Unit A1.5, Systems of Linear Equations and Inequalities: Notes **Answer Key**

Lessons 1–2: Introduction to Elimination

Summary

A *system of equations* is two or more equations representing constraints on shared variables.

Elimination is a method of solving systems of equations where you add or subtract the equations to produce a new equation with fewer variables.

Here are two systems of equations that have been solved by elimination.

Use the elimination method to solve the final system.

$2x + y = 30$ $-\ (x + y = 23)$ $x + 0 = 7$ $x = 7$ $(7) + y = 23$ $y = 16$	$-2x + y = 9$ $+\ (8x - y = 3)$ $6x + 0 = 12$ $x = 2$ $-2(2) + y = 9$ $y = 13$	$x + 2y = 10$ $-\ (x - y = 7)$ $0 + 3y = 3$ $y = 1$ $x - (1) = 7$ $x = 8$

Describe how to use elimination to solve a system of equations.

***Responses vary.* If two variables share the same coefficient, you can eliminate them by subtraction. If the coefficient is the same number but they have opposite signs, you can add the equations to eliminate the variable.**

Things I Want to Remember

Unit A1.5, Systems of Linear Equations and Inequalities: Notes

Answer Key

Lessons 1–2: Introduction to Elimination

Try This!

Circle whether adding, subtracting, or either can be used to eliminate a variable.

1.1 $3x + 5y = 20$
$x + 5y = 4$

Adding [✓ Subtracting] Either

1.2 $-2x + 3y = 12$
$2x - 6y = 3$

[✓ Adding] Subtracting Either

1.3 $-9x + y = 5$
$9x + y = 3$

Adding Subtracting [✓ Either]

2. Solve this system of equations:

$$6x + 2y = 4$$
$$2x + 2y = 12$$

$x = -2$ $y = 8$

☐ I can solve shape puzzles.
☐ I can solve systems of equations by adding or subtracting the equations to eliminate a variable.
☐ I recognize that adding or subtracting the equations in a system creates a new equation that shares a solution with that system.

Unit A1.5, Systems of Linear Equations and Inequalities: Notes **Answer Key**

Lesson 3: Elimination Using Equivalent Equations

Summary

Writing equivalent equations can be useful when using elimination to solve systems of equations.

Here is one student's first step. Why do you think they got stuck?

$$\begin{array}{r} 9x - 4y = 2 \\ -(3x + y = 10) \\ \hline 6x - 5y = -8 \end{array}$$

Here is how a different student eliminated x.

$$\begin{array}{r} 9x - 4y = 2 \\ -3\,(3x + y = 10) \end{array}$$

$$\begin{array}{r} 9x - 4y = 2 \\ +\ \ -9x - 3y = -30 \\ \hline 0 - 7y = -28 \\ \boxed{y = 4} \end{array}$$

$$\begin{array}{r} 3x + (4) = 10 \\ 3x = 6 \\ \boxed{x = 2} \end{array}$$

Why do you think they multiplied the second equation by −3?

Responses vary. **So that the coefficient of the x in both equations was the same just with opposite signs, so when they are added, they eliminated x.**

Try eliminating y instead.

$$\begin{array}{r} 9x - 4y = 2 \\ 4\,(3x + y = 10) \end{array}$$

$$\begin{array}{r} 9x - 4y = 2 \\ +\ \ 12x + 4y = 40 \\ \hline 21x + 0 = 42 \\ \boxed{x = 2} \end{array}$$

$$\begin{array}{r} 9(2) - 4y = 2 \\ 18 - 4y = 2 \\ -4y = -16 \\ \boxed{y = 4} \end{array}$$

How did you decide what to multiply by?

Responses vary. **To match the -4 of the first equation, I multiplied by 4, so that when I added both equations together, the y would be eliminated.**

Things I Want to Remember

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Unit A1.5, Systems of Linear Equations and Inequalities: Notes **Answer Key**

Lesson 3: Elimination Using Equivalent Equations

Try This!

Show the first step that you would use to solve each system using elimination.

1.1 *

$$2x - y = 5$$
$$2\,(-x + 3y = 5)$$

$$2x - y = 5$$
$$+\ -2x + 6y = 10$$

1.2 *

$$-3x + 12y = 15$$
$$-4\,(4x + 3y = -1)$$

$$-3x + 12y = 15$$
$$+\ -16x - 12y = 4$$

Responses vary.

Solve this system of equations using elimination:

2.1 $4x + 3y = 3$

$8x + y = 1$

$x = 0$ $y = 1$

☐ I can solve a system of equations using elimination.
☐ I recognize that multiplying an equation by a number creates a new system with the same solution.

Unit A1.5, Systems of Linear Equations and Inequalities: Notes **Answer Key**

Lesson 4: Solving Systems by Substitution

Summary

Substitution is a method of solving systems of equations where you replace a variable with an expression it is equal to.

Here is a complete example of the substitution method and an example with the first step shown.

Complete the second example using substitution.

$$y = \boxed{-4x + 6} \qquad y = 3x - 15$$

$$-4x + 6 = 3x - 15$$

$$-7x = -21$$

$$\boxed{x = 3}$$

$$y = 3(3) - 15$$

$$\boxed{y = -6}$$

$$y = \boxed{2x - 5} \qquad -3x - 2y = 3$$

$$-3x - 2(2x - 5) = 3$$

$$-3x - 4x + 10 = 3$$

$$-7x + 10 = 3$$

$$-7x = -7$$

$$\boxed{x = 1}$$

$$y = 2(1) - 5$$

$$\boxed{y = -3}$$

What is important to remember when using substitution to solve a system of equations?

Responses vary. **You have to remember to distribute the coefficient to every term in the expression.**

When might using substitution be a useful strategy for solving a system?

Responses vary. **When one variable is by itself on one side of the equation.**

Things I Want to Remember

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Unit A1.5, Systems of Linear Equations and Inequalities: Notes **Answer Key**

Lesson 4: Solving Systems by Substitution

Try This!

Show the first step you would complete to solve each system using substitution.

1.1 * $y = \boxed{-3x - 10}$ $x + 2y = 10$

x + 2(-3x - 10) = 10

1.2 * $y = \boxed{-x + 3}$ $y = 2x - 12$

-x + 3 = 2x - 12

Responses vary.

Solve the system of equations using substitution.

2.1 $y = 2x - 9$ $y = -x + 9$

$$2x - 9 = -x + 9$$
$$3x = 18$$
$$x = 6$$

$$y = 2(6) - 9$$
$$y = 3$$

$x = 6$ $y = 3$

2.2 $b = 3a - 8$ $2a + b = 2$

$$2a + (3a - 8) = 2$$
$$2a + 3a - 8 = 2$$
$$5a = 10$$
$$a = 2$$

$$b = 3(2) - 8$$
$$b = -2$$

$a = 2$ $b = -2$

☐ I can solve a system of equations using substitution.

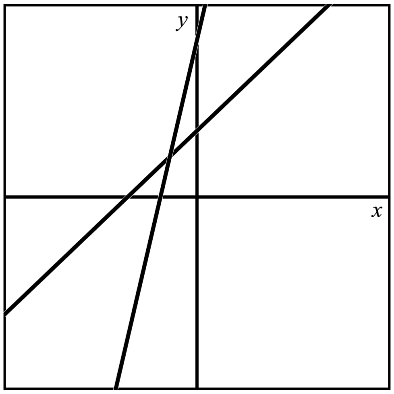
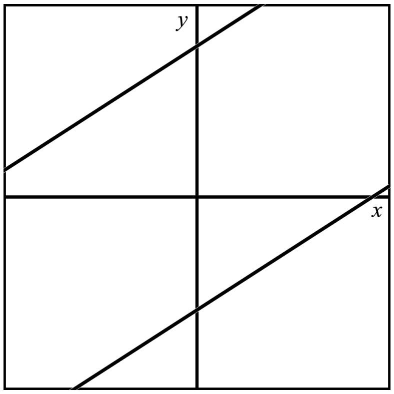
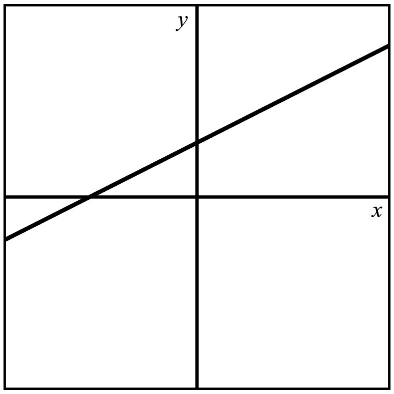
desmos Science Mom Lesson 49

Unit A1.5, Systems of Linear Equations and Inequalities: Notes **Answer Key**

Lesson 5: Graphing Systems of Linear Equations

Summary

A system of equations can have one solution, no solutions, or infinite solutions.

Number of Solutions	**One Solution**	**No Solutions**	**Infinite Solutions**
Graph			
How can you tell from a graph?	***Responses vary.*** **If two lines intersect.**	***Responses vary.*** **If two lines are parallel.**	***Responses vary.*** **If two lines are the same or directly on top of each other.**
Equation	$y = 2x + \frac{1}{4}$ $y = 4x + \frac{1}{4}$	$y = \frac{2}{3}x + 10$ $y = \frac{2}{3}x - 7$	$y = \frac{1}{2}x + 3$ $2y = x + 6$
How can you tell from the equations?	***Responses vary.*** **If two lines have different slopes.**	***Responses vary.*** **If two lines have the same slope and different y-intercepts.**	***Responses vary.*** **If both equations are equivalent.**

Things I Want to Remember

Lesson 5: Graphing Systems of Linear Equations

Try This!

One equation in a system is $y = 7x - 12$. Write another equation to create a system with:

1.1 No solutions. $y = 7x + 5$ *

1.2 One solution. $y = -x + 10$ *

1.3 Infinite solutions. $2y = 14x - 24$ *

**** Responses vary.***

Determine if each system has no solutions, one solution, or infinite solutions. Circle your choice.

2.1 $y = 2x + 3$, $3y = 6x + 9$ No solutions One solution [✓ Infinite solutions]

2.2 $y = 2x + \frac{1}{2}$, $y = \frac{1}{2}x + 2$ No solutions [✓ One solution] Infinite solutions

2.3 $y = 3x + 6$, $y = 3x - 6$ [✓ No solutions] One solution Infinite solutions

☐ I can solve a system of linear equations using a graph.
☐ I can determine if a system of linear equations has no solutions, one solution, or infinite solutions by looking at a graph or at the equations.

Unit A1.5, Systems of Linear Equations and Inequalities: Notes **Answer Key**

Lesson 8: Strategically Solving Systems of Equations

Summary

Elimination or substitution can each be used to solve systems of linear equations.

$$\begin{array}{r} 3x + 4y = 3 \\ + \; -3x + 3y = 18 \\ \hline 7y = 21 \\ y = 3 \end{array}$$

$3x + 4(3) = 3$

$x = -3$ $\boxed{(3, -3)}$

$y = \boxed{3x + 6}$

$2x + 2y = 20$

$2x + 2y = 20$

$2x + 2(3x+6) = 20$

$2x + 6x + 12 = 20$

$8x + 12 = 20$

$8x = 8$

$x = 1$

$y = 3(1) + 6$

$y = 9$

$\boxed{(1, 9)}$

Why do you think elimination was used here?

***Responses vary.* Both equations are in standard form, so I would use elimination.**

Why do you think substitution was used here?

***Responses vary.* One of the questions has an isolated variable, so I would use substitution.**

Choose either elimination or substitution to solve this system of equations.
Solve the system and explain your choice.

$$y = 2x - 4$$
$$y = 0.5x + 5$$

(6, 8)

Responses vary.

- **I used substitution because the y variables were both isolated.**
- **I used elimination because y could be eliminated by subtraction.**

Things I Want to Remember

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Unit A1.5, Systems of Linear Equations and Inequalities: Notes **Answer Key**

Lesson 8: Strategically Solving Systems of Linear Equations

Try This!

Circle which method (elimination, substitution, or either) you would use to solve each system.

1.1	$y = 3x - 10$ $2x - 3y = 16$	Elimination	[✓ Substitution]	Either
1.2	$y = 4x - 10$ $y = x + 2$	[✓ Elimination]	[✓ Substitution]	[✓ Either]
1.3	$9x + y = 25$ $3x + 2y = 5$	[✓ Elimination]	Substitution	Either
1.4	$y = -2x + 3$ $x + 2y = 9$	Elimination	[✓ Substitution]	Either

Pick one system and solve using the method of your choice.

Solution:

1.1 $(2, -4)$

1.2 $(4, 6)$

1.3 $(3, -2)$

1.4 $(-1, 5)$

☐ I can solve systems of equations using substitution and elimination.

Unit A1.5, Systems of Linear Equations and Inequalities: Notes **Answer Key**

Lessons 9–10: Solutions to Systems of Inequalities

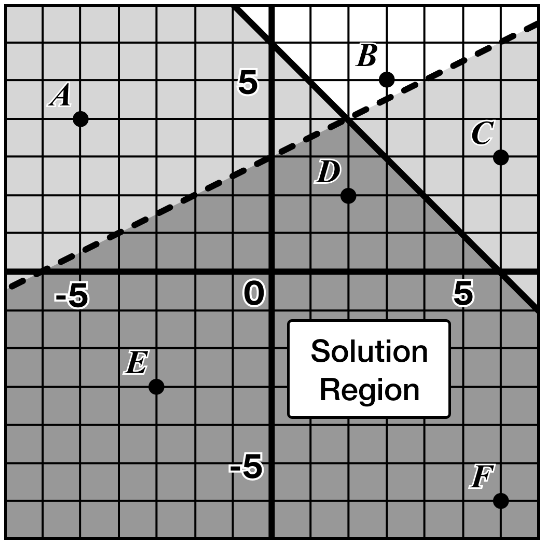
Summary

The *solutions to a system of inequalities* are all the points that make both inequalities true. The solutions can be seen in the region where the graphs overlap, called the *solution region*.

$$y \leq -x + 6$$
$$-2x + 4y < 12$$

Which point(s) are in the solution region?

D, E, and F are in the solution region.

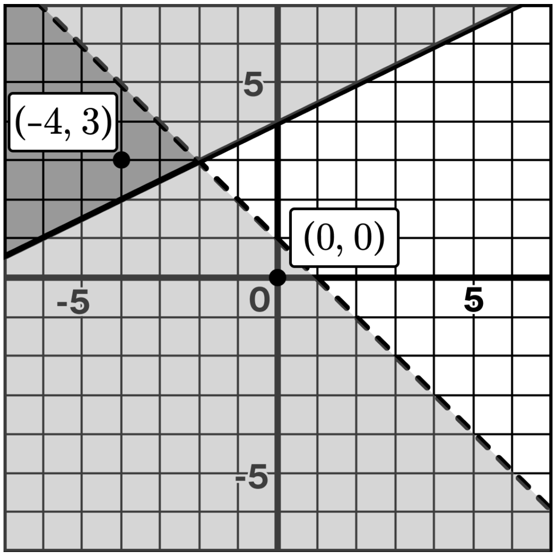


Testing points can be helpful to determine the solution region. The point $(0, 0)$ was tested in the system of inequalities shown.

Test the point $(-4, 3)$ and mark the solution region on the graph.

$$x + y < 1$$
$$y \geq \frac{1}{2}x + 4$$

$(0, 0)$	$(-4, 3)$
$x + y < 1$ $0 + 0 < 1$ $0 < 1$ True ✓ $y \geq \frac{1}{2}x + 4$ $0 \geq \frac{1}{2}(0) + 4$ $0 \geq 4$ False ✗ Not in the solution region.	$x + y < 1$ $-4 + 3 < 1$ $-1 < 1$ **True ✓** $y \geq \frac{1}{2}x + 4$ $3 \geq \frac{1}{2}(-4) + 4$ $0 \geq 2$ **True ✓** $(-4, 3)$ **is in the solution region.**



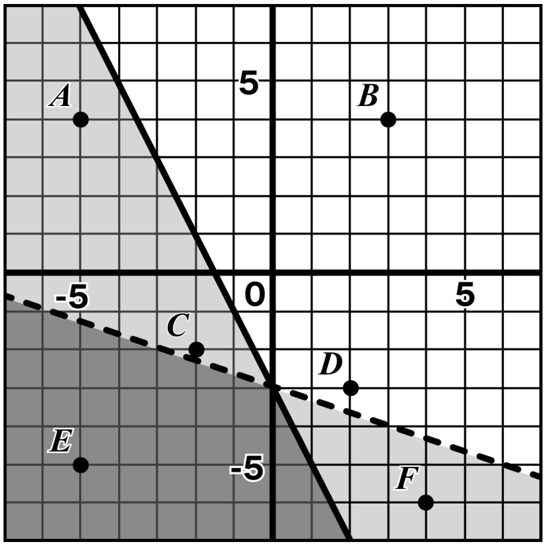
Things I Want to Remember

Lessons 9–10: Solutions to Systems of Inequalities

Try This!

The graph for this system is shown.

$$-2x - y \geq 3$$
$$y < -\frac{1}{3}x - 3$$



Identify a point that is:

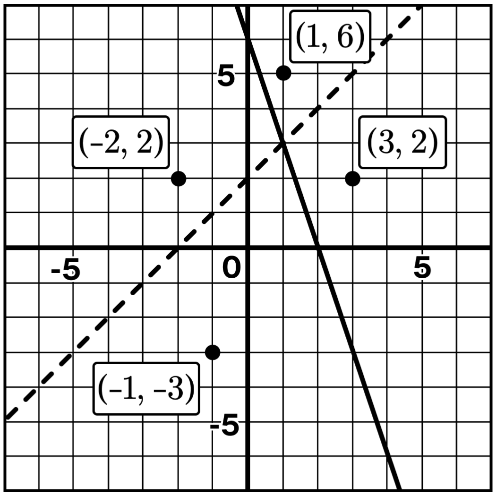
1.1 In the solution region: E

1.2 A solution to **no** inequalities: B, D

1.3 A solution to only **one** inequality: A, C, F

Determine if each point is in the solution region.

$$3x + y \geq 6$$
$$y > x + 2$$



2.1 Is $(3, 2)$ in the solution region?

Yes [✓ No]

2.2 Is $(-1, -3)$ in the solution region?

Yes [✓ No]

2.3 Is $(1, 6)$ in the solution region?

[✓ Yes] No

2.4 Is $(-2, 2)$ in the solution region?

Yes [✓ No]

☐ I can use a graph to determine if a point is a solution to a system of inequalities.
☐ I can connect coordinate pairs to constraints in a situation.
☐ I can describe that solutions to a system of inequalities are all points that make both inequalities true and the region where the graphs overlap.
☐ I can determine the solution region of a system of inequalities given the boundary lines.

Unit A1.6, Exponential Functions: Notes Name ______________________

Lessons 1–3: Connecting Representations of Linear and Exponential Functions

Summary

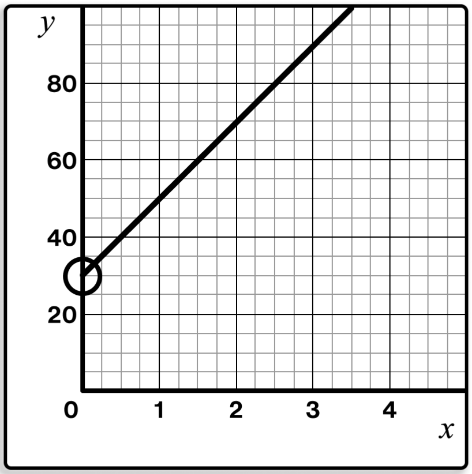
Here are two examples of *linear* and *exponential* relationships. Let's compare the two types.
Fill in the missing information in each example.

Linear

x	y
0	30
1	50
2	70
3	90

$y = 30 + 20x$

At first there are 30 globs. 20 more globs are added each month.

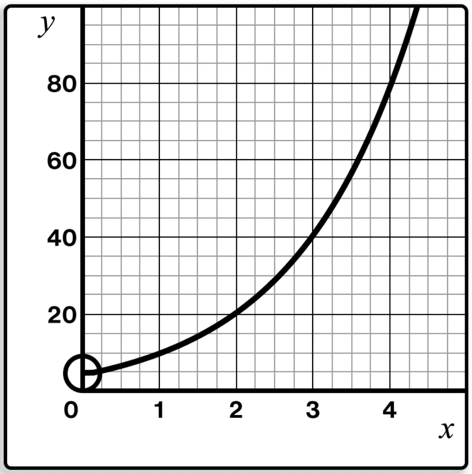


Exponential

x	y
0	5
1	10
2	20
3	40

$y = 5 \cdot (2)^x$

At first there are 5 globs. Then they double each month.



Circle where you see the initial value in each representation.

Describe or show where you see either the rate of change or growth factor in the table and equation.

Responses vary. **For the linear example, the rate of change is 20 in the equation, and the outputs increase by 20 as the input increases by 1. For the exponential example, the growth factor is 2 in the equation, and the outputs in table grow by a factor of 2 as the input increases by 1.**

Will the linear growth or exponential growth produce more globs after 10 months?
Show or explain your thinking.

Responses vary. **Exponential growth because $5 \cdot (2)^{10} > 30 + 20(10)$.**

Things I Want to Remember

Unit A1.6, Exponential Functions: Notes Name ____________________

Lessons 1–3: Connecting Representations of Linear and Exponential Functions

Try This!

Determine if each table shows a linear relationship, exponential relationship, or something else. Circle your choice.

1.1

x	y
3	5
4	15
5	45

Linear / [✓ Exponential] / Something else

1.2

x	y
1	1
2	11
3	111

Linear / Exponential / [✓ Something else]

1.3

x	y
2	25
3	50
4	75

[✓ Linear] / Exponential / Something else

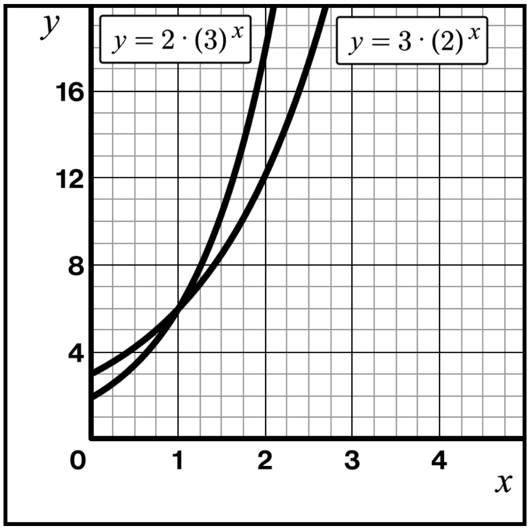
2.1 Select one equation and sketch its graph. (Circle it.)

$f(x) = 2 \cdot (3)^x$

x	y
0	2
1	6
2	18

$g(x) = 3 \cdot (2)^x$

x	y
0	3
1	6
2	12



2.2 Is $f(6) < g(6)$ true? Explain how you know.

False. ***Explanations vary.*** **Because** $f(6) = 2 \cdot (3)^6 = 1458$ **is greater than** $g(6) = 3 \cdot (2)^6 = 192$**.**

☐ I understand that linear patterns have a constant rate of change and exponential patterns have a constant growth factor.
☐ I can determine whether a pattern represents a linear or exponential relationship.
☐ I can use graphs and equations of linear and exponential functions to make predictions.
☐ I can describe exponential functions using their key features and statements written in function notation.
☐ I can create equations, tables, and graphs for linear and exponential functions.

Unit A1.6, Exponential Functions: Notes Name ______________________________

Lesson 4: Introducing Simple and Compound Interest

Summary

Here are examples of *simple interest* and *compound interest*. Let's compare the two.

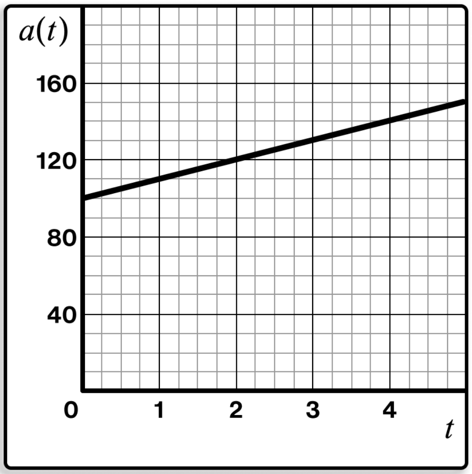
Fill in the missing information (table, graph, description) for each type of interest account.

Account A: Simple Interest

t	$a(t)$
0	100
1	110
2	120
3	130

$$a(t) = 100 + 10t$$

Account A starts with $100 and $10 is added each year.

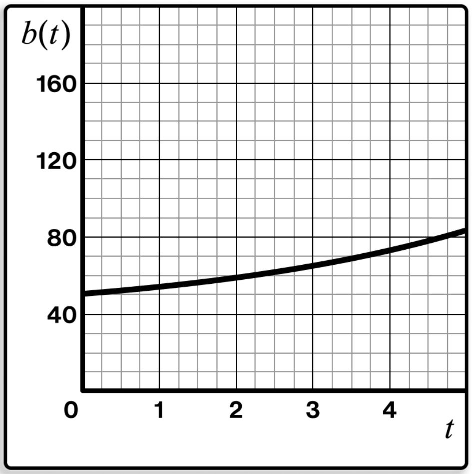


Account B: Compound Interest

t	$b(t)$
0	50
1	55
2	60.5
3	66.55

$$b(t) = 50 \cdot (1.1)^t$$

Account B starts with $50 and increases by 10% each year.



Use a calculator to determine how long it would take each account to reach $200.

Account A would take 10 years. Account B would take 14.55 years.

How do simple interest and compound interest relate to linear and exponential relationships?

***Responses vary.* Simple interest is a linear relationship because it changes by a constant difference. Compound interest is an exponential relationship because it changes by a constant growth factor.**

Things I Want to Remember

Unit A1.6, Exponential Functions: Notes Name ______________________________

Lesson 4: Introducing Simple and Compound Interest

Try This!

Adah invests \$75 in an account that earns 5% simple interest yearly. The function $a(t) = 75 + 3.75t$ models the account balance after t years.	Jamir invests \$75 in an account that earns 3% compound interest yearly. The function $j(t) = 75(1.03)^t$ models the account balance after t years.

Determine the balance of each account at:

1.1 10 years Adah: ____\$112.50_____ Jamir: ____\$100.79_____

1.2 12 years Adah: ______\$120_______ Jamir: ____\$106.93_____

Determine how many years it will take for each account to reach a balance of:

1.3 \$100 Adah: ____6.67 **years**___ Jamir: ____9.73 **years**___

1.4 \$200 Adah: ___33.33 **years**___ Jamir: ____33.18 **years**___

1.5 Which person's account would you prefer to have? Justify your choice.

Responses vary.

- **Since I plan to have my money invested for a short amount of time, about 5 years, I would prefer Adah's account because it would have a higher balance.**
- **Since I plan to only take out my money once I retire in 35 years, I would prefer Jamir's account because it would have a higher balance.**

☐ I can make connections between simple and compound interests, and linear and exponential functions.
☐ I can use a graphing calculator to solve problems about linear and exponential functions.

Unit A1.6, Exponential Functions: Notes Name ______________________________

Lesson 5: Evaluating Exponential Functions

Summary

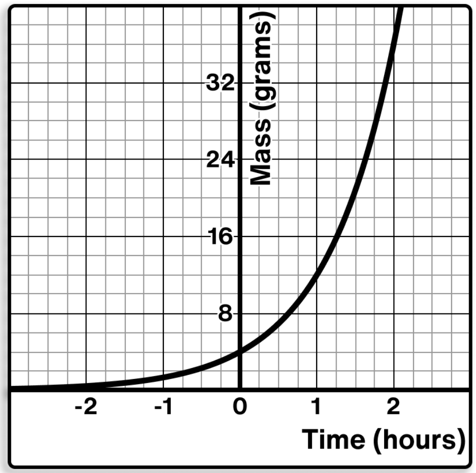
Exponential functions can be used to model exponential behavior.

Let’s compare the two examples. Fill in the missing parts of the representations.

t	$m(t)$
-2	$\frac{4}{9}$
-1	$\frac{4}{3}$
0	4
1	12
2	36

Carlos bought a new toy fish that triples every hour when placed in water.

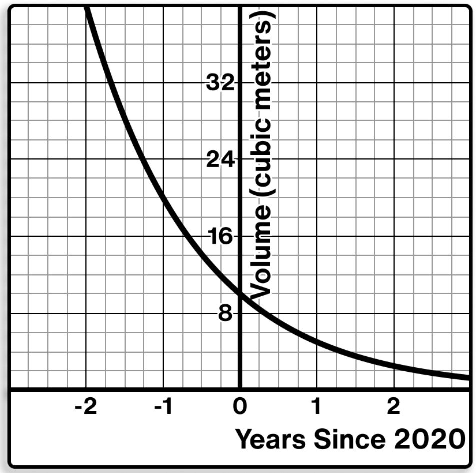
$$m(t) = 4 \cdot (3)^t$$



t	$v(t)$
-2	40
-1	20
0	10
1	5
2	2.5

In 2020, a coral reef had a volume of 10 cubic meters and decreased by a factor of $\frac{1}{2}$.

$$v(t) = 10 \cdot \left(\frac{1}{2}\right)^t$$



Describe how you evaluate exponential functions for negative inputs and zero.

Responses vary. **Any value to the zero power is one ($n^0 = 1$). For negative inputs, rewrite the growth factor (for example, $(\frac{1}{2})^{-1}$ can be rewritten as 2), then multiply.**

Compare and contrast the domains of $m(t)$ and $v(t)$.

Responses vary. **For negative inputs, such as $v(-1)$, it would not make sense to calculate the mass of the toy fish before it was placed in the water. But $m(-1)$ would represent the volume of the coral reef 1 year before 2020, which is a value that makes sense to calculate.**

Things I Want to Remember

Unit A1.6, Exponential Functions: Notes Name ______________________

Lesson 5: Evaluating Exponential Functions

Try This!

Marc's video channel had 18 subscribers in 2020. The function $m(x)$ represents his total subscriber count x years after 2020.

$$m(x) = 18(\frac{3}{2})^x$$

1.1 Complete the table.

x	$m(x)$
–2	8
–1	12
0	18
1	27

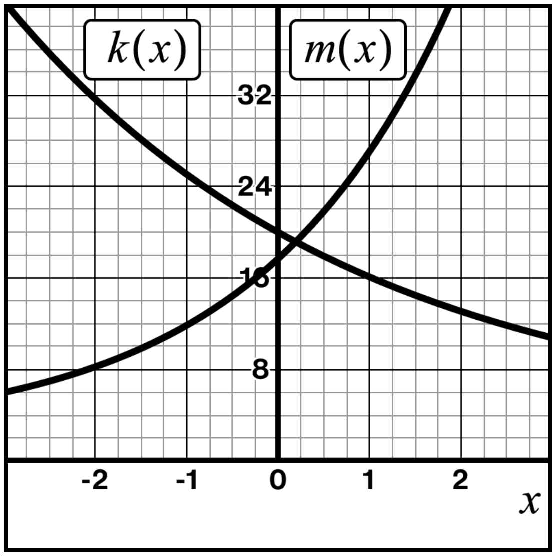
Kiana drank a beverage that had 20 milligrams of caffeine. The function $k(x)$ represents the total caffeine in Kaina's system x hours after she consumed the drink.

$$k(x) = 20(\frac{4}{5})^x$$

1.2 Complete the table.

x	$k(x)$
–2	31.25
–1	25
0	20
1	16

1.3 Sketch the graph of either $m(x)$ or $k(x)$.



1.4 Explain whether $m(-1)$ and $k(-1)$ make sense in their situation.

Responses vary. **$m(-1)$ represents the total subscriber count in the year 2019, which makes sense.**

It would not make sense to calculate how much caffeine is in Kaina's system 1 hour before she consumed the drink, which is what $k(-1)$ represents.

☐ I can evaluate exponential functions with positive, zero, and negative inputs.
☐ I can describe the domain of an exponential function in context.

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Unit A1.6, Exponential Functions: Notes Name ______________________________

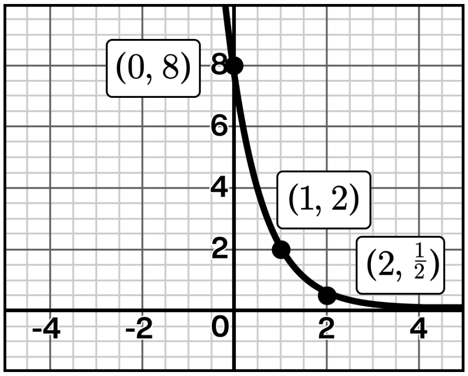
Lesson 6: Writing Equations of Exponential Functions

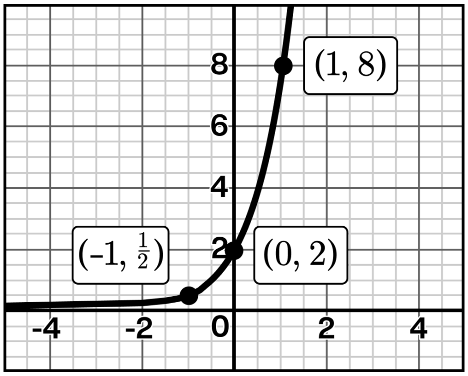
Summary

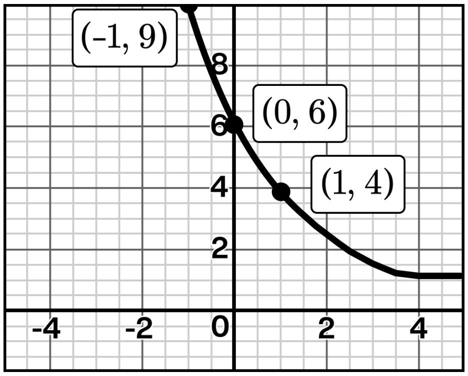
Exponential functions can be written in the form $f(x) = a \cdot b^{x}$, where:

- The a-value is the y-intercept or initial value.
- The b-value is the growth factor.

Write the exponential equation for each graph. Show your thinking.







x	$f(x)$
0	8
1	2
2	$\frac{1}{2}$

x	$g(x)$
−1	$\frac{1}{2}$
0	2
1	8

x	$h(x)$
−1	9
0	6
1	4

$f(x) = 8 \cdot (\frac{1}{4})^{x}$

$g(x) = 2 \cdot (4)^{x}$

$h(x) = 6 \cdot (\frac{2}{3})^{x}$

Describe how to determine the a- and b-value of an exponential function given a graph.

***Responses vary.* The a-value is the y-coordinate of the y-intercept. To find the b-value, identify two coordinates that are 1 x-value apart, then divide the second y-coordinates by the first.**

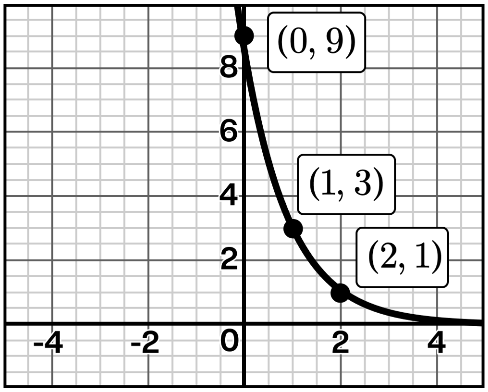
Things I Want to Remember

Lesson 6: Writing Equations of Exponential Functions

Try This!

Select the equation that matches each exponential graph. Use a table if it helps with your thinking.

1.1



A. $y = 9(\frac{2}{3})^x$

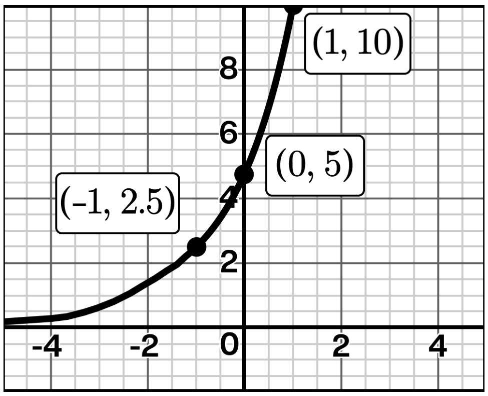
B. $y = 9(\frac{1}{3})^x$

C. $y = 9(3)^x$

D. $y = 9(-3)^x$

x	y

1.2



A. $y = 5(\frac{1}{2})^x$

B. $y = -2.5(2)^x$

C. $y = 2.5(4)^x$

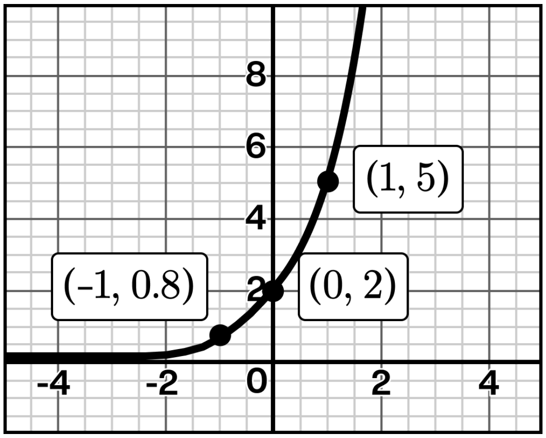
D. $y = 5(2)^x$

x	y

2. Write the exponential equation that represents this graph. Use the table if it helps with your thinking.

x	y

$y = 2(\frac{5}{2})^x$



☐ I can write equations of exponential functions from a graph.

☐ I can describe how changing a and b in $f(x) = a \cdot b^x$ affects its graph.

Unit A1.6, Exponential Functions: Notes Name ______________________________

Lessons 7–9: Percent and Fractional Increase and Decrease

Summary

We can use the growth factor, b, in an exponential equation like $y = a \cdot b^x$ to determine the type of exponential relationship: growth or decay.

- When $b > 1$, this is an example of exponential **growth**.
- When $0 < b < 1$, this is an example of exponential **decay**.

Let's look at a bank account that starts with $\$100$. Complete the matching information for each row of the table. The first row is done for you.

Description	Growth or Decay?	Equation
Doubles	Growth	$y = 100(2)^x$
Increases by 5% every year	**Growth**	$y = 100(1 + 0.05)^x$ (or equivalent)
Keeps 97% every year (or equivalent)	**Decay**	$y = 100(0.97)^x$
Triples every year	**Growth**	$y = 100(3)^x$ (or equivalent)
Retains 63% each year	**Decay**	$y = 100(0.63)^x$ (or equivalent)
Decreases by 3% every year (or equivalent)	**Decay**	$y = 100(1 - 0.03)^x$
Halves every year	**Decay**	$y = 100(0.5)^x$ (or equivalent)

How can you use the description of an exponential situation to write an equation?

Responses vary. **If the description is a percent increase, add the percentage to 100%. If it's a decrease, subtract the percentage from 100%, then convert that to a decimal to use as a growth factor of your exponential equation. The a-value is the initial value.**

Which two situations are equivalent?

Keeping 97% and decreasing by 3% are both decay situations that have equivalent equations $y = 100(0.97)^x$ and $y = 100(1 - 0.03)^x$.

Things I Want to Remember

Lessons 7–9: Percent and Fractional Increase and Decrease

Try This!

Determine which equation matches each situation. Circle your choice.

1.1 The value of a Desmon card collection increases 4% every year. In 2020, the price of the collection was valued at $\$500$.

Use y to represent the cost of the collection and x to represent time in years since 2020.

A. $y = 500(0.4)^x$
B. $y = 500(0.04)^x$
C. $y = 500(1.4)^x$
D. $y = 500(1.04)^x$

1.2 A potted plant receives 24 mL of fertilizer. It will lose 1% of the fertilizer every hour.

Use y to represent the amount of fertilizer left in the potted plant and x to represent time in hours since receiving the fertilizer.

A. $y = 24(0.01)^x$
B. $y = 24(1.01)^x$
C. $y = 24(0.99)^x$
D. $y = 24(0.1)^x$

A laptop battery can currently stay on for 480 minutes. The battery's capacity is decreasing by 8% each year.

2.1 Write a function that represents the battery's capacity, $b(x)$, after x years.

$b(x) = 480(1 - 0.08)^x$ ***(or equivalent)***

2.2 Use the function you wrote to determine how long the laptop battery will stay on after 5 years.

≈ 316 **minutes**

☐ I can write equations of exponential functions from a graph.
☐ I can describe how changing a and b in $f(x) = a \cdot b^x$ affects its graph.
☐ I can compare and write equivalent equations of functions that represent exponential decay.
☐ I can calculate the percent change of an exponential function given a graph.
☐ I can use equations of exponential functions to solve problems in context.
☐ I can interpret what the values of a, b, and k mean for exponentials in the form $y = a \cdot b^x$ and $y = b^x + k$.

desmos Science Mom Lesson 59

Unit A1.6, Exponential Functions: Notes Name ______________________________

Lessons 10–11: Different Compounding Intervals

Summary

Compound interest can be earned in different time intervals, such as daily, monthly, and annually.

Write three equivalent expressions that will calculate the total value of a $1 000 loan with a 7% monthly interest rate after 2 years, with no additional payments.

$1000 \cdot (1.07)^{24}$*	$1000 \cdot (1.07^{12})^{2}$*	$1000 \cdot (2.25219)^{2}$*

Or equivalent.

When you take out a loan from the bank, the annual interest rate may be compounded at different intervals.
Complete the table for the total value of the $1 000 loan for different compounding intervals.

Interest	Owed In	Compounded Monthly	Compounded Quarterly	Compounded Annually
15% annually	5 years	$1000(1 + \frac{0.15}{12})^{12 \cdot 5}$ $\approx \$2\,107.18$	$1000(1 + \frac{0.15}{4})^{4 \cdot 5}$* $\approx \$2\,088.15$	$1000(1 + 0.15)^{5}$* $\approx \$2\,011.36$
9% annually	6 months	$1000(1 + \frac{0.09}{12})^{6}$* $\approx \$1\,045.85$	$1000(1 + \frac{0.09}{4})^{4 \cdot 0.5}$ $\approx \$1\,045.51$	$1000(1 + 0.09)^{0.5}$* $\approx \$1\,044.03$

Responses vary.

How does the compounding interval affect the total value?

***Responses vary.* The more compounding intervals there are, the greater the total value.**

The formula $P(1 + \frac{r}{n})^{nt}$ calculates the total amount in an account with compound interest.

Select **two** variables from the formula and describe what they represent.

***Responses vary.* r: the interest rate. t: amount of time you want to calculate. n: compounding interval size. P: the initial investment or loan.**

Things I Want to Remember

Name ________________________

Lessons 10–11: Different Compounding Intervals

Try This!

For each situation, select **all** the expressions that can be used to calculate the balance after 5 years. Assume that no additional payments, deposits, or withdrawals are made.

1.1 A savings account has a starting balance of $100 and earns 4% annual interest compounded annually.

- ✓ $100(1.04)^{5}$
- ✓ $100(1+0.04)^{5}$
- ☐ $100(1.1699)^{5}$
- ☐ $100(1+0.04^{5})$
- ☐ $100(1.04)^{20}$

1.2 A $100 loan with a monthly interest rate of 8%.

- ☐ $100(1.08)^{5}$
- ☐ $100(1+0.08)^{5}$
- ✓ $100(1.08^{12})^{5}$
- ✓ $100(1.08)^{60}$
- ✓ $100(2.51817)^{5}$

A bank offers different interest rates for their checking accounts.

- Option A: 2% annual interest rate compounded daily.
- Option B: 3% annual interest rate compounded semi-annually.

2. If you deposit $900, which option will give you a greater balance in 4 years? Show your reasoning.

Option A's balance would be $900(1+\frac{0.02}{365})^{4 \cdot 365} \approx \$974.96$.

Option B's balance would be $900(1+\frac{0.03}{2})^{4 \cdot 2} \approx \$1\,013.84$.

So Option B would have a greater balance.

☐ I can write and interpret exponential functions that represent compound interest.
☐ I can use the properties of exponents to interpret equivalent exponential expressions.
☐ I can interpret exponential expressions for accounts that compound at different intervals.
☐ I can calculate balances given an annual interest rate and a compounding interval.

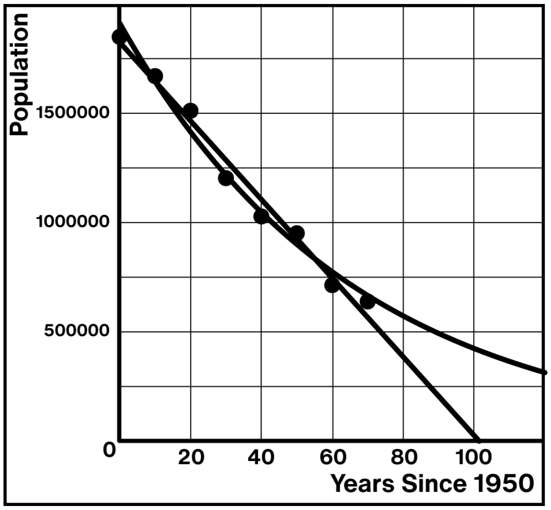
Unit A1.6, Exponential Functions: Notes Name ______________________________

Lessons 13–14: Modeling Exponential Data

Summary

Similar to a line of best fit, a calculator can compute an equation for an exponential curve of best fit.

Kyrie used the population of Detroit from 1950–2020 data to generate a line and curve of best fit.



$y_1 \sim mx_1 + b$

PARAMETERS

$m = -17\,989.3$

$b = 1\,825\,460$

$y_1 \sim a \cdot b^{x_1}$

PARAMETERS

$a = 1\,912\,612$

$b = 0.985051$

Describe how to determine whether to use a line or exponential curve of best fit.

***Responses vary.* Examine the shape of the pattern of the data. If the data does not seem to change by a constant difference, an exponential curve might work better.**

Describe what the slope and growth factor means in each.

Slope of linear model:

This means… **for every 1 year, the population of Detroit is predicted to decrease by about $17\ 989$ people.**

Growth factor of exponential model:

This means…**for every 1 year, the population of Detroit decreases by about 1.56%.**

Use Kyrie's graphs to estimate the population of Detroit in 2040 (90 years after 1950).

Linear model prediction: $\sim 230\ 000$ **people**

Exponential model prediction: $\sim 500\ 000$ **people**

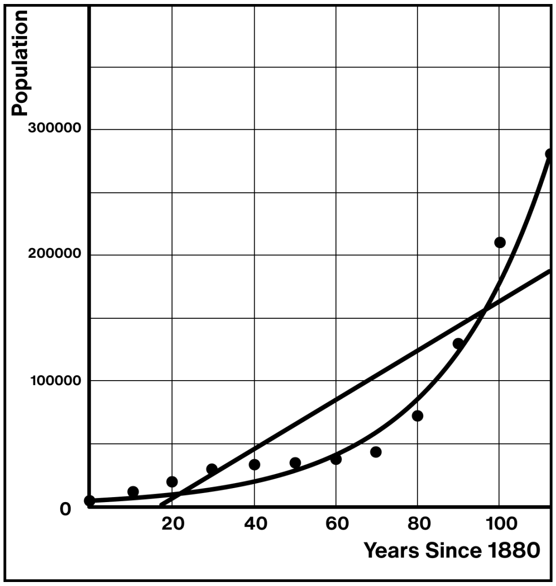
Things I Want to Remember

Lessons 13–14: Modeling Exponential Data

Try This!

Aki was curious about the population changes in Colorado. They generated this line and exponential curve of best fit.

Describe what the slope and growth factor means in each.



$y_1 \sim mx_1 + b$

PARAMETERS

$m = 2075.25$

$b = -38082.20$

$y_1 \sim a \cdot b^{x_1}$

PARAMETERS

$a = 3991.89$

$b = 1.03967$

1.1 **Slope of linear model:**

This means…**for every 1 year, the population of Colorado is predicted to increase by about $2\ 075$ people.**

1.2 **Growth factor of exponential model:**

This means…**for every 1 year, the population of Colorado increases by about 4%.**

1.3 Which model do you think better fits the data for the population of Colorado from 1880–1990?

***Responses vary.* The exponential model fits better because the data follows the shape of an exponential curve more closely than it follows a line.**

Use Aki's graphs to estimate the population of Colorado in 1945 (65 years after 1980).

1.4 **Linear model prediction**: $\sim 100\ 000$ **people**

1.5 **Exponential model prediction**: $\sim 50\ 000$ **people**

☐ I can fit an exponential or linear function to data.
☐ I can interpret and use models to make predictions.
☐ I can use a graphing calculator to create a linear or exponential model to fit a function to data.
☐ I can informally assess the fit of a function to data.
☐ I can use a linear or exponential model to make predictions and solve problems in context.

desmos Science Mom Lesson 62

Unit A1.7, Quadratic Functions: Notes **Answer Key**

Lessons 1–2: Quadratic Visual Patterns

Summary

Let's explore a new type of relationship. Here is a visual pattern.

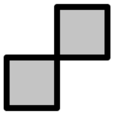


Figure 1

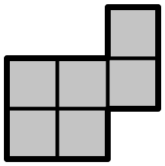


Figure 2

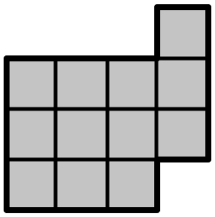


Figure 3

Figure	Number of Tiles
1	$1^2 + 1$
2	$2^2 + 2$
3	$3^2 + 3$
4	$4^2 + 4$
10	$10^2 + 10$
n	$n^2 + n$

Sketch the pattern for figure 4.

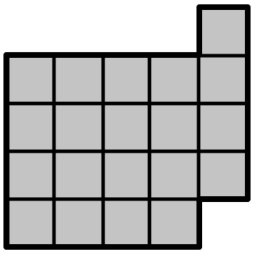


Figure 4

The relationship between figure number and number of tiles is *quadratic* because . . .

. . . each figure contains a square that is related to the figure number $(0^2, 1^2, 2^2, 3^2, 4^2)$**.**

Things I Want to Remember

Lessons 1–2: Quadratic Visual Patterns

Try This!

1.1 Sketch or describe what figure 10 would look like.

Explanations vary. **Figure 10 would have a 10-by-10 square with one tile sticking out of 3 of its corners.**

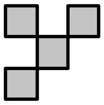


Figure 1

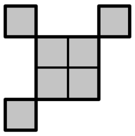


Figure 2

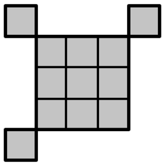


Figure 3

1.2 How many tiles would figure n have?

Figure n would have $n^2 + 3$ tiles.

1.3 Is this relationship quadratic? Explain your thinking.

Yes. ***Explanations vary.*** **Each figure includes a square that is related to the figure number.**

2.1 Sketch or describe what figure 10 would look like.

It would have 10 rows of 5 squares each, with 2 tiles on the top.

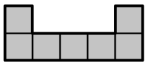


Figure 1

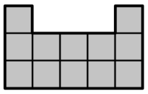


Figure 2

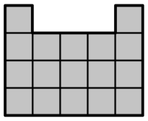


Figure 3

2.2 How many tiles would figure n have?

Figure n would have $5n + 2$ tiles.

2.3 Is this relationship quadratic? Explain your thinking.

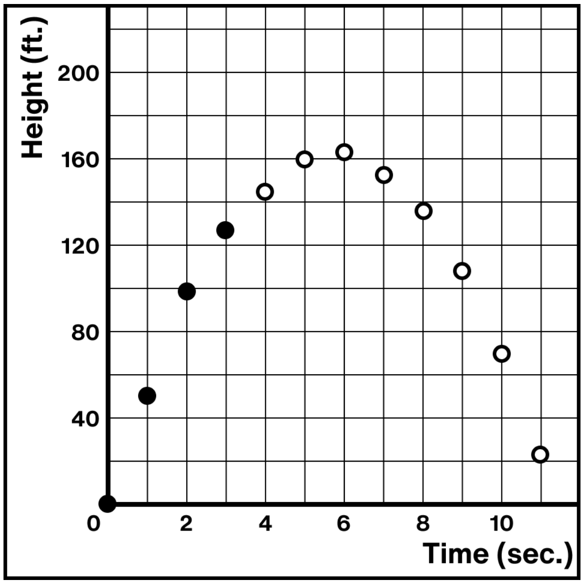
No. ***Explanations vary.*** **There is a constant difference of 5 tiles (there are 5 more tiles in each new figure), so this relationship is linear.**

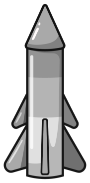
- ☐ I can describe a nonlinear pattern using words or an expression.
- ☐ I can compare linear and quadratic relationships.
- ☐ I can write an expression for a quadratic relationship from a pattern.
- ☐ I understand that quadratic relationships contain a square or squared term.

Lessons 4–5: Quadratic Relationships in Tables and Graphs

Summary

Use symmetry and *constant second differences* to complete the table and graph for this rocket.





Time (sec.)	Height (ft.)
0	0
1	52
2	94
3	126
4	$126 + 22 = 148$
5	$148 + 12 = 160$

Graphs of quadratic relationships have a *line of symmetry* that is . . .

A line so that if you fold a parabola along this line, you get two identical halves.

Things I Want to Remember

Lessons 4–5: Quadratic Relationships in Tables and Graphs

Try This!

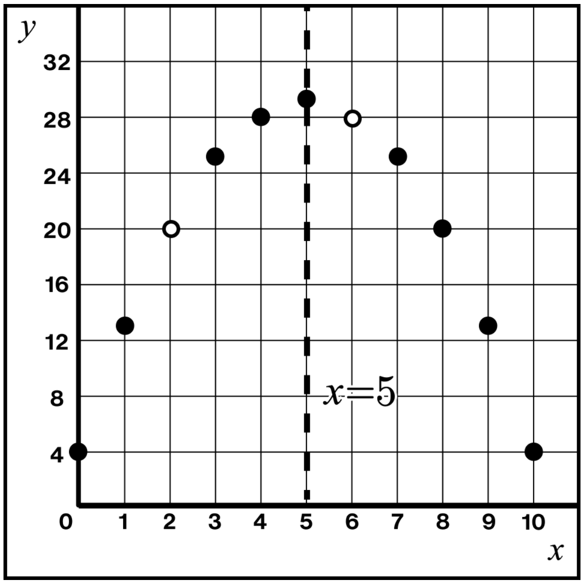
Here are some of the points on a parabola.

1.1 Draw the line of symmetry on the graph.

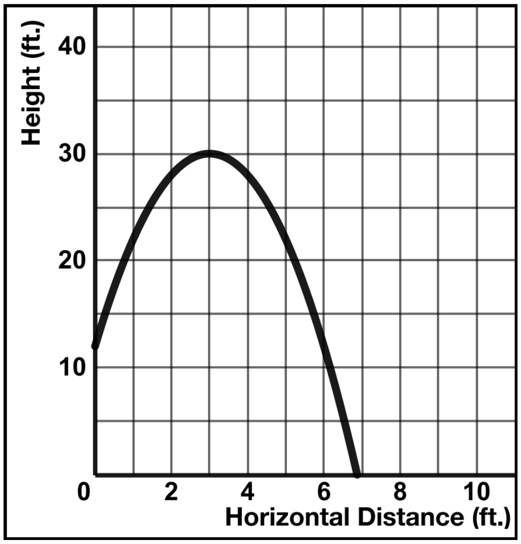
1.2 Use the line of symmetry to determine 2 more points that fall on this parabola.

Plot the points on the graph **and** list them below.

x	y
2	20
6	28



Balloon A



Balloon B

Horizontal Distance (ft.)	Height (ft.)
1	28
2	29
3	28
4	25
5	20

2. Which water balloon went higher?

Balloon A

Explain or show how you know.

The highest point in the graph is 30 ft, but the highest value in the table is 30 ft.

☐ I can create tables and graphs to represent a quadratic relationship in context.
☐ I can describe that parabolas are symmetrical around a line of symmetry.
☐ I can use tables and graphs to make predictions about quadratic relationships in context.

Unit A1.7, Quadratic Functions: Notes

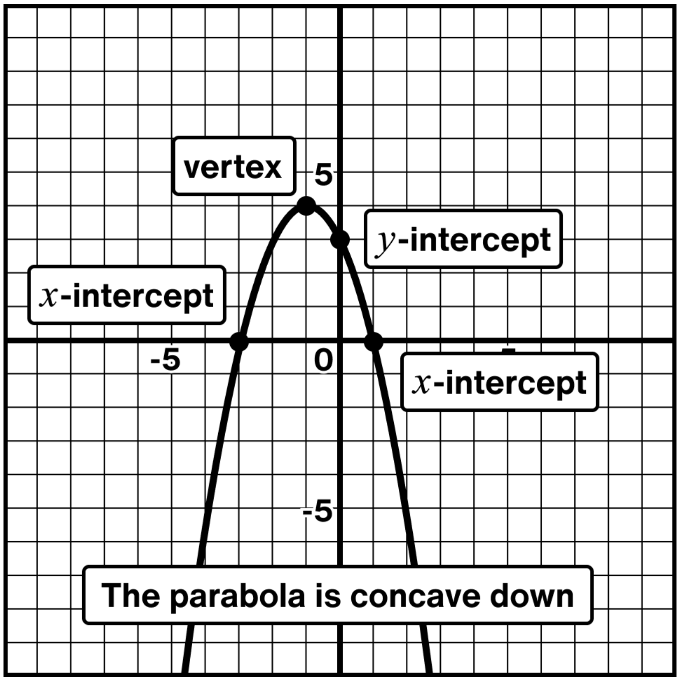
Answer Key

Lessons 6–7: Key Features of Parabolas

Summary

Quadratic functions have many key features we have used for other functions and some new ones.

Label each of these terms on the parabola: x-intercept(s), y-intercept, vertex, concave up/down.



Write a definition of each new term.

Vertex:

The point where a parabola changes from increasing to decreasing, or vice versa.

It is the highest or lowest point on the graph.

Line of symmetry:

A line so that if you fold a parabola along this line, you get two identical halves.

Concave up:

A parabola that opens upward.

Concave down:

A parabola that opens downward.

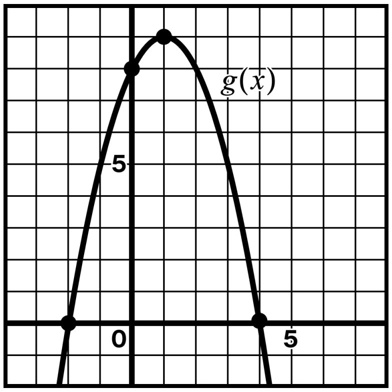
Things I Want to Remember

Lessons 6–7: Key Features of Parabolas

Try This!

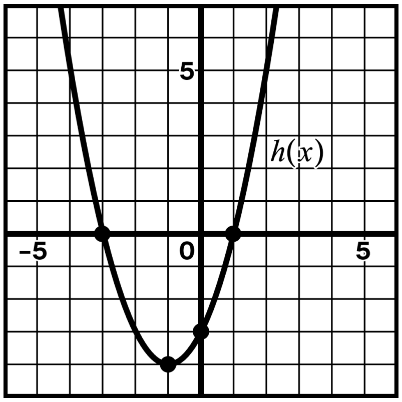
Describe each parabola using the terms x-intercept(s), y-intercept, vertex, concave up/down.

1.1



x-intercept(s) $(-2,\ 0)$ **&** $(4,\ 0)$	Concave Up/Down **concave down**
y-intercept $(0,\ 8)$	Vertex $(1,\ 9)$

1.2



x-intercept(s) $(-3,\ 0)$ **&** $(1,\ 0)$	Concave Up/Down **concave up**
y-intercept $(0,\ -3)$	Vertex $(-1,\ -4)$

The path of a stomp rocket is modeled by this graph. Which key feature describes:

2.1 The starting height of the stomp rocket?

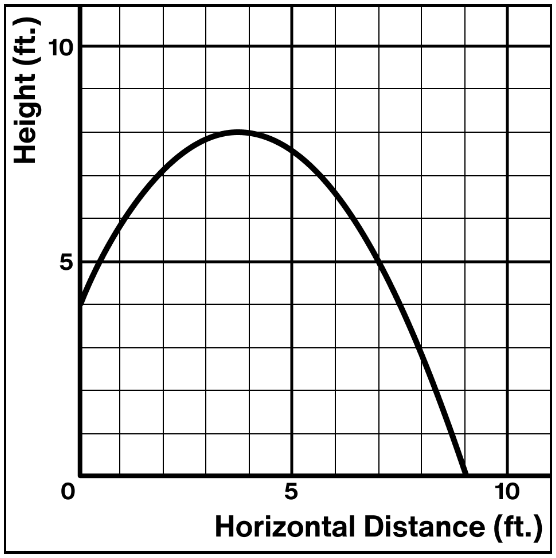
The y-intercept

2.2 The maximum height of the stomp rocket?

The vertex

2.3 Where the stomp rocket lands?

The rightmost x-intercept



☐ I can describe the key features of a parabola.
☐ I can create graphs when given a description of a parabola.
☐ I can compare properties of quadratic functions represented in different ways.

Science Mom Lesson 68

Unit A1.7, Quadratic Functions: Notes

Answer Key

Lesson 10: Intercepts in Factored and Standard Form

Summary

Different forms of quadratic equations help us see different key features of a parabola.

Factored form: $p(x) = (2x - 10)(x + 3)$

x	$(2x - 10)$	$(x + 3)$	$(2x - 10)(x + 3)$
-3	-16	0	$(-16)(0) = 0$
5	0	8	$(0)(8) = 0$
0	-10	3	$(-10)(3) = -30$

How are the x-intercepts related to the equation in factored form?

The x-intercepts are the values that make each factor equal to 0.

For example, $2(5) - 10 = 0$.

Standard form: $p(x) = 2x^2 - 4x - 30$

x	$2x^2$	$4x$	-30	$2x^2 - 4x - 30$
0	0	0	-30	$0 - 0 - 30 = -30$

How is the y-intercept related to the equation in standard form?

The y-intercept is the constant term in the equation.

For example, the constant term of $p(x) = 2x^2 - 4x - 30$ is -30.

Things I Want to Remember

Unit A1.7, Quadratic Functions: Notes **Answer Key**

Lesson 10: Intercepts in Factored and Standard Form

Try This!

1. The same function is written in factored and standard form.

   Factored form: $f(x) = (2x - 2)(x + 5)$ **Standard form:** $2x^2 + 8x - 10$

   Determine the x- and y-intercepts of $f(x)$.

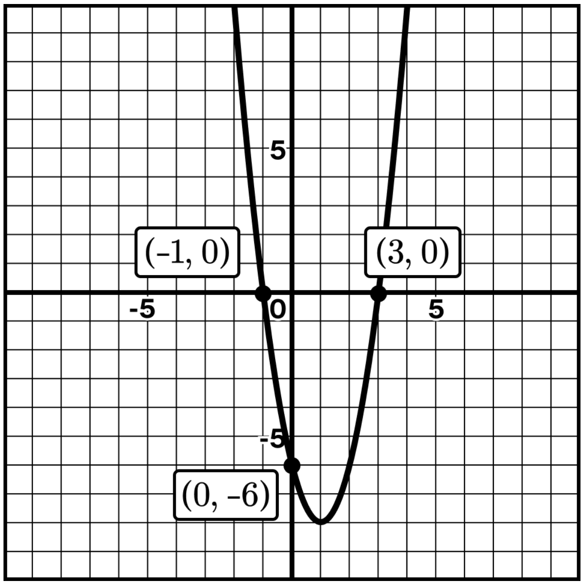
   The x-intercepts are $(1, 0)$ and $(-5, 0)$, because $2x - 2$ is equal to 0 when $x = 1$, and $x + 5$ is equal to 0 when $x = -5$.

   The x-value of the y-intercept is 0 so $f(0) = -10$. The y-intercept is $(0, 10)$.

Here is a function: $g(x) = (x - 3)(2x + 2)$.

2. Sketch a graph of $g(x)$ on the coordinate plane.

   Include the x- and y-intercepts.



☐ I can determine the x-intercepts of a parabola from its equation in factored form.
☐ I can determine the y-intercept of a parabola from its equation in standard form.

Lessons 11–12: Graphing Parabolas in Factored Form

Summary

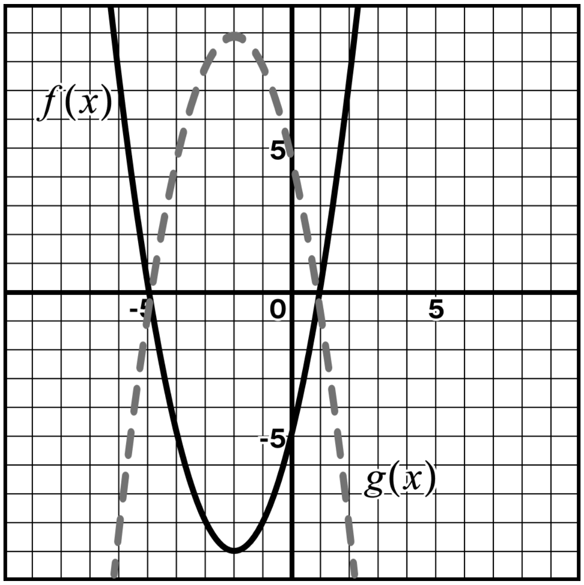
We can use what we know about key features in factored form to create graphs.

Here is a function: $f(x) = (x + 5)(x - 1)$. Its x-intercepts are (,) and (,).

The vertex is in the middle of the two x-intercepts.

This means the x-value of the vertex is _–2_ because . . .

-2 is the same distance (3 units) from each x-intercept.



A table can help determine the y-value of the vertex.

x	$(x + 5)$	$(x - 1)$	$(x + 5)(x - 1)$
–5	0	–6	0
1	6	0	0
–2	3	–3	(3)(–3) = –9

How is the graph of $g(x) = -(x + 5)(x - 1)$ different from the graph of $f(x) = (x + 5)(x - 1)$?

$g(x)$ is concave down and $f(x)$ is concave up.

Sketch both graphs on the axes above. Label each function.

See above.

Things I Want to Remember

Unit A1.7, Quadratic Functions: Notes **Answer Key**

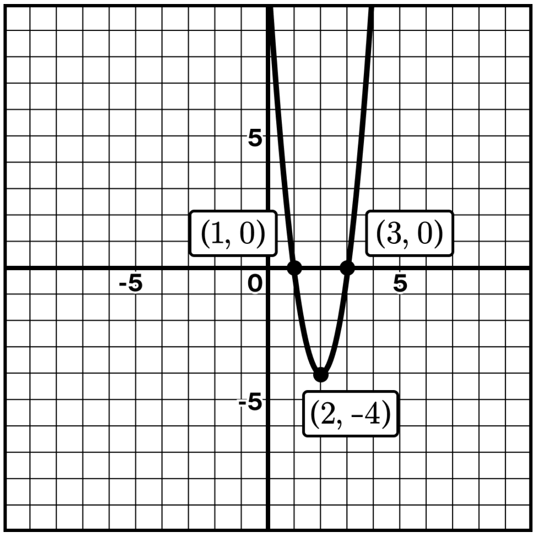
Lessons 11–12: Graphing Parabolas in Factored Form

Try This!

1. Sketch a graph of $f(x) = (4x - 4)(x - 3)$.

Include the x-intercepts and vertex.

See the graph on the right.



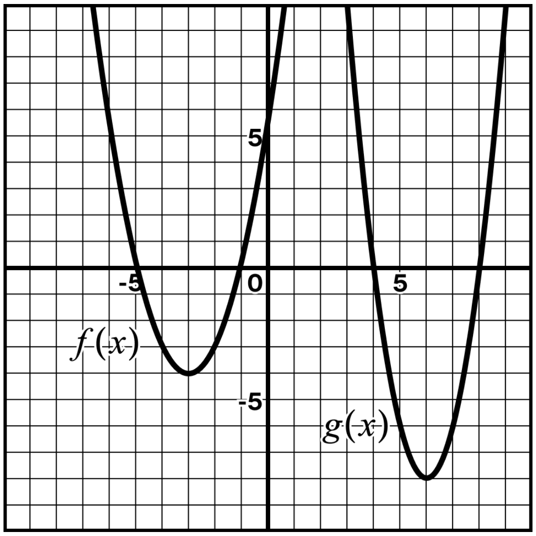
Write a quadratic equation to match each graph.

2.1 $f(x) = (x + 5)(x + 1)$

See the graph on the right.

2.2 $g(x) = 2(x - 4)(x - 8)$

See the graph on the right.



- ☐ I can explain how to determine the vertex of a parabola given its equation in factored form.
- ☐ I can use key features to graph quadratic functions in factored form.
- ☐ I can write a quadratic equation in factored form when given its graph.
- ☐ I can describe what the value of a in $a(x - m)(x - n)$ tells us about a parabola.

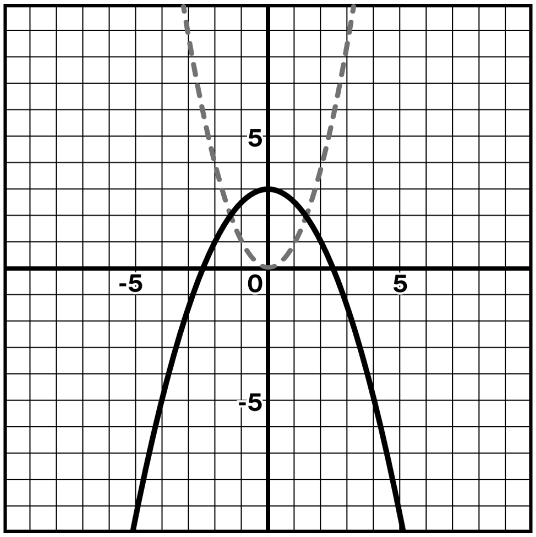
Lesson 14: Vertical Translations and Stretches of Quadratic Functions

Summary

Two ways to change parabolas are using *translations* and *vertical stretches.*

The dashed line is a graph of $y = x^2$. The solid line is a graph of $y = -\frac{1}{2}x^2 + 3$.

x	x^2	$-\frac{1}{2}x^2$	$-\frac{1}{2}x^2 + 3$
-2	4	-2	1
-1	1	$-\frac{1}{2}$	$2\frac{1}{2}$
0	0	0	3
1	1	$-\frac{1}{2}$	$2\frac{1}{2}$
2	4	-2	1



Describe how the graph of $y = -\frac{1}{2}x^2 + 3$ compares to the graph of $y = x^2$. Where do you see each transformation in the table?

Responses vary. **The $-\frac{1}{2}$ tells me that the parabola opens downwards, and also that it is growing slowly compared to $y = x^2$. The 3 tells me that the vertex of the parabola is at $(0, 3)$, which is shifted 3 units up from $y = x^2$.**

Things I Want to Remember

Lesson 14: Vertical Translations and Stretches of Quadratic Functions

Try This!

1. Describe how the graph of $y = -x^2 + 8$ compares to the graph of $y = x^2$.

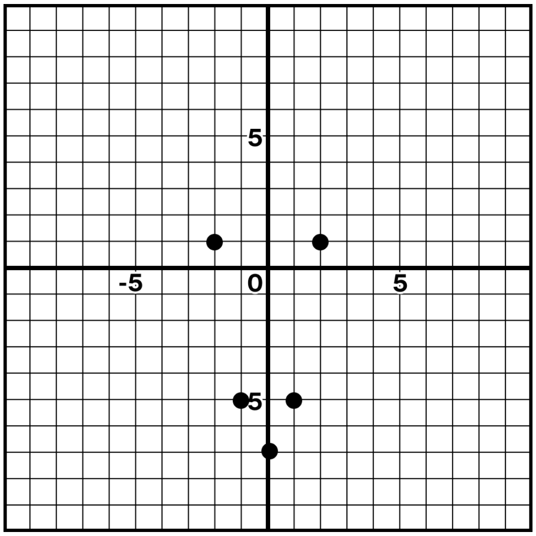
   Responses vary. **The – multiplies all the values by -1, which makes the parabola concave down. The $+ 8$ adds 8 to all the values, so the vertex of the parabola is at $(0, 8)$ instead of $(0, 0)$. This is shifted 8 units up from $y = x^2$.**

x	x^2	$-x^2$	$-x^2 + 8$
-2	4	-4	4
-1	1	-1	7
0	0	0	8
1	1	-1	7
2	4	-4	4

2. Write an equation for this transformation of $y = x^2$.

   Use a table if it helps you with your thinking.

x	x^2	$2x^2$	$2x^2 - 7$
-2	4	8	1
-1	1	2	-5
0	0	0	-7
1	1	2	-5
2	4	8	1



Equation: $y = 2x^2 - 7$

☐ I can describe what vertical translations and stretches do to the graph of a quadratic function.
☐ I can write equations of the form $y = ax^2 + k$ to model a quadratic function.

desmos Science Mom Lesson 72

Unit A1.7, Quadratic Functions: Notes **Answer Key**

Lesson 15: Vertex Form

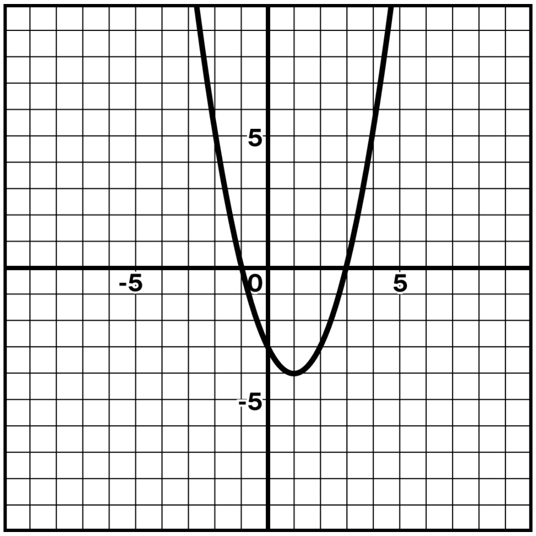
Summary

There are three common forms of quadratic equations. Here is the same function in all three forms:

Standard form: $f(x) = x^2 - 2x - 3$

Factored form: $f(x) = (x + 1)(x - 3)$

Vertex form: $f(x) = (x - 1)^2 - 4$



A table can help show why the vertex of $f(x)$ is $(1, -4)$.

x	$x - 1$	$(x - 1)^2$	$(x - 1)^2 - 4$
1	0	0	−4

The minimum has to be -4 because . . .

Explanations vary. **0 is the smallest possible squared number, so any other x-value would create a larger output than** -4.

Write a different equation in vertex form with a vertex at $(1, -4)$. Show or explain your thinking.

Equations and explanations vary. $g(x) = 5(x - 1)^2 - 4$ **also has a vertex of** $(1, -4)$ **because multiplying by** 5 **doesn't change the vertex. It only changes the vertical stretch.**

Things I Want to Remember

Unit A1.7, Quadratic Functions: Notes **Answer Key**

Lesson 15: Vertex Form

Try This!

1.1 Determine the vertex of $f(x) = (x + 3)^2 + 1$. Use a table if it helps you with your thinking.

x	$x + 3$	$(x + 3)^2$	$(x + 3)^2 + 1$
-3	0	0	1

The vertex is $(-3, 1)$**.**

1.2 Determine the vertex of $y = -2(x - 4)^2 - 8$. Use a table if it helps you with your thinking.

x	$x - 4$	$(x - 4)^2$	$-2(x - 4)^2$	$-2(x - 4)^2 - 8$
4	0	0	0	-8

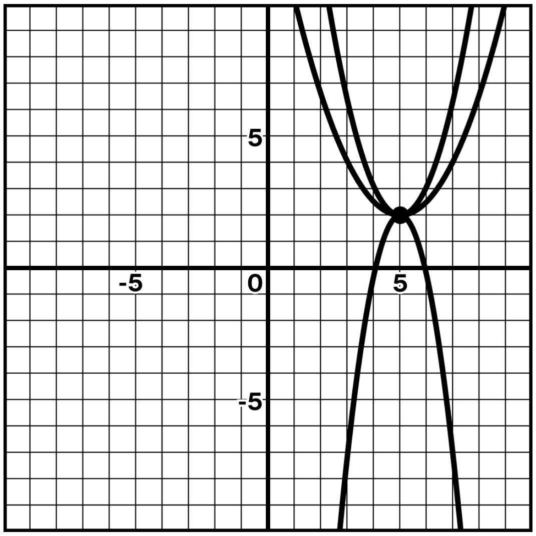
The vertex is $(4, -8)$**.**

2. Write an equation of a parabola that has a vertex at $(5, 2)$.

Use the graph if it helps you with your thinking.

Equations vary.

- $y = (x - 5)^2 + 2$
- $y = -2(x - 5)^2 + 2$
- $y = 0.5(x - 5)^2 + 2$



☐ I can identify the maximum, minimum, or vertex from a quadratic function in vertex form.

Unit A1.7, Quadratic Functions: Notes

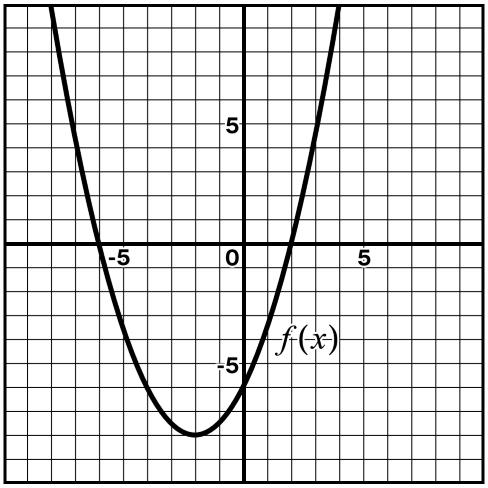
Answer Key

Lessons 16–17: Writing Equations of Quadratic Functions

Summary

We can use what we know about the key features of a parabola to write equations of its graph.

Here are two different equations of the parabola $f(x)$.



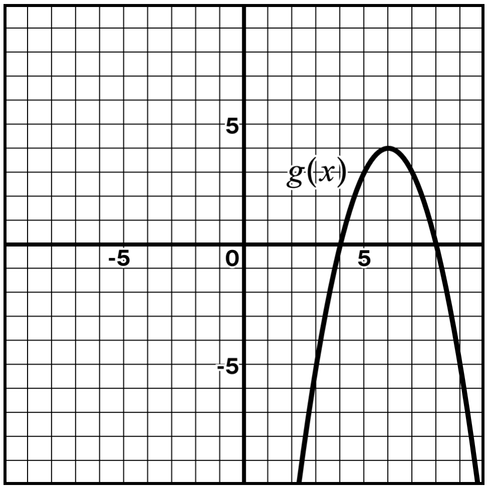
Factored form: $f(x) = \frac{1}{2}(x + 6)(x - 2)$

Factored form helps us see the x-intercepts of a parabola.

Vertex form: $f(x) = \frac{1}{2}(x + 2)^2 - 8$

Vertex form helps us see the vertex of a parabola.

Use the key features to write two equations of $g(x)$.



Factored form:

$g(x) = -(x - 4)(x - 8)$

Vertex form:

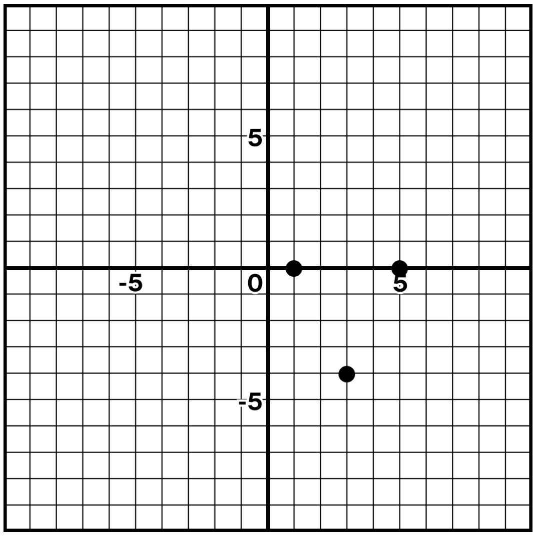
$g(x) = -(x - 6)^2 + 4$

Things I Want to Remember

Lessons 16–17: Writing Equations of Quadratic Functions

Try This!

1. Write two **different** quadratic equations whose graph passes through these points.



Equations vary.

- $f(x) = (x - 3)^2 - 4$
- $f(x) = (x - 1)(x - 5)$

2.1 Write an equation of a parabola that is concave down with a vertex at $(-3, 5)$.

Equations vary. $g(x) = -(x + 3)^2 + 5$

2.2 Write an equation of a parabola that is concave up with x-intercepts at -3 and 5.

Equations vary. $g(x) = (x + 3)(x - 5)$

☐ I can write quadratic functions in vertex and factored forms.
☐ I can use quadratic equations, tables, and graphs to analyze an issue in society.

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Unit A1.8, Quadratic Equations: Notes **Answer Key**

Lessons 1–2: Patterns in Factored-Form and Standard-Form Expressions

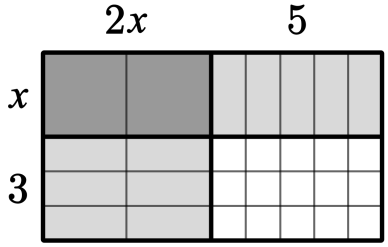
Summary

Quadratic expressions written in *factored form*, which look like $a(x - m)(x - n)$, can be rewritten in *standard form*, which looks like $ax^2 + bx + c$.

Where on the area model do you see the expression's factored form? What about its standard form?

***Responses vary.* The side lengths of the area model are the factors in factored form. The inside tiles of the area model add up to the expression in standard form.**

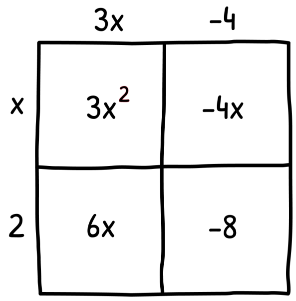
Factored Form	**Standard Form**
$(x + 3)(2x + 5)$	$2x^2 + 11x + 15$



An area model can be useful in rewriting a quadratic expression from factored form to standard form.

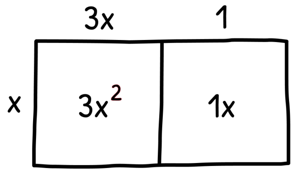
Draw an area model of each quadratic expression. Then rewrite the expressions in standard form.

$(3x - 4)(x + 2)$



Standard form: $3x^2 + 2x - 8$

$x(3x + 1)$



Standard form: $3x^2 + x$

Describe how to write a factored-form expression in standard form.

***Responses vary.* To write a factored-form expression in standard form, you can use a diagram. You write factored form on the outside of the diagram and standard form on the inside. Then you can find the area of the smaller sections and combine like terms to write the expression in standard form.**

Things I Want to Remember

Lessons 1–2: Patterns in Factored-Form and Standard-Form Expressions

Try This!

Rewrite the expressions in standard form. Use the diagrams if they help with your thinking.

1.1 $(2x - 5)(x + 3)$

	$2x$	-5
x	$2x^2$	$-5x$
3	$6x$	-15

Standard form: $2x^2 + x - 15$

1.2 $(3x - 4)(2x + 1)$

	$3x$	-4
$2x$	$6x^2$	$-8x$
1	$3x$	-4

Standard form: $6x^2 - 5x - 4$

Identify the a-, b-, and c- values of these standard-form quadratic expressions:

2.1 $5x^2 + 9x - 15$

$a = 5 \quad b = 9 \quad c = -15$

2.2 $x^2 - 1$

$a = 1 \quad b = 0 \quad c = -1$

2.3 $3x^2 + x$

$a = 3 \quad b = 1 \quad c = 0$

- ☐ I can use area models and diagrams to rewrite a factored-form quadratic expression in standard form.
- ☐ I can identify the a-, b-, and c-values in a standard-form quadratic expression.
- ☐ I can use a factored-form expression to predict what an equivalent expression in standard form will look like.

Lessons 3–4: Factoring Quadratic Expressions

Summary

When factoring quadratic expressions, you can look to the values of the terms to determine potential strategies.

Let’s look at two different quadratic expressions. Complete the diagrams, then rewrite the expressions in factored form.

$3x^2 + 14x + 15$

	$3x$	5
x	$3x^2$	$5x$
3	$9x$	15

Factored form: $(3x + 5)(x + 3)$

$x^2 - 9$

	x	3
x	x^2	$3x$
-3	$-3x$	-9

Factored form: $(x - 3)(x + 3)$

If there is a common factor in a quadratic expression, it may be helpful to write an equivalent expression first. For example, rewriting $2x^2 + 4x - 16$ as $2(x^2 + 2x - 8)$.

Rewrite $3x^2 - 9x - 30$ as an equivalent expression, then factor it. Use the diagram if it helps your thinking.

Factored form: $3(x - 5)(x + 2)$

	x	-5
x	x^2	$-5x$
2	$2x$	-10

Describe how to write a standard-form expression in factored form.

Responses vary. **Try to divide by a common factor first, if possible. You can then use a diagram by putting the remaining ax^2 in the top-left corner of the diagram and c in the bottom right. Try different pairs of numbers that multiply to the terms on the inside until you get an expression equivalent to the standard-form expression.**

Things I Want to Remember

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Unit A1.8, Quadratic Equations: Notes **Answer Key**

Lessons 3–4: Factoring Quadratic Expressions

Try This!

Factor the expressions. Use the diagrams if they help with your thinking.

1.1 $x^2 + 10x + 21$

	x	7
x	x^2	$7x$
3	$3x$	21

Factored form: $(x + 7)(x + 3)$

1.2 $2x^2 + 3x - 27$

	$2x$	9
x	$2x^2$	$9x$
-3	$-6x$	-27

Factored form: $(2x + 9)(x - 3)$

1.3 $x^2 - 81$

	x	9
x	x^2	$9x$
-9	$-9x$	-81

Factored form: $(x - 9)(x + 9)$

1.4 $2x^2 + 22x + 60$

	x	5
x	x^2	$5x$
6	$6x$	30

Factored form: $2(x + 5)(x + 6)$ **(or equivalent)**

☐ I can factor to rewrite standard-form quadratic expressions in factored form.
☐ I can use a standard-form expression to predict what an equivalent expression in factored form will look like.

Answer Key

Lessons 5–6: Solving Quadratic Equations Using the Zero-Product Property

Summary

The *zero-product property* says that if the product of two or more factors is 0, then at least one of the factors is 0. This property can be used to determine the x-intercepts of a function or the solutions to quadratic equations.

Describe how to determine the x-intercepts of quadratic functions using the zero-product property. Use the examples if they help with your thinking.

$$f(x) = (x-3)(x+9)$$

$$g(x) = x^2 + 9x + 18$$

Responses vary. **Because the x-intercepts are where the y-value is 0, I can set the function equal to 0. I can use the zero-product property to find the x-intercepts of quadratic functions by first factoring a quadratic function and then setting each factor equal to 0.**

Describe how to solve quadratic equations using the zero-product property. Use the examples if they help with your thinking.

$$x^2 + 12x + 20 = 0$$

$$x^2 + 8x = 33$$

Responses vary. **I can set the equation equal to 0. Then, I can factor the equation and set each factor equal to 0. Finally, I solve my equations.**

What are the similarities and differences between solving for the x-intercepts of a quadratic function and solving a quadratic equation?

Responses vary.

- **The process of solving for x-intercepts of a quadratic function and solving a quadratic equation are similar because they both involve factoring and setting the factors equal to 0.**
- **They are different because when solving for the x-intercepts, the answer is a coordinate point. When solving the equation, it is just the x-value.**

Things I Want to Remember

Unit A1.8, Quadratic Equations: Notes **Answer Key**

Lessons 5–6: Solving Quadratic Equations Using the Zero-Product Property

Try This!

Determine the x-intercepts of the quadratic functions. Use the diagram if it helps with your thinking.

1.1 $h(x) = (x - 1)(x + 6)$

x-intercepts: $(-6, 0)$ **and** $(1, 0)$

1.2 $k(x) = x^2 - 6x - 40$

	x	-10
x	x^2	$-10x$
4	$4x$	-40

x-intercepts: $(10, 0)$ **and** $(-4, 0)$

Determine the solutions to the quadratic equations. Use the diagram if it helps with your thinking.

2.1 $(5x - 3)(2x + 3) = 0$

Solution: $x = \frac{3}{5}$ **and** $x = \frac{-3}{2}$

2.2 $2x^2 - x = 21$

	$2x$	-7
x	$2x^2$	$-7x$
3	$6x$	-21

Solution: $x = -3$ **and** $x = \frac{7}{2}$

- ☐ I can determine the x-intercepts of quadratic functions written in factored form and standard form.
- ☐ I understand the zero-product property, which states that if the product of two or more factors is 0, then at least one of its factors is 0.
- ☐ I can use factoring and the zero-product property to solve quadratic equations.

Lesson 7: Solving Equations by Reasoning

Summary

The structure of a quadratic equation can help determine the number of solutions.

Here are examples of quadratic equations with no solutions, one solution, and two solutions. Write an additional example of each type of equation.

No Solution	One Solution	Two Solutions
$x^2 + 4 = 0$ $x^2 = -25$ $(x - 3)^2 + 1 = 0$	$(x + 4)^2 = 0$ $x^2 + 9 = 9$ $(x - 3)(x - 3) = 0$	$(x + 4)^2 = 1$ $x^2 - 12 = -3$ $(x - 3)(x - 3) = 1$
Responses vary. $x^2 + 7 = 0$	***Responses vary.*** $(x - 8)^2 = 0$	***Responses vary.*** $(x - 1)(x - 1) = 2$

How can you determine if a quadratic equation has no solutions? One solution? Two solutions?

Responses vary. **The structure of a quadratic equation will tell you the number of solutions it has. If $(x + \_\_)^2$ equals a negative number, then there will be no solutions. If $(x + \ \_\_)^2$ equals 0, then there is one solution. If $(x + \_\_)^2$ equals a positive number, then there are two solutions.**

How can you check if your solution solves the equation? How can you avoid missing a solution?

Responses vary. **To check your solution, substitute your answer back in for x and see if you get a true equation. To avoid missing a solution, remember to check for both the positive and negative values that can make this equation true.**

Things I Want to Remember

Unit A1.8, Quadratic Equations: Notes **Answer Key**

Lesson 7: Solving Equations by Reasoning

Try This!

Circle whether each equation has no solutions, one solution, or two solutions. Solve the equation if there are solutions.

1.1 $x^2 + 10 = 110$ No solutions One solution [✓ **Two solutions**]

$x = –10$ **and** $x = 10$

1.2 $(x - 8)^2 = 0$ No solutions [✓ **One solution**] Two solutions

$x = 8$

1.3 $x(x + 1) = 6$ No solutions One solution [✓ **Two solutions**]

$x = –3$ **and** 2

1.4 $(x + 3)(x + 3) = –9$ [✓ **No solutions**] One solution Two solutions

$x =$ ________________

☐ I can solve quadratic equations by reasoning.
☐ I can determine whether a quadratic equation has zero, one, or two solutions.

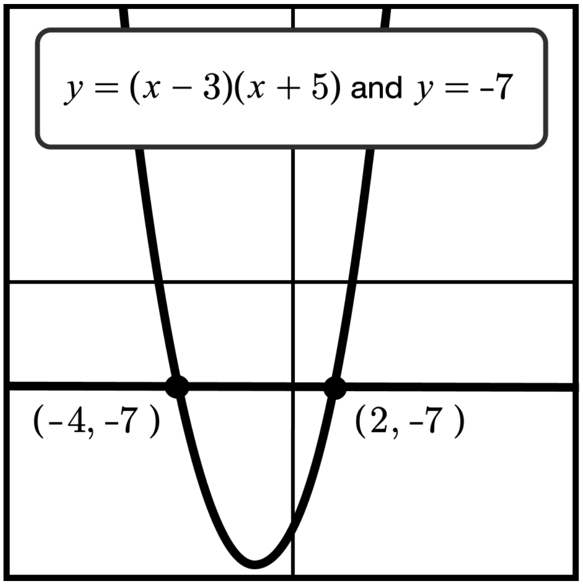
Lesson 8: Solving Quadratic Equations by Graphing

Summary

The graph of a quadratic equation can help determine the solutions.

Let's look at how a graph can help solve the equation $(x - 3)(x + 5) = -7$.

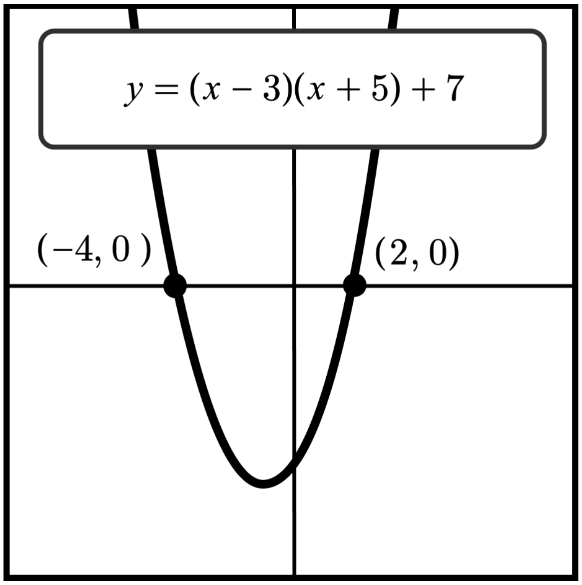
Identify the solutions and describe the strategy used for each approach.



Solutions: $x = -4$ **and** $x = 2$

Strategy:

Responses vary. **Graph both sides of the equation as two separate graphs. Find the x-coordinates where the graphs intersect.**



Solutions: $x = -4$ **and** $x = 2$

Strategy:

Responses vary. **Rearrange the equation so that it equals 0. Then graph the equation. The solutions will be at the x-intercepts.**

Things I Want to Remember

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Unit A1.8, Quadratic Equations: Notes **Answer Key**

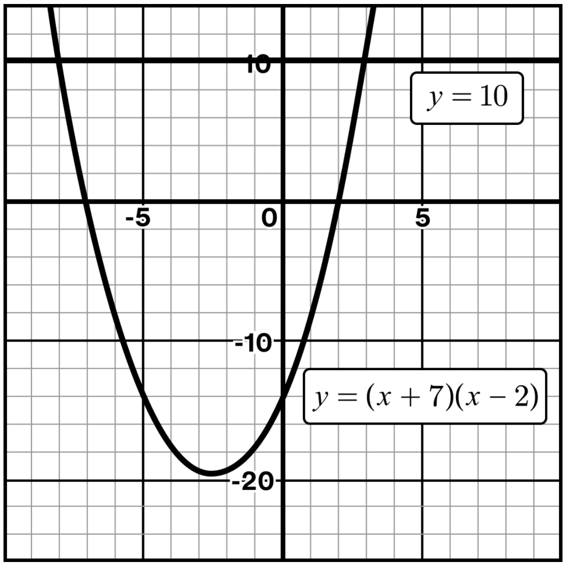
Lesson 8: Solving Quadratic Equations by Graphing

Try This!

Here is the graph that Peter made to solve $(x + 7)(x - 2) = 10$.

1. Determine the solution(s) to this equation.

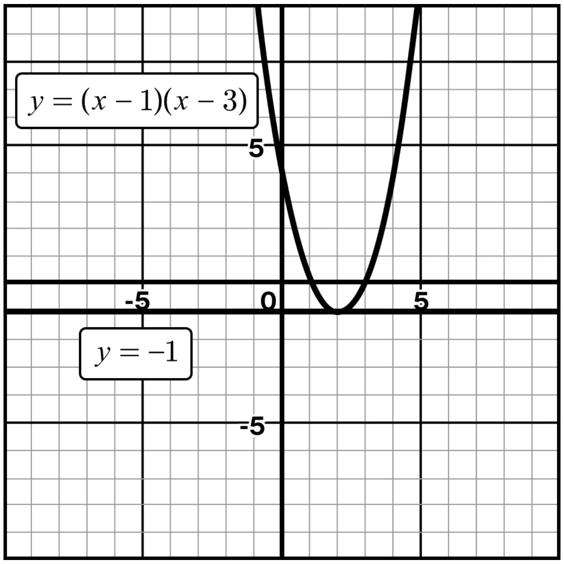
   $x = -8$ **and** $x = 3$



Here is the graph that Peter made to solve $(x - 1)(x - 3) = -1$.

2. Determine the solution(s) to this equation.

   $x = 2$



☐ I can solve quadratic equations by graphing.
☐ I can use graphs to determine whether a quadratic equation has zero, one, or two solutions.

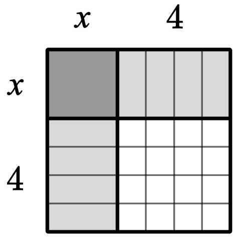
Unit A1.8, Quadratic Equations: Notes **Answer Key**

Lessons 10–11: Solving by Completing the Square

Summary

A quadratic expression is a *perfect square* if it can be represented as something multiplied by itself.

For example, $(x + 4)^2$ and $x^2 + 8x + 16$ are both perfect squares.



What are other examples of perfect square quadratic expressions?

Responses vary. $(x - 5)^2$ **and** $x^2 - 10x + 25$

Equations in the form $(x + \_\_\_)^2 = \_\_\_$ can be solved by taking the square root. *Completing the square* is the process of rewriting a quadratic expression or equation to include a perfect square.

How do you know what constant value to add to make a perfect square? Use the example if it helps with your thinking.

$x^2 + 12x = 24$

$x^2 + 12x + 36 = 24 + 36$

$(x+6)^2 = 60$

Responses vary. **I can determine the constant value to add by dividing the linear coefficient in half and then squaring that number. In this example, half of 12 is 6, and 6 squared is 36.**

Solve the equation $x^2 - 18x = 10$ by completing the square.

$x^2 - 18x + 81 = 10 + 81$

$(x - 9)^2 = 91$

$x - 9 = \pm\sqrt{91}$

$x = 9 \pm \sqrt{91}$

Things I Want to Remember

Unit A1.8, Quadratic Equations: Notes **Answer Key**

Lessons 10–11: Solving by Completing the Square

Try This!

Fill in the blanks to make each expression a perfect square.

1.1 $x^2 - 14x + 49$

1.2 $x^2 + 18x + 81$
or $x^2 + -18x + 81$

1.3 $x^2 + 6x + 9$

Solve the equations by completing the square.

2.1 $x^2 + 10x = 2$

$x^2 + 10x + 25 = 2 + 25$
$(x + 5)^2 = 27$
$x + 5 = \pm\sqrt{27}$
$x = -5 \pm \sqrt{27}$

$x = -5 \pm \sqrt{27}$

2.2 $x^2 + 20x - 25 = 0$

$x^2 + 20x = 25$
$x^2 + 20x + 100 = 25 + 100$
$(x + 10)^2 = 125$
$x + 10 = \pm\sqrt{125}$
$x = -10 \pm \sqrt{125}$

$x = -10 \pm \sqrt{125}$

- ☐ I can justify whether a quadratic expression is a perfect square.
- ☐ I can determine missing values to make perfect square quadratic expressions.
- ☐ I can solve quadratic equations by completing the square.

desmos Science Mom Lesson 85

Unit A1.8, Quadratic Equations: Notes **Answer Key**

Lessons 13–15: Solving Quadratic Equations Using the Quadratic Formula

Summary

The quadratic formula comes from completing the square.

The solutions to any quadratic equation $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

This formula is known as the quadratic formula.

The solutions to the equation $x^2 - 4x - 12 = 0$ are $x = -2$ and $x = 6$.

Use the quadratic formula to show that the solutions are correct.

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-12)}}{2(1)} \rightarrow x = \frac{4 \pm \sqrt{64}}{2} \rightarrow x = \frac{4 \pm 8}{2} \rightarrow x = -2 \text{ and } x = 6$$

You can write quadratic equations to represent a situation and solve problems about it.

For example, the function $h(t) = -1.5t^2 + 12t + 8$ represents the height, in meters, of a stomp rocket t seconds after it has been launched.

Explain why the equation $0 = -1.5t^2 + 12t + 8$ will help us determine when the rocket will touch the ground.

Responses vary. **Since $h(t)$ represents the height, if we set that equal to 0, then solving this equation will let us know the time when the rocket's height is 0 or touches the ground.**

Xavier solved the equation and got the solutions $x \approx -0.619$ and $x \approx 8.619$. Do both of these solutions answer the question of when the rocket will hit the ground? Explain your thinking.

Responses vary. **No, only 8.619 makes sense in this situation. It is not possible for the rocket to touch the ground at -0.619 seconds.**

Things I Want to Remember

Unit A1.8, Quadratic Equations: Notes **Answer Key**

Lessons 13–15: Solving Quadratic Equations Using the Quadratic Formula

Try This!

Use the quadratic formula to solve the equations.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1.1 $0 = x^2 + 6x + 3$

$a = 1 \quad b = 6 \quad c = 3$

$$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(1)(3)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{24}}{2}$$

1.2 $0 = 3x^2 - 5x + 2$

$a = 3 \quad b = -5 \quad c = 2$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(2)}}{2(3)}$$

$$x = \frac{5 \pm \sqrt{1}}{6}$$

$x = \frac{2}{3}$ and $x = 1$

The function $h(t) = -2.5t^2 + 6t - 8$ represents the height, in meters, of a stomp rocket t seconds after it has been launched.

2.1 Write an equation that can be solved to determine when the rocket will return to its original height of –8 meters.

$-8 = -2.5t^2 + 6t - 8$ **or equivalent**

2.2 How long will it take for the rocket to return to its original height of –8 meters?

2.4 **seconds**

☐ I understand that the quadratic formula is based on the process of completing the square.
☐ I can use the quadratic formula to solve quadratic equations.
☐ I can write quadratic equations to solve problems about a situation.
☐ I can solve quadratic equations and explain what the solutions mean in a situation.

Lesson 17: Solving Systems of Linear and Quadratic Equations

Summary

A system of equations contains two or more equations, each of which represents different constraints on shared variables. A system can contain a variety of equations, including linear and quadratic equations.

Describe a strategy for solving a system.
Use the example if it helps with your thinking.

$y = \boxed{2x - 7} \qquad y = x^2 - 5x + 3$

Responses vary.

- **Use substitution or elimination to create an equation with only one variable.**
- **Solving for x, factoring and using the zero-product property, solving with square roots, completing the square, and using the quadratic formula can be helpful strategies.**
- **After determining the value of one variable, substitute it into one of the equations to determine the other. I like to pick the equation that looks easier to me.**
- **Solutions to systems of equations are points of intersection on the graph.**

$2x - 7 = x^2 - 5x + 3$

$0 = x^2 - 7x + 10$

$0 = (x - 5)(x - 2)$

$\boxed{x = 5} \qquad \boxed{x = 2}$

$y = 2(5) - 7 \qquad y = 2(2) - 7$

$\boxed{y = 3} \qquad \boxed{y = -3}$

Points of intersection:

$\boxed{(5, 3)} \qquad \boxed{(2, -3)}$

What are the similarities and differences between solving a system of linear equations and solving a system that includes a quadratic equation?

Responses vary.

- **The solutions to both are written as coordinates.**
- **The solutions are at the points of intersection when graphed.**
- **A system that has a quadratic equation can have a maximum of two solutions compared to a maximum of one solution for a system of linear equations.**

Things I Want to Remember

Unit A1.8, Quadratic Equations: Notes **Answer Key**

Lesson 17: Solving Systems of Linear and Quadratic Equations

Try This!

Solve these systems of equations:

1.1 $y = 9x^2 - 7$
$y = -3$

$(\frac{-2}{3}, -3)$ **and** $(\frac{2}{3}, -3)$

1.2 $y = x^2 - 3x$
$y = 2x - 6$

$(2, -2)$ **and** $(3, 0)$

☐ I can solve systems of linear and quadratic equations.