $5^2$ ,  $\sqrt{90}$ ,  $8$ ,  $3^3$ ,  $\sqrt{27}$ .

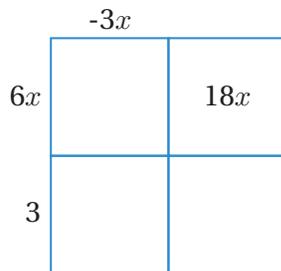
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Least

Greatest

**Problems 9–10:** Complete each diagram. Then write the corresponding quadratic expression in factored form and standard form.

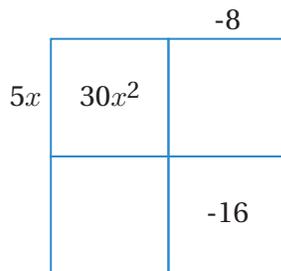
9.



Factored form:

Standard form:

10.



Factored form:

Standard form:

## Reflection

1. Put a question mark next to a problem you're feeling unsure of.
2. Use this space to ask a question or share something you're proud of.

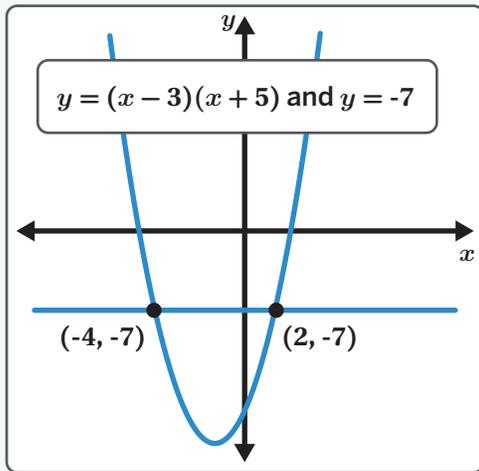
Lesson Summary

Graphs can be used to determine the solutions to a quadratic equation.

Here are two strategies using graphs to solve  $(x - 3)(x + 5) = -7$ .

Strategy 1

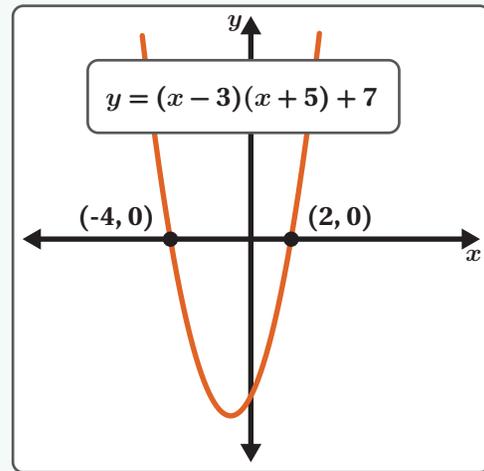
- Graph both sides of the equation as two separate graphs.
- Determine the  $x$ -coordinates where the graphs intersect.



Solutions:  $x = -4$  and  $x = 2$

Strategy 2

- Rewrite the equation so that it equals 0.
- Graph the equation. The solutions will be at the  $x$ -intercepts.



Solutions:  $x = -4$  and  $x = 2$

Things to Remember:

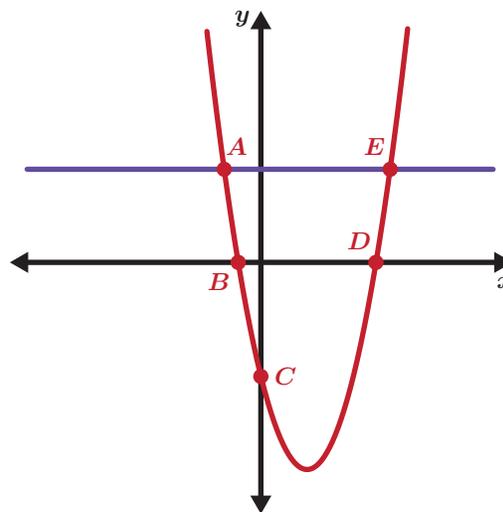
# Lesson Practice

A1.8.09

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

1. Here is a graph of  $y = x^2 - 4x - 5$  and  $y = 4$ .  
Select *all* the points that are solutions to  $x^2 - 4x - 5 = 4$ .

- A. Point A
- B. Point B
- C. Point C
- D. Point D
- E. Point E



**Problems 2–5:** Circle how many solutions each equation has. Record any solutions.

- |    |                          |                 |                 |                  |  |
|----|--------------------------|-----------------|-----------------|------------------|--|
| 2. | $-2x^2 + 2 = -6$         | No<br>solutions | One<br>solution | Two<br>solutions | $x = \dots\dots\dots$<br>$x = \dots\dots\dots$ |
| 3. | $x(x + 3) = 4$           | No<br>solutions | One<br>solution | Two<br>solutions | $x = \dots\dots\dots$<br>$x = \dots\dots\dots$ |
| 4. | $0 = (x - 2)(x + 4) + 9$ | No<br>solutions | One<br>solution | Two<br>solutions | $x = \dots\dots\dots$<br>$x = \dots\dots\dots$ |
| 5. | $2x(x + 1) = -1$         | No<br>solutions | One<br>solution | Two<br>solutions | $x = \dots\dots\dots$<br>$x = \dots\dots\dots$ |

**Problems 6–7:** Fill in the blank so the equation has:

6. One solution

$3x(x + 2) = \dots\dots\dots$

7. No solutions

$(x - 2)(x + \dots\dots\dots) = -4$



**Lesson Summary**

A quadratic expression is a **perfect square** if it can be represented as something multiplied by itself, like  $(x + \dots)^2$ .

You can determine the missing constant value to add to make a perfect square by dividing the linear coefficient in half and then squaring that number.

Here are two examples of filling in the blanks to make each expression a perfect square.

$$x^2 + 14x + \dots$$

$$x^2 - \dots + 81$$

In perfect square expressions, the  $b$ -value, 14, is always double the constant in the factored form  $(x + 7)^2$ .

The  $c$ -value, 81, is a perfect square so the factored form expression must be  $(x - 9)^2$ .

So the missing number must be the constant 7 squared, which is 49.

Rewrite the factored form to standard by using an area model or distributive property. The missing number must be  $-18x$ .

$$x^2 + 14x + 49$$

	$x$	$7$
$x$	$x^2$	$7x$
$7$	$7x$	$49$

$$x^2 - 18x + 81$$

	$x$	$-9$
$x$	$x^2$	$-9x$
$-9$	$-9x$	$81$

**Things to Remember:**

# Lesson Practice

## A1.8.11

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

1. Which expression is equivalent to  $(x - 6)^2$ ?

A.  $(x + 6)^2$

B.  $x^2 - 12x - 36$

C.  $x^2 - 36$

D.  $x^2 - 12x + 36$

2. Write each perfect square expression in factored form.

a  $x^2 + 6x + 9$

b  $x^2 - 16x + 64$

c  $x^2 - 12x + 36$

d  $x^2 + 5x + \frac{25}{4}$

3. Select *all* the expressions that are perfect squares.

A.  $x^2 + 10x + 25$

B.  $x^2 - 10x + 25$

C.  $x^2 + 4$

D.  $(x + 3.5)(3.5 + x)$

E.  $(x - 10)(10 - x)$

F.  $x^2 + \frac{1}{2}x + \frac{1}{16}$

4. Erendirani says that if a perfect square expression is written in the form  $x^2 + bx + c$ , the value of  $c$  cannot be negative. Why is this true?

**Problems 5–8:** Fill in the blanks to complete each perfect square.

5.  $x^2 + 24x$  \_\_\_\_\_

6.  $x^2 - 2x$  \_\_\_\_\_

7.  $x^2 -$  \_\_\_\_\_  $+ 64$

8.  $x^2 + \frac{2}{3}x$  \_\_\_\_\_

# Lesson Practice

## A1.8.11

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

9. The expressions  $(x - 4)^2$  and  $(4 - x)^2$  are both perfect squares. Are they equivalent to one another? Explain your thinking.

10. LaShawn claims that  $x^2 + bx + \frac{b^2}{4}$  is a perfect square. Do you agree? Explain your thinking.

### Spiral Review

**Problems 11–12:** The equations  $y = x^2 + 5x + 6$  and  $y = (x + 2)(x + 3)$  are equivalent.

11. Which equation would you use to determine the  $x$ -intercepts? Explain your thinking.

12. Which equation would you use to determine the  $y$ -intercept? Explain your thinking.

13. Without using a graphing calculator, select *all* the equations with a positive  $y$ -intercept.

A.  $y = x^2 + 3x - 2$

B.  $y = (x + 1)(x + 5)$

C.  $y = x^2 - 10x$

D.  $y = (x - 3)^2$

E.  $y = -5x^2 + 3x - 12$

### Reflection

1. Circle the problem you feel least confident about.

2. Use this space to ask a question or share something you're proud of.

**Lesson Summary**

You can solve quadratic equations by graphing, factoring, or **completing the square**, which is the process of rewriting a quadratic expression or equation to include a perfect square. You can analyze the structure of the equation to help you decide which strategy to use.

Here is an example:  $x^2 + 10x = 2$ . Solving by graphing will not produce exact solutions and factoring is not possible for this equation, so we can solve by completing the square:

**Work**

$$x^2 + 10x + 25 = 2 + 25$$

$$(x + 5)^2 = 27$$

$$x + 5 = \pm \sqrt{27}$$

$$x = -5 \pm \sqrt{27}$$

**Explanation**

$x^2 + 10x + 25$  is a perfect square, so add the constant value 25 to both sides of the equation.

Rewrite the perfect square  $x^2 + 10x + 25$  in factored form.

Take the square root and include both possibilities by writing  $\pm$ .

Solve for  $x$ .

---

**Things to Remember:**

# Lesson Practice

## A1.8.12

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

1. Select *all* the expressions that are perfect squares.

A.  $(x + 5)(5 + x)$

B.  $(x - 3)^2$

C.  $x^2 - 3^2$

D.  $x^2 + 8x + 64$

E.  $x^2 + 10x + 25$

2. Add the number that would make the expression a perfect square. Then write an equivalent expression in factored form.

$x^2 - 6x + \square$   
.....

$x^2 + 2x + \square$   
.....

$x^2 - 14x + \square$   
.....

3. Match each equation to an equivalent equation.

a.  $x^2 - 12x = 6$

.....  $(x - 6)^2 = 30$

b.  $x^2 - 12x + 6 = 0$

.....  $(x - 3)^2 = 42$

c.  $x^2 - 6x = 6$

.....  $(x - 6)^2 = 42$

d.  $x^2 - 6x = 33$

.....  $(x - 3)^2 = 15$

4. Alexis solved the equation  $x^2 + 12x = 13$  by completing the square, but some parts are blank. Fill in the blanks.

$$x^2 + 12x = 13$$

$$\square$$

$$(x + 6)^2 = 49$$

$$x + 6 = \pm 7$$

$$x = \square \text{ and } x = \square$$

# Lesson Practice

A1.8.12

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

**Problems 5–7:** Solve each equation by completing the square.

5.  $x^2 - 2x = 8$

6.  $7 = x^2 + 4x - 1$

7.  $x^2 - 18x + 60 = -11$

8. Write a quadratic equation of the form  $x^2 + bx + c = 0$  with solutions that are  $x = 5 - \sqrt{2}$  and  $x = 5 + \sqrt{2}$ .

## Spiral Review

9. For each equation, determine the number of solutions.

Equation	Number of Solutions
$x^2 + 144 = 0$	
$x^2 - 144 = 0$	
$(x - 7)^2 = 0$	

10. The graph of  $y = (x - 1)^2 + 4$  is the same as the graph of  $y = x^2$ , but:

- A. It is shifted 1 unit to the right and 4 units up.
- B. It is shifted 1 unit to the left and 4 units up.
- C. It is shifted 1 unit to the right and 4 units down.
- D. It is shifted 1 unit to the left and 4 units down.

## Reflection

- 1. Put a heart next to the problem you found most interesting.
- 2. Use this space to ask a question or share something you're proud of.

## Lesson Summary

You can use the **quadratic formula** to solve quadratic equations written in standard form. The quadratic formula states that the solutions to any quadratic equation  $ax^2 + bx + c = 0$  are  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ .

The quadratic formula comes from completing the square.

Here is an example of completing the square and using the quadratic formula to solve  $x^2 + 8x + 13 = 0$ . Even though the answers look different, they are equivalent.

**Completing the Square**

$$\text{Solve } x^2 + 8x + 13 = 0$$

$$x^2 + 8x = -13$$

$$x^2 + 8x + 16 = -13 + 16$$

$$x^2 + 8x + 16 = 3$$

$$(x + 4)^2 = 3$$

$$x + 4 = \pm \sqrt{3}$$

$$x = -4 \pm \sqrt{3}$$

**Quadratic Formula**

$$\text{Solve } x^2 + 8x + 13 = 0$$

$$a = 1, b = 8, c = 13$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(8) \pm \sqrt{(8)^2 - 4(1)(13)}}{2(1)}$$

$$x = \frac{-8 \pm \sqrt{12}}{2}$$

---

Things to Remember:

# Lesson Practice

## A1.8.14

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

1. The quadratic formula is derived by solving  $ax^2 + bx + c = 0$  by . . .
- A. Factoring                      B. Completing the square                      C. Graphing                      D. Elimination

2. The quadratic formula can be used to find the solutions to any quadratic equation in the form  $ax^2 + bx + c = 0$ .

### Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Which equation represents the solutions to

$$2x^2 - x + 13 = 0?$$

- A.  $x = \frac{1 \pm \sqrt{(-1)^2 - 4(2)(13)}}{2(2)}$     B.  $x = \frac{-2 \pm \sqrt{(2)^2 - 4(13)(-1)}}{2(13)}$     C.  $x = \frac{-1 \pm \sqrt{(-1)^2 - 4(2)(13)}}{2(-1)}$

**Problems 3–5:** The quadratic equation  $x^2 + 7x + 10 = 0$  is in the form  $ax^2 + bx + c = 0$ .

3. What are the values of  $a$ ,  $b$ , and  $c$ ?

$$a =$$

$$b =$$

$$c =$$

4. Substitute the values of  $a$ ,  $b$ , and  $c$  into the quadratic formula. (You do not need to perform any operations.)

5. Explain how the expression you wrote is related to solving  $x^2 + 7x + 10 = 0$  by completing the square.

6. Part of the quadratic formula is  $\pm \sqrt{b^2 - 4ac}$ . What value must this equal to have exactly one solution? Explain your thinking.

# Lesson Practice

## A1.8.14

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

7. Kiri is using a quadratic equation to figure out the quadratic formula. Here are her first few steps.

Why did Kiri add  $\left(\frac{b}{2a}\right)^2$  in the bottom row?

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

### Spiral Review

**Problems 8–10:** Rewrite each equation in the form  $(x + \underline{\hspace{1cm}})^2 = \underline{\hspace{1cm}}$ .

An example has been done for you.

Example:

$$x^2 + 10x = 4$$

$$(x + 5)^2 = 29$$

8.  $x^2 + 6x = -2$

9.  $x^2 + 12x + 3 = -7$

10.  $x^2 - 32 = -20x$

**Problems 11–13:** Solve each equation for  $x$ .

11.  $3x^2 - 5 = 0$

12.  $-7x^2 + 2 = 0$

13.  $ax^2 + c = 0$

### Reflection

1. Star a problem you're still feeling confused about.
2. Use this space to ask a question or share something you're proud of.

## Lesson Summary

You can solve *any* quadratic equation written in standard form,  $ax^2 + bx + c = 0$ , using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here is an example of using the quadratic formula to solve the equation  $2x^2 - 9x + 6 = 0$ .

**Work**

$$a = 2, b = -9, c = 6$$

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(2)(6)}}{2(2)}$$

$$x = \frac{9 \pm \sqrt{33}}{4}$$

$$x = \frac{9 + \sqrt{33}}{4} \text{ and } x = \frac{9 - \sqrt{33}}{4}$$

**Explanation**

Identify the  $a$ -,  $b$ -, and  $c$ -values from the standard form quadratic equation.

Substitute the  $a$ -,  $b$ -, and  $c$ -values into the quadratic formula.

Use order of operations to simplify the expression.

Write the solutions.

---

Things to Remember:

# Lesson Practice

## A1.8.15

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

**Problems 1–2:** Each equation is in the form  $ax^2 + bx + c = 0$ . Determine the values of  $a$ ,  $b$ , and  $c$ .

1.  $x^2 - 5x + 9 = 0$

$a = \dots$   $b = \dots$   $c = \dots$

2.  $-x^2 + 8 = 0$

$a = \dots$   $b = \dots$   $c = \dots$

**Problems 3–5:** Determine the exact solution(s) of each equation.

3.  $2x^2 - 7x = 15$

4.  $2x^2 + 5x - 1 = 0$

5.  $4x^2 = 5$

6. Santiago determined that the solutions to  $3x^2 - 6x - 9 = 0$  are  $x = 3$  and  $x = -1$ . Is Santiago correct? Explain your thinking.

7. Write a quadratic equation that would be simpler to solve *without* using the quadratic equation.

# Lesson Practice

A1.8.15

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

8. Write a quadratic function with the following  $x$ -intercepts.

$$\left(\frac{2 \pm \sqrt{40}}{6}, 0\right)$$

## Spiral Review

**Problems 9–10:** Here is the function  $f(x) = (x + 1)(x + 5)$ .

9. What are the coordinates of the  $x$ -intercepts?

10. What are the coordinates of the vertex? Explain your thinking.

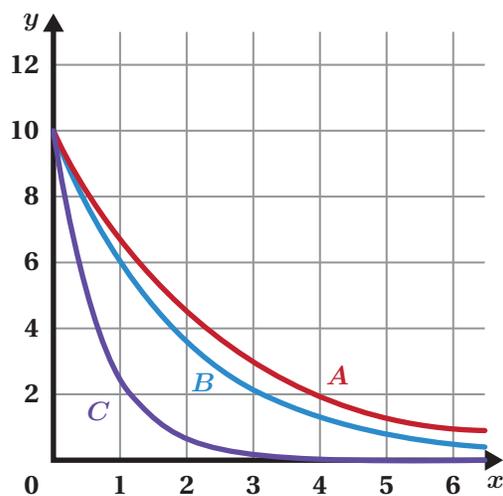
11. Here are the graphs of three equations.

Match each graph with its equation.

.....  $y = 10\left(\frac{2}{3}\right)^x$

.....  $y = 10\left(\frac{1}{4}\right)^x$

.....  $y = 10\left(\frac{3}{5}\right)^x$



## Reflection

1. Circle the problem you feel least confident about.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

You can use quadratic functions to represent the paths of objects that are launched in the air, such as a stomp rocket. You can use functions to answer questions about the launch by:

1. Substituting the given information into the equation.
2. Solving for the missing variable using any strategy (e.g., the quadratic formula or any other available strategy).
3. Interpreting if the solution(s) make sense in the situation.

The function  $h(t) = -2.5t^2 + 6t + 8$  represents the height, in meters, of a stomp rocket  $t$  seconds after it has been launched.

When will the rocket touch the ground?	When will the rocket be at a height of 10 meters?	Will the rocket reach a height of 15 meters?
<p>Given information: <math>h(t) = 0</math></p> $0 = -2.5t^2 + 6t + 8$ <p>Using the quadratic formula, I get the following solutions:</p> $t = -0.954 \text{ and } t = 3.354$ <p>Going back in time does not make sense in this situation, so the only answer is 3.354 seconds.</p>	<p>Given information: <math>h(t) = 10</math></p> $10 = -2.5t^2 + 6t + 8$ $0 = -2.5t^2 + 6t - 2$ <p>Using the quadratic formula, I get the following solutions:</p> $t = 0.4 \text{ and } t = 2$ <p>The rocket will be at a height of 10 meters at both .4 seconds and 2 seconds.</p>	<p>Given information: <math>h(t) = 15</math></p> $15 = -2.5t^2 + 6t + 8$ $0 = -2.5t^2 + 6t - 7$ <p>Using the quadratic formula, I get no solution:</p> $t = \frac{-6 \pm \sqrt{-34}}{-5}$ <p>This means the rocket will never reach a height of 15 feet.</p>

Things to Remember:

# Lesson Practice

## A1.8.16

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

**Problems 1–3:** The function  $h(t) = -75t^2 + 60t$  models the height, in inches, of a jumping frog, where  $t$  is the number of seconds after it jumped.

1. Solve the equation  $-75t^2 + 60t = 0$ .
2. What do the solutions tell us about the jumping frog?
3. When does the frog reach its maximum height of 12 inches?

**Problems 4–5:** The function  $f(t) = 4 + 12t - 16t^2$  models the height of a tennis ball, in feet,  $t$  seconds after it was hit.

4. Select *all* of the solutions to the equation  $0 = 4 + 12t - 16t^2$ .

A.  $-\frac{1}{4}$        B.  $\frac{1}{4}$        C. 4       D. 1       E. -1

5. How many seconds until the tennis ball hits the ground? Explain how you know.

**Problems 6–8:** Katie is planning to go skydiving. She writes the function  $h(t) = -16t^2 + 13500$  to represent her height, in feet,  $t$  seconds after jumping out of the airplane.

6. According to Katie's function, how high is the airplane when she jumps?
7. It's recommended that skydivers open their parachutes at 5000 feet. Use  $h(t)$  to approximate how many seconds after jumping Katie should open her parachute.
8. When Katie actually jumps, do you think she will reach 5000 feet in less time, more time, or exactly the amount of time you approximated? Explain your thinking.

# Lesson Practice

A1.8.16

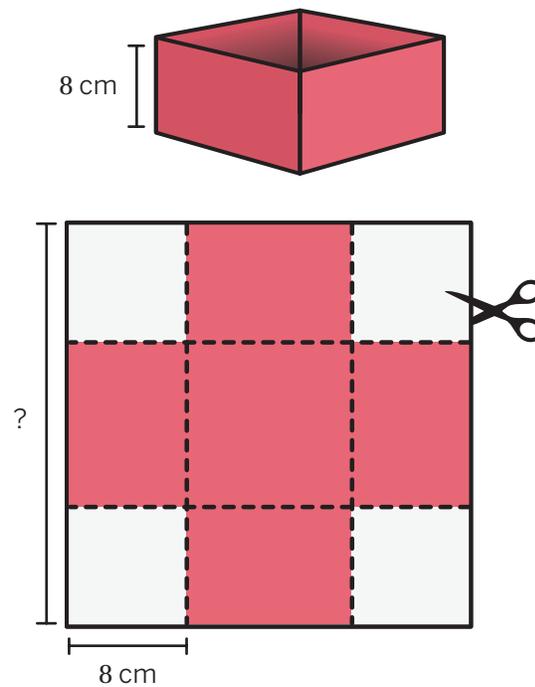
Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

9. A company wants to make a square box with no top. The requirements are:

- It must be 8 cm tall.
- Its volume must be 1,000 cubic cm.

The boxes are made by cutting four corners from a square piece of cardboard and folding the flaps up.

What should the length of the starting square be? Show or explain your thinking.



## Spiral Review

10. Select *all* the irrational numbers.

A.  $\sqrt{13}$

B.  $-\frac{53}{11}$

C.  $-\sqrt{17}$

D.  $-\sqrt{\frac{28}{7}}$

E.  $\sqrt{81}$

F.  $(-\sqrt{8})^2$

11. Write an equation of each type that starts with  $x^2 + 8x = \dots$

No Solutions	$x^2 + 8x = \dots$
One Solution	$x^2 + 8x = \dots$
Two Solutions	$x^2 + 8x = \dots$

## Reflection

1. Put a star next to a problem you want to understand better.
2. Use this space to ask a question or share something you're proud of.

Lesson Summary

Rational numbers are numbers that you can write as a fraction with an integer numerator and denominator. Irrational numbers are numbers that are not rational, which means you cannot write them as a fraction with an integer numerator and denominator.

Here are some properties of sums and products of rational and irrational numbers.

	Sums		Products	
Properties	The sum of two rational numbers is always rational.	The sum of a rational number and an irrational number is always irrational.	The product of two rational numbers is always rational.	The product of a nonzero rational number and an irrational number is always irrational.
Examples	$\frac{1}{2} + 3$ $8 + \sqrt{25}$	$\sqrt{7} + 2$ $\frac{1}{2} + \sqrt{44}$	$\frac{2}{3} \cdot 0.\bar{1}$ $\frac{4}{5} \cdot \left(-\frac{7}{11}\right)$	$\pi \cdot \frac{1}{3}$ $2 \cdot \sqrt{5}$

Things to Remember:

# Lesson Practice

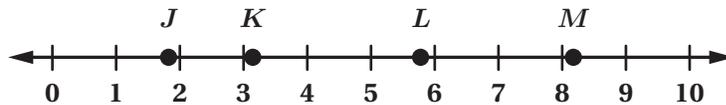
## A1.8.17

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

1. Select *all* the numbers that are rational.

A.  $-2$        B.  $\frac{1}{3}$        C.  $\sqrt{24}$        D.  $\sqrt{25}$        E.  $\sqrt{-9}$

2. Here is a number line with four points on it. Match each point to the appropriate expression.



$\sqrt{10}$  \_\_\_\_\_       $\sqrt{34}$  \_\_\_\_\_       $5 + \sqrt{10}$  \_\_\_\_\_       $5 - \sqrt{10}$  \_\_\_\_\_

3. Select *all* the expressions that are rational.

A.  $3 + \frac{8}{7}$        B.  $\sqrt{2} \cdot 9$        C.  $\frac{8}{3} \cdot \sqrt{7}$   
 D.  $\sqrt{16} - \sqrt{25}$        E.  $\sqrt{-9} + 1.\bar{6}$

4. Here are the solutions to some quadratic equations. Decide if the solutions are rational or irrational.

<b>a</b>	$-2 \pm \sqrt{9}$	Rational	Irrational
<b>b</b>	$3 \pm \sqrt{2}$	Rational	Irrational
<b>c</b>	$\frac{1}{2} \pm \frac{3}{2}$	Rational	Irrational
<b>d</b>	$10 \pm 0.3$	Rational	Irrational
<b>e</b>	$\frac{1 \pm \sqrt{8}}{2}$	Rational	Irrational
<b>f</b>	$-7 \pm \sqrt{\frac{4}{9}}$	Rational	Irrational

5. Consider the statement: *An irrational number multiplied by an irrational number always makes an irrational product.* Select *all* the examples that show that this statement is false.

A.  $\sqrt{4} \cdot \sqrt{4}$        B.  $\sqrt{5} \cdot \sqrt{5}$        C.  $\sqrt{3} \cdot \sqrt{5}$   
 D.  $\sqrt{0} \cdot \sqrt{5}$        E.  $\frac{1}{\sqrt{5}} \cdot \sqrt{5}$

# Lesson Practice

A1.8.17

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

6. Mark the statements based on whether they are *always*, *sometimes*, or *never* true.

Statement	Always true	Sometimes true	Never true
a The sum of two rational numbers is rational.			
b The sum of a rational number and an irrational number is rational.			
c The product of two rational numbers is rational.			
d The product of a rational number and an irrational number is irrational.			

7. Using the digits 0 to 9 without repeating, fill in the boxes to create a true equation.

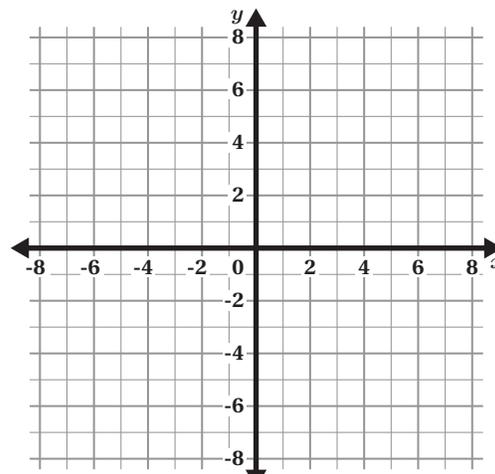
$$\square \sqrt{\square} \cdot \square \sqrt{\square} = \square \square$$

## Spiral Review

Problems 8–9: Use the coordinate plane.

8. Graph  $y = -\frac{1}{2}x - 5$ .

9. Graph  $y = x^2 + 7x + 6$ .



## Reflection

- Put a star next to a problem you're still wondering about.
- Use this space to ask a question or share something you're proud of.