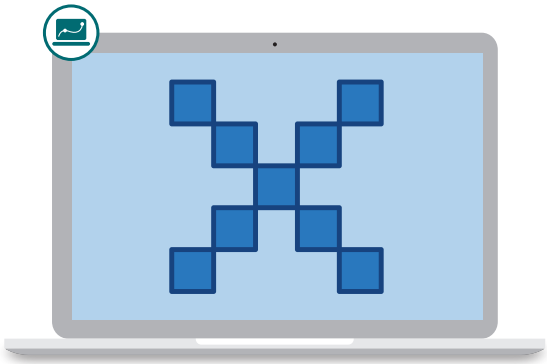
Unit 1
Lesson
1

Name: ..... Date: ..... Period: .....

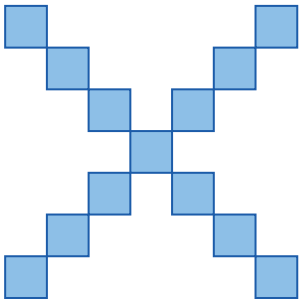


Visual Patterns

Let's explore visual patterns.

Warm-Up

1 Without counting one by one, determine how many tiles are in this figure.



Show or explain your thinking.

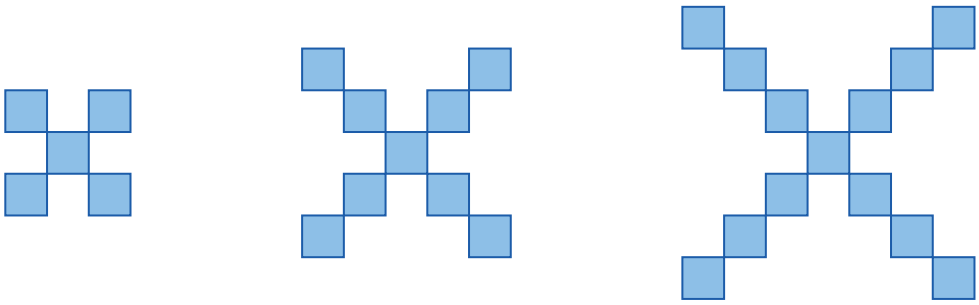
Activity 1

Name: _____ Date: _____ Period: _____

Pattern A

2 The figure in the Warm-Up is part of a visual pattern.

Figure 1 **Figure 2** **Figure 3**



a **Discuss:** What do you think Figure 4 will look like?

b How many tiles will there be in Figure 4?

3 Here are the number of tiles in Figures 1–3.

How many tiles will there be in Figure 10?

Figure	Number of Tiles
1	5
2	9
3	13

Name: _____ Date: _____ Period: _____

Pattern B

4 Here is a new visual pattern.

Figure 1 **Figure 2** **Figure 3**

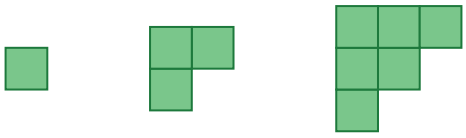


Figure	Number of Tiles
1	1
2	3
3	6

Matias says Figure 4 will have $6 + 4$ tiles.

Do you agree? Circle one.

Yes No I'm not sure

Explain your thinking.

5 Draw Figure 5 of the pattern.

6 How many tiles will there be in Figure 10?

Name: _____ Date: _____ Period: _____

Pattern C

7 Here are the two visual patterns we've seen.

Pattern A

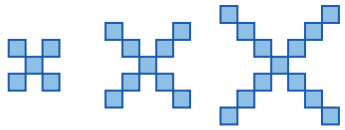


Figure 1 Figure 2 Figure 3

Figure	Number of Tiles
1	5
2	9
3	13
4	17

Pattern B

Figure 1 Figure 2 Figure 3

Figure	Number of Tiles
1	1
2	3
3	6
4	10

How are these patterns alike? How are they different?

Alike	Different

8 Here is a new visual pattern.

Figure 1

Figure 2

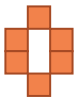


Figure 3

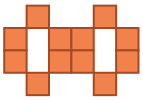


Figure 4

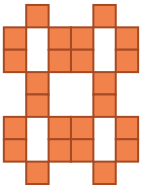


Figure	Number of Tiles
1	3
2	6
3	12
4	24

How many tiles will there be in Figure 7?

Name: _____ Date: _____ Period: _____

Pattern C (continued)

9 Here are the number of tiles in Figures 1–3 of another new visual pattern.

Figure	Number of Tiles
1	4
2	7
3	10

a Draw three figures to match the pattern in the table.

Figure 1	Figure 2	Figure 3

b How many tiles will there be in Figure 8?

Explore More

10 There are many possibilities for Figure 3 in this visual pattern.

Make two different versions that each continue the pattern in some way.

Figure 1	Figure 2	Figure 3
		

Figure 1	Figure 2	Figure 3
		

11 Synthesis

Describe *at least one* strategy for determining the number of tiles in Figure 7 of a visual pattern. Use these examples if they help with your thinking.

Pattern B

Figure 1 Figure 2 Figure 3

Figure	Number of Tiles
1	1
2	3
3	6
4	10

Pattern C

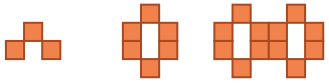


Figure 1 Figure 2 Figure 3

Figure	Number of Tiles
1	3
2	6
3	12
4	24

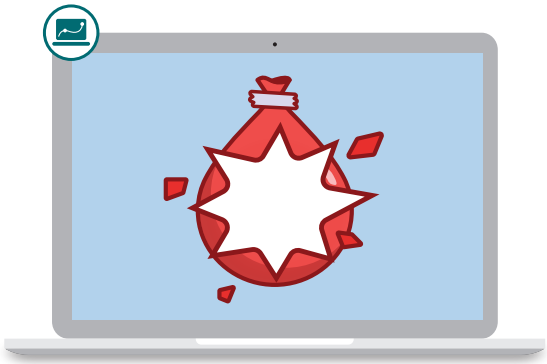
Things to Remember:

Unit 1
Lesson
2

Name: ..... Date: ..... Period: .....

Sequence Carnival

Let's explore sequences.



Warm-Up

1 Here is a **sequence**: a list of numbers in a particular order.

Let's watch an animation to see how the sequence is made.

What do you notice? What do you wonder?

I notice:

I wonder:

Activity 1

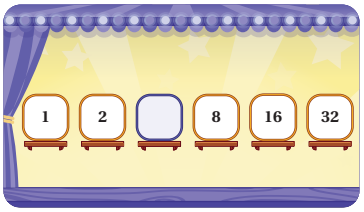
Name: _____ Date: _____ Period: _____

Seeing Sequences

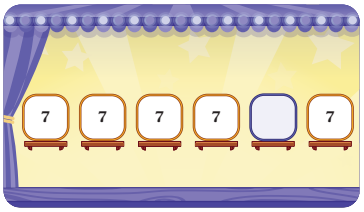
2 Each sequence follows a pattern and has a missing term. Write the missing term for each sequence.

Sequence 1

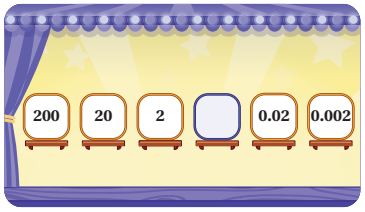
Sequence 2



Sequence 3



Sequence 4



Sequence 5

Sequence 6

3 Here are two new sequences.

A **5, 8, 11, 14, 17, . . .**

B **5, 15, 45, 135, 405, . . .**

How are they alike? How are they different?

Alike	Different

4 Sequence A changes by a **constant difference**. Sequence B changes by a **constant ratio**.

What type of change does Sequence C show? Circle one.

C **40, 20, 10, 5, . . .**

Constant difference Constant ratio Neither

Explain your thinking.

Activity 2

Name: _____ Date: _____ Period: _____

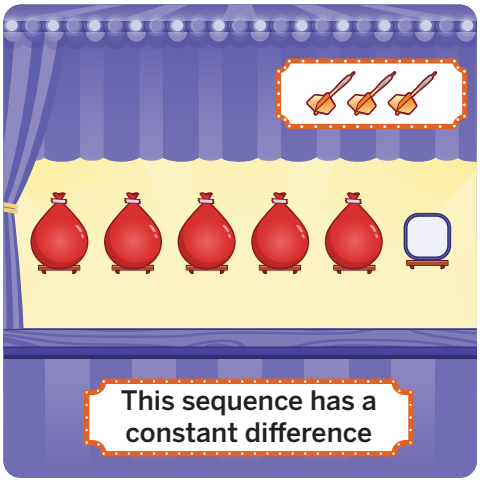
Sequence Challenges

5 Let's look at a new sequence.

This sequence has a constant difference. The terms are hidden by balloons.

As a class, decide which balloons to pop. You can pop up to *three* balloons.

What is the missing term?

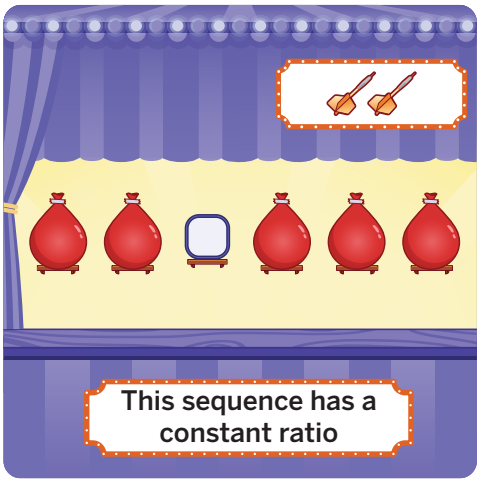


6 Let's look at a new sequence.

This sequence has a constant ratio.

As a class, decide which balloons to pop. You can pop up to *two* balloons.

What is the missing term?



7 Moon and Gabriel looked at this sequence.

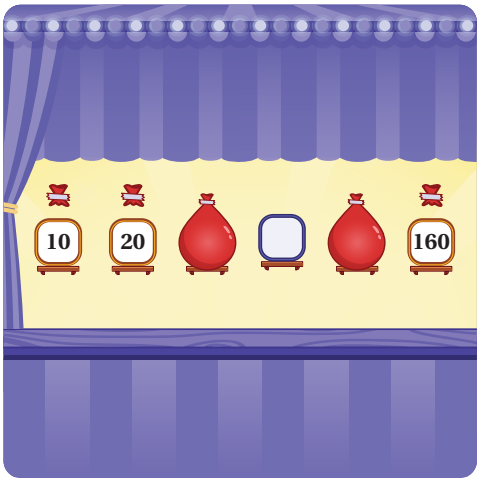
Moon said: *This sequence has a constant difference of* 10.

Gabriel said: *This sequence has a constant ratio of* 2.

Who is correct? Circle one.

Moon Gabriel Both Neither

Explain your thinking.



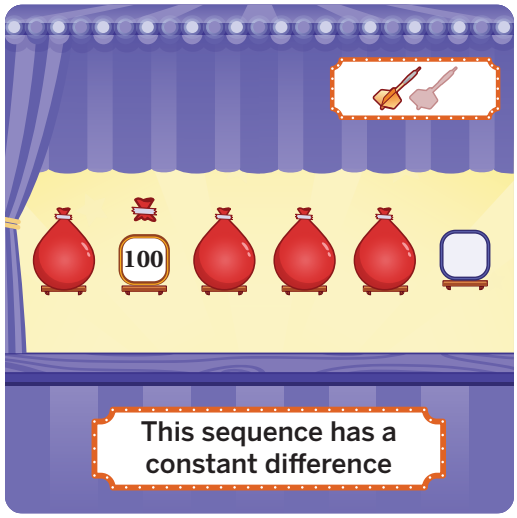
Name: _____ Date: _____ Period: _____

Sequence Challenges (continued)

8 Laila is working on this sequence, which has a constant difference.

She can pop *one more* balloon.

Which balloon do you think she should pop? Explain your thinking.

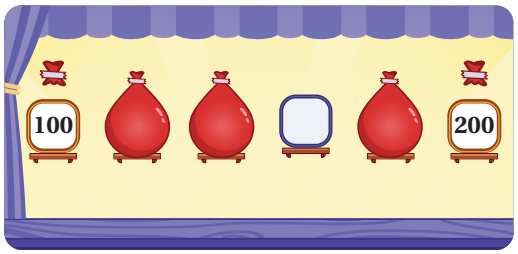


9 Let's see which balloon Laila chose to pop.

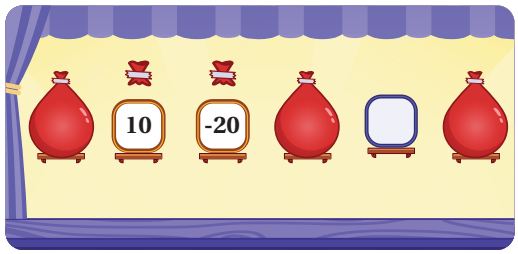
What is the missing term?

10 What is the missing term in each sequence?

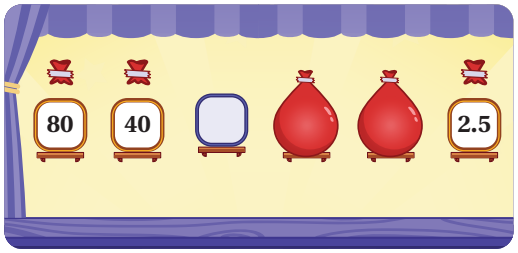
a This sequence has a constant difference.



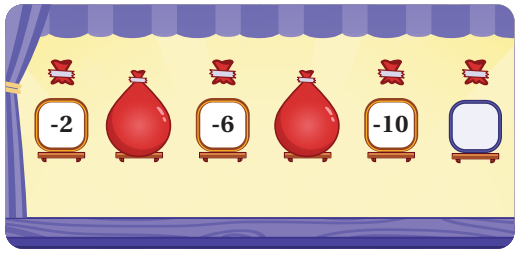
b This sequence has a constant ratio.



c This sequence has either a constant ratio *or* a constant difference.



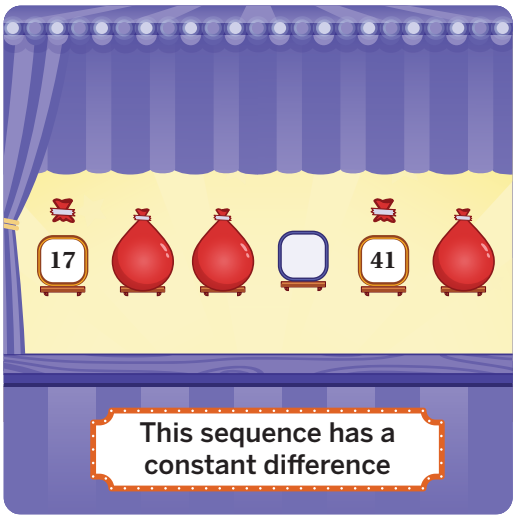
d This sequence has either a constant ratio *or* a constant difference.



11 Synthesis

Describe a strategy for finding a missing term in a sequence.

Use the example if it helps with your thinking.



Things to Remember:

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Name: ..... Date: ..... Period: .....

Recursion Machine

Let's write recursive definitions of sequences to meet certain requirements.

Warm-Up

1 a Let's watch an animation to see how a machine created this sequence.

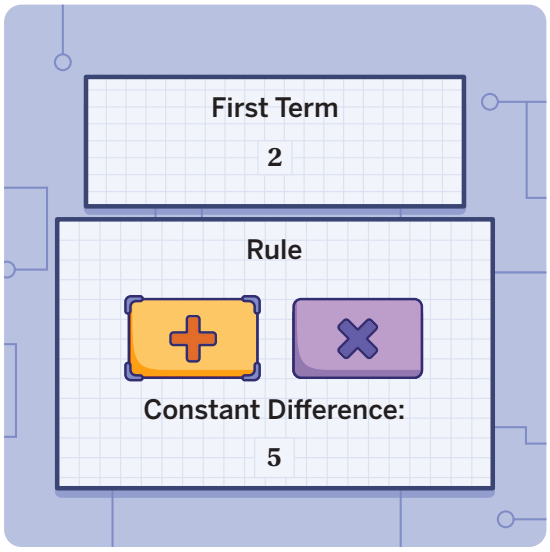
b How would you describe what the machine is doing?

Activity 1

Name: _____ Date: _____ Period: _____

Recursive Definitions

2 The machine creates a sequence using a first term and a rule. Together, a first term and a rule make a **recursive definition**.



a Take a look at these sequences that were made with different *recursive definitions*.

Sequence	First Term	Rule
2, 7, 12, 17, 22	2	Constant difference: 5
3, 6, 12, 24, 48	3	Constant ratio: 2
48, 24, 12, 6, 3	48	Constant ratio: $\frac{1}{2}$

b Create as many sequences as you can that include the number 12.

Sequence	First Term	Rule
____, ____, ____, ____, ____		
____, ____, ____, ____, ____		
____, ____, ____, ____, ____		

Activity 2

Name: _____ Date: _____ Period: _____

Recursive Challenges

3 In this sequence, the first term is 1,600. Create a rule that will produce this sequence.

Sequence	First Term	Rule
1600, 400, 100, 25, 6.25	1,600	

4 In this sequence, the first term is 3. Create a rule that will make the fourth term 24.

Sequence	First Term	Rule
3, ____, ____, 24, ____	3	

5 Create a recursive definition for a sequence that makes the second term 25 and the fourth term 1.

Sequence	First Term	Rule
____, 25, ____, 1, ____		

Name: _____ Date: _____ Period: _____

Recursive Challenges (continued)

6 Troy made a mistake when he created a recursive definition on the previous problem.

a **Discuss:** How do you think Troy created this recursive definition?

Troy

_____, 25, _____, 1, _____

First term: 49

Rule: Constant difference of -24

b What is something Troy can improve on?

7 Create a recursive definition for a sequence that makes the fourth term -40. Try to complete this challenge in different and interesting ways!

Sequence	First Term	Rule
_____, _____, _____, -40, _____		
_____, _____, _____, -40, _____		
_____, _____, _____, -40, _____		

Activity 3

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Challenge Creator

8 Now it's your turn to design your own sequence challenge.

a **Make it!**

- Write up to three terms anywhere in the sequence.
- Write a recursive definition for your sequence.

My Sequence	First Term	Rule
____, ____, ____, ____, ____		

b **Swap it!**

- Share your three terms and where they are in the sequence with a partner. Keep your recursive definition a secret!
- Write a recursive definition that completes your partner's sequence.

Partner's Sequence	First Term	Rule
____, ____, ____, ____, ____		

Partner's Sequence	First Term	Rule
____, ____, ____, ____, ____		

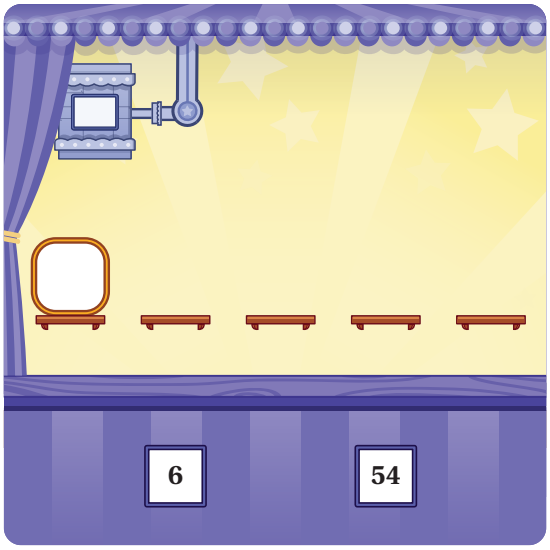
Partner's Sequence	First Term	Rule
____, ____, ____, ____, ____		

9 Synthesis

We learned that a recursive definition of a sequence includes a first term and a rule.

Describe a strategy for determining a recursive definition of a sequence that meets certain requirements.

Use the example if it helps with your thinking.



Things to Remember:

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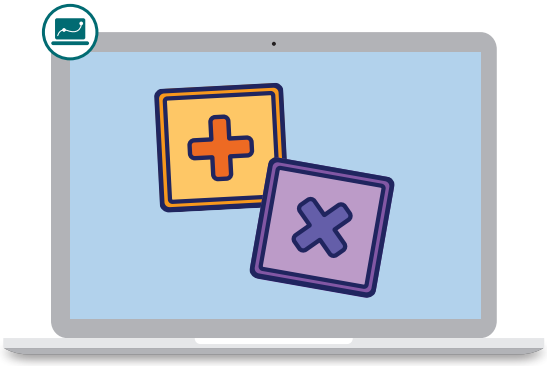
See the Sequence

Let's compare sequences using tables and graphs.



Warm-Up

1 Here are two sequences.

a Let's watch an animation to see how the machines create the first four terms of each sequence.

b What do you notice? What do you wonder?

I notice:

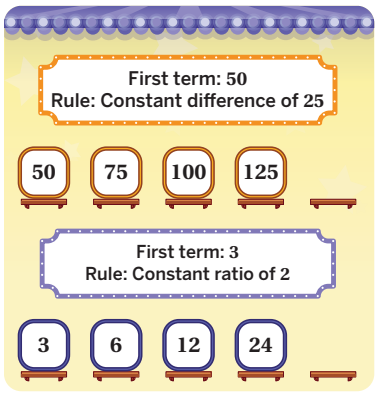
I wonder:

Activity 1

Name: _____ Date: _____ Period: _____

Sequence Types

2 The top sequence changes by a constant difference. We call that an **arithmetic sequence**. The bottom sequence changes by a constant ratio. We call that a **geometric sequence**. Which do you think will have a greater 10th term?



A. The arithmetic sequence

B. The geometric sequence

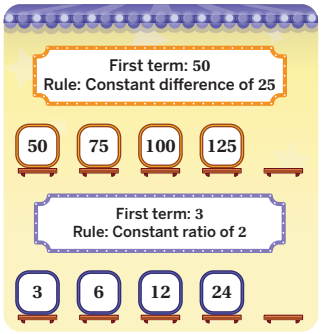
C. They will be the same

D. Not enough information

Explain your thinking.

3 Sequences can be represented in multiple ways.

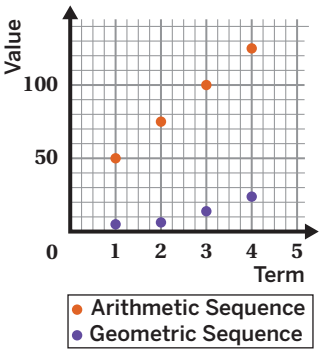
Recursive Definition



Table

Term	Arithmetic Sequence	Geometric Sequence
1	50	3
2	75	6
3	100	12
4	125	24

Graph



a **Discuss** What are the advantages and disadvantages of each representation?

b Choose one representation. Explain how it could be used to help determine which sequence has the greater 10th term.

Name: _____ Date: _____ Period: _____

Sequence Types (continued)

4 **a** Let's see which sequence has the greater 10th term.

b **Discuss** What do you notice about the graphs of the two sequences?

5 **a** Group together the cards that represent the same sequence.

Card A

Term	Value
1	3
2	1
3	-1
4	-3
5	-5

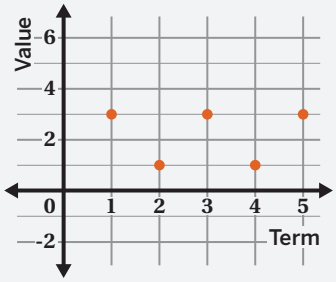
Card B

Term	Value
1	3
2	6
3	12
4	24
5	48

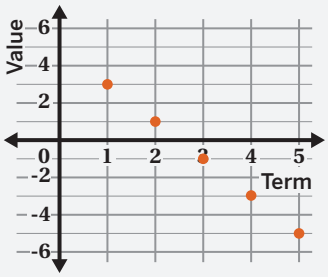
Card C

Term	Value
1	3
2	1
3	3
4	1
5	3

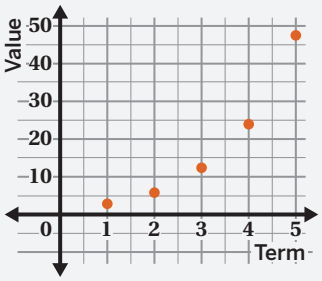
Card D



Card E



Card F



Sequence 1	Sequence 2	Sequence 3
First term: 3 **Rule:** Multiply the previous term by 2	**First term:** 3 **Rule:** Add -2 to the previous term	**First term:** 3 **Rule:** Alternate between 3 and 1

b **Discuss:** Is each sequence arithmetic, geometric, or neither?

Activity 2

Name: _____ Date: _____ Period: _____

Make Your Own Sequences

6 You will use Screen 6 to complete this activity.

a Create several different sequences and compare their tables and graphs. Then describe something that you found interesting or surprising.

b Decide whether each statement is *always*, *sometimes*, or *never* true. Explain your thinking.

Statement	Always, Sometimes, Never	Explanation
When you graph an arithmetic sequence, the points lie on a line.		
The graph of a geometric sequence curves upward.		
The graph of a geometric sequence with a positive first term will stay above the x-axis.		

c Write a recursive definition of a sequence that meets each set of criteria.

Criteria	Recursive Definition
Its graph lies on a horizontal line.	
It approaches 0 but never reaches it.	
Its 6th term is negative and 7th term is positive.	

Name: _____ Date: _____ Period: _____

Make Your Own Sequences (continued)

Explore More

7 Malik notices he can make an arithmetic sequence and a geometric sequence that have the same first two terms.

a In the table, continue the sequence in two ways: assuming it is *arithmetic* and assuming it is *geometric*.

b How long do you think it would take each sequence to reach 1,000?

Term	Arithmetic Sequence	Geometric Sequence
1	8	8
2	12	12
3		
4		
5		

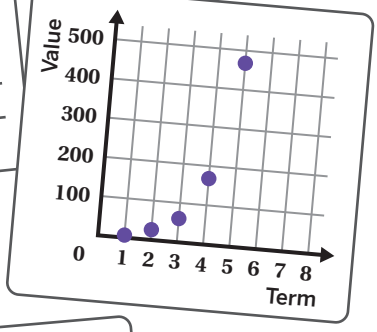
8 Synthesis

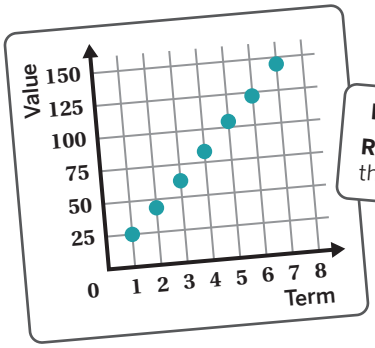
Sequences can be represented in multiple ways. What are some clues that a sequence might be arithmetic? Geometric?

Arithmetic:

Geometric:

Term	Value
1	6
2	18
3	54
4	162





First Term: 25
Rule: Add 20 to the previous term.

Things to Remember:

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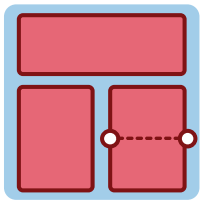
Paper Patterns



Let's represent situations in different ways.

Warm-Up

1. Match each *expression* with an equivalent one.

a. $10 + 3 + 3 + 3 + 3 + 3$ ..... $10 \cdot 3^5$

b. $10 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$ ..... $10 \cdot \left(\frac{1}{3}\right)^5$

c. $10 - 3 - 3 - 3 - 3 - 3$ ..... $10 + 5(-3)$

d. $10 \div 3 \div 3 \div 3 \div 3 \div 3$ ..... $10 + 5 \cdot 3$

2. **Discuss:** Why might it be useful to have multiple forms of an expression?

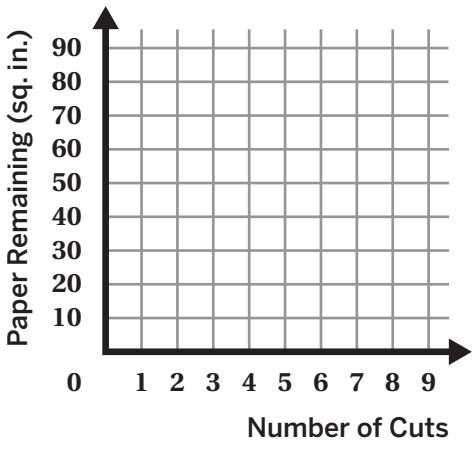
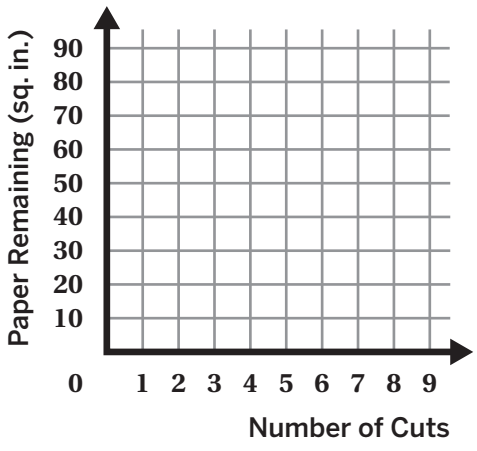
Activity 1

Name: _____ Date: _____ Period: _____

Paper Cutting

Each of these patterns starts with a piece of paper that measures 8-by-10 inches.

3. Complete each representation.

Pattern 1	Pattern 2
Situation Cut off half of the paper and discard. Repeat.	**Situation** Cut a 1-by-8-inch strip off the paper and discard. Repeat.
Table Number of Cuts / Paper Remaining (sq. in.) 0 / 80 1 / 2 / 3 /	**Table** Number of Cuts / Paper Remaining (sq. in.) 0 / 80 1 / 2 / 3 /
Graph 	**Graph** 

4. For each pattern, what is the area of the paper remaining after 9 cuts?

Activity 2

Name: Date: Period:

Explicit Expressions

5. Let's look at Wohali's and Kadeem's strategies for Pattern 1.

Kadeem wrote the expression $80 \cdot \left(\frac{1}{2}\right)^4$ to calculate the area of paper remaining after 4 cuts.

a What does each part of his expression represent in this situation?

80 represents . . .

$\frac{1}{2}$ represents . . .

4 represents . . .

b Write an expression that represents the area of the paper remaining after n cuts. This expression is an example of an **explicit definition**.

6. Let's think about Pattern 2.

a Select *all* the expressions that could be used to determine the area of the paper remaining after 9 cuts.

- ☐ **A.** $80 - 8 - 8 - 8 - 8 - 8 - 8 - 8 - 8 - 8$
- ☐ **B.** $80 + 9(8)$
- ☐ **C.** $80 \cdot (-8)^9$
- ☐ **D.** $80 - 9(8)$
- ☐ **E.** $80 \cdot 0.5^9$

b Write an explicit expression to determine the area of the paper remaining after n cuts.

c What does each part of your expression represent in this situation?

7. How are the explicit expressions for the two patterns alike? How are they different?

Name: _____ Date: _____ Period: _____

More Representations

8. You will use a set of cards to complete this activity. Group the cards based on whether they have a constant difference, a constant ratio, or neither.

Constant Difference	Constant Ratio	Neither

9. Choose one card with a constant difference.

a Create the matching representations.

Situation

Explicit Expression

Table

0	
1	
2	
3	
…	…
10	

b What does each part of your expression represent in this situation?

Name: _____ Date: _____ Period: _____

More Representations (continued)

10. Choose one card with a constant ratio.

a Create the matching representations.

Situation

Explicit Expression

Table

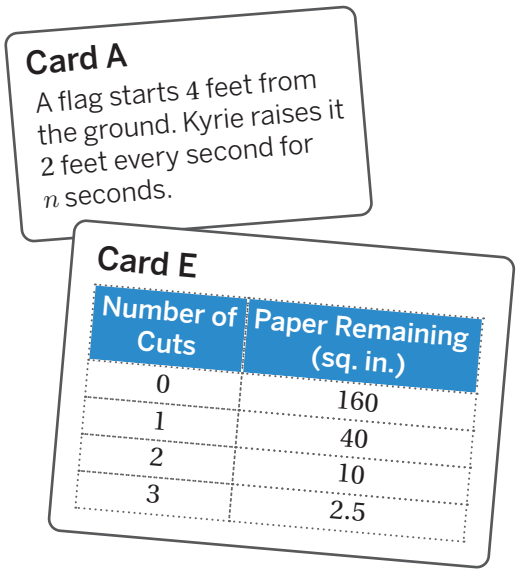
0	
1	
2	
3	
…	…
10	

b What does each part of your expression represent in this situation?

Synthesis

11. What information from a situation can help you write an explicit expression?

Use the cards if they help with your thinking.



Things to Remember:

Lesson 1.05
Activity 3

More Representations

Directions: Make one copy per pair of students. Then pre-cut the the cards and give each pair of students one set.

Card A

A flag starts 4 feet from the ground. Kyrie raises it 2 feet every second for n seconds.

Card B

The number of fish in a lake doubles each year for n years. The lake starts with 4 fish.

Card C

$$120 - 10n$$

Card D

$$5 \cdot 3^n$$

Card E

Number of Cuts, n	Paper Remaining (sq. in.)
0	160
1	40
2	10
3	2.5

Card F

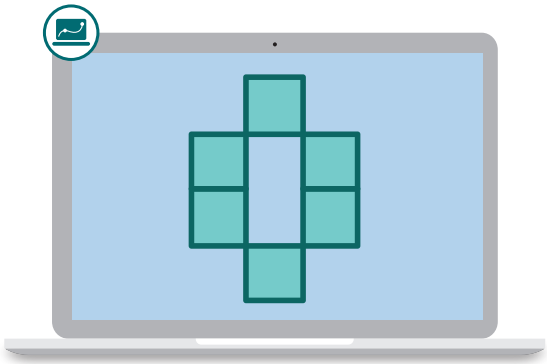
Years, n	Number of Fish
0	15
1	18
2	22
3	27

Unit 1
Lesson
6

Name: ..... Date: ..... Period: .....

More Visual Patterns

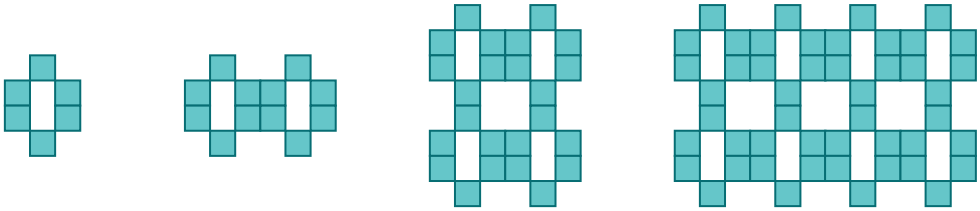
Let's write explicit expressions for arithmetic and geometric sequences.



Warm-up

1 How do you see this pattern growing?

Figure 1 Figure 2 Figure 3 Figure 4



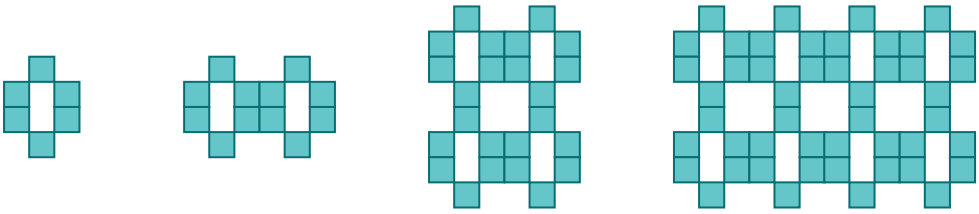
Activity 1

Name: _____ Date: _____ Period: _____

How Many Tiles?

2 Here is the pattern from the Warm-Up.

Figure 1 **Figure 2** **Figure 3** **Figure 4**



a How many tiles will there be in Figure 7?

b **Discuss** How could you determine the number of tiles in Figure 15?

3 Here is Zoe's strategy for determining the number of tiles in Figure 15.

Figure 1 **Figure 2** **Figure 3** **Figure 4**

6 tiles | 12 tiles | 24 tiles | 48 tiles

6 | 6·2 | 6·2·2 | 6·2·2·2

Figure 15: $6 \cdot 2^{15}$ tiles

a What did Zoe do well?

b What was Zoe's mistake?

4 Amir and Maria want to write an explicit expression for the number of tiles in Figure n.

Amir writes: $6 \cdot 2^{(n-1)}$ Maria writes: $3 \cdot 2^n$

Which explicit expression is correct? Explain your thinking.

A. $6 \cdot 2^{(n-1)}$ **B.** $3 \cdot 2^n$ **C.** Both **D.** Neither

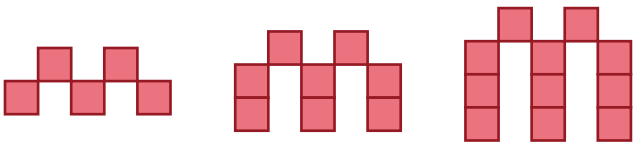
Activity 2

Name: _____ Date: _____ Period: _____

A New Pattern

5 Here is a new pattern.

Figure 1 Figure 2 Figure 3



How many tiles will there be in Figure 4 and Figure 15?

Figure	Number of Tiles
1	5
2	8
3	11
4	
...	...
15	

6 Here are Omari's and Ivory's responses for Figure 15.

Figure	Number of Tiles (Omari)	Number of Tiles (Ivory)
15	$5 + 3(14)$	$2 + 3(15)$

Discuss:

- Where did the numbers in Omari's and Ivory's work come from?

- How might each student determine the number of tiles in Figure 50?

7 Write an explicit expression for the number of tiles in Figure n.

Activity 3

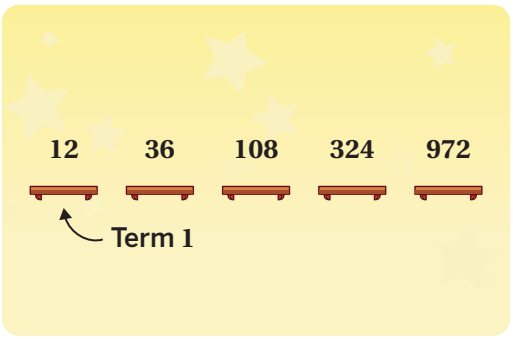
Name: _____ Date: _____ Period: _____

Sequences Only

8 Let's look at a sequence *without* a visual pattern.

Here are the first five terms of the sequence.

Write an explicit expression for Term n.

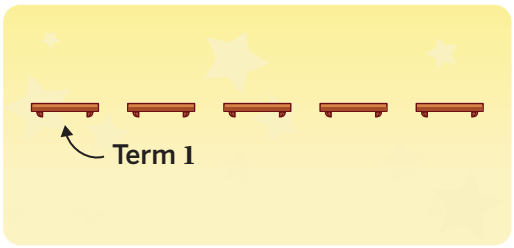


9 You will be designing a challenge for your classmates to solve.

a **Make it!**

- Write the first five terms of your arithmetic or geometric sequence.
- Write an explicit expression for Term n of your sequence.

My Challenge **Explicit Expression**



b **Solve it!**

- Share your sequence with a classmate. Keep your explicit expression a secret!
- Write an explicit expression for Term n of their sequence.

_____'s Challenge **Explicit Expression**

_____, _____, _____, _____, _____

_____'s Challenge **Explicit Expression**

_____, _____, _____, _____, _____

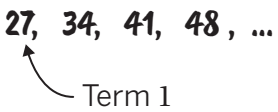
_____'s Challenge **Explicit Expression**

_____, _____, _____, _____, _____

10 Synthesis

Describe one strategy for writing an explicit expression for Term n of a sequence.

Use the example if it helps with your thinking.

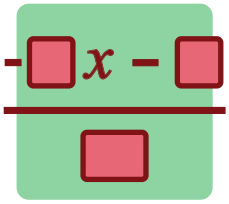


Things to Remember:

Unit 2
Lesson
2

Name: ..... Date: ..... Period: .....

Solving Strategies



Let's explore strategies for solving equations.

Warm-Up

Equivalent equations have the exact same solution.

1. **Discuss:** Which of these equations are equivalent to each other? How do you know?

Equation A	Equation B
$-2(2x + 3) = 10$	$2x + 3 = -5$
Equation C	**Equation D**
$2x + 3 = 12$	$-4x - 6 = 10$

Activity 1

Name: Date: Period:

Equivalent or Not?

For each pair of equations, determine whether they are equivalent.

- If they're equivalent, explain how to get from one equation to the other.
- If they're not equivalent, explain how you know.

2. $5x = 24 + 2x$

$3x = 24$

Are the equations equivalent?

Explanation:

3. $-3(2x + 9) = 12$

$-4 = 2x + 9$

Are the equations equivalent?

Explanation:

4. $\frac{1}{2}x - 8 = 9$

$x - 8 = 18$

Are the equations equivalent?

Explanation:

5. **a** Write one equation that is equivalent to $12 = 5x - 30 + x$ and one equation that is not.

b Trade papers with a classmate. Then circle the equation they wrote that is equivalent to $12 = 5x - 30 + x$. Show or explain your thinking on their paper.

Activity 2

Name: _____ Date: _____ Period: _____

Step It Up

Here are Sadia's and Amir's steps for correctly solving the same equation.

Sadia

$$6 - 7x = \frac{-15x - 12}{3}$$
$$6 - 7x = -5x - 4$$
$$10 - 7x = -5x$$
$$10 = 2x$$
$$5 = x$$

Amir

$$6 - 7x = \frac{-15x - 12}{3}$$
$$18 - 21x = -15x - 12$$
$$18 - 6x = -12$$
$$-6x = -30$$
$$x = 5$$

6. **Discuss:** How did each student solve the equation?

7. How did Amir and Sadia take different first steps but have the same solution?

8. Caleb and Roberto also tried to solve the equation but made some errors. For each student's work:

a **Discuss:** What is correct? What is incorrect?

b Write a question to help each student see how they could revise their work.

Caleb

$$6 - 7x = \frac{-15x - 12}{3}$$
$$-1x = -5x - 4$$
$$4x = -4$$
$$x = -1$$

Roberto

$$6 - 7x = \frac{-15x - 12}{3}$$
$$6 - 7x = -5x - 4$$
$$2 - 7x = -5x$$
$$2 = 2x$$
$$1 = x$$

Activity 3

Name: Date: Period:

The Choice Is Yours

9. Examine these equations. Organize the equations into two or three groups based on patterns you notice.

Equation A

$2(2a + 1.5) = 17 - 3a$

Equation B

$-\frac{1}{2}(b + 3) - 5 = -\frac{7}{2}$

Equation C

$\frac{-6 + 4c}{2} = 3(2c + 1)$

Equation D

$4d + 1 = -2(d - 5)$

Equation E

$\frac{x}{4} - 2 = 2x + 5$

Equation F

$9f + 3 - (f - 1) = 2(3f + 1)$

Equation G

$g - 4 = -\frac{8 + 4g}{8}$

Equation H

$h + 5h + 20 = h - 6 + h$

Group A	Group B	Group C

10. **Discuss:** How did you group the equations?

11. Choose *three* equations to solve. (Choose at least one equation from each group.) Show your thinking.

Explore More

12. Two of the equations in this activity are equivalent. Identify the two equivalent equations and explain your thinking.

Synthesis

13. **a** Write an equation you think is challenging to solve.

b What makes your equation challenging to solve?

c What are some strategies or tips for solving equations like this?

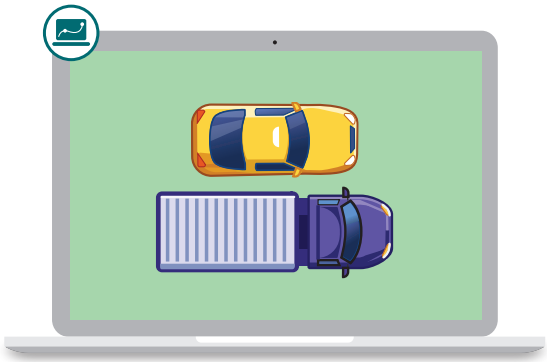
Things to Remember:

Unit 2
Lesson
3

Name: ..... Date: ..... Period: .....

Same Position

Let's explore how many solutions are possible for a one-variable equation.



Warm-Up

1 Let's watch the animation.

Write a story about what you see.

Activity 1

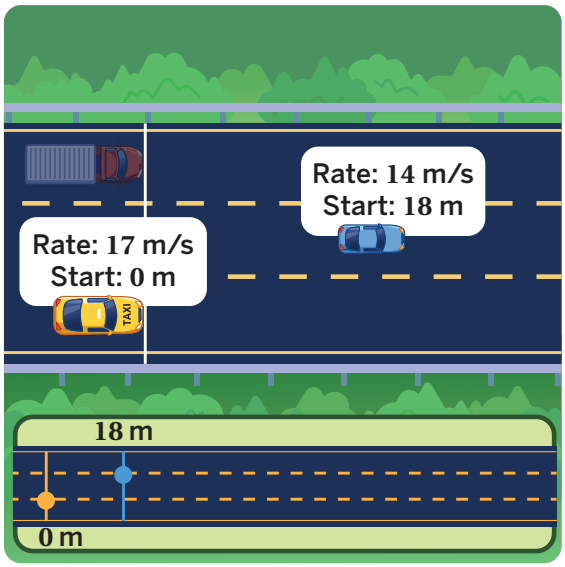
Name: _____ Date: _____ Period: _____

Same Position

2 The car and the taxi are moving at constant speeds.

Car Position Expression	Taxi Position Expression
$14t + 18$	$17t$

At what time, t, will the car and the taxi be in the same position?

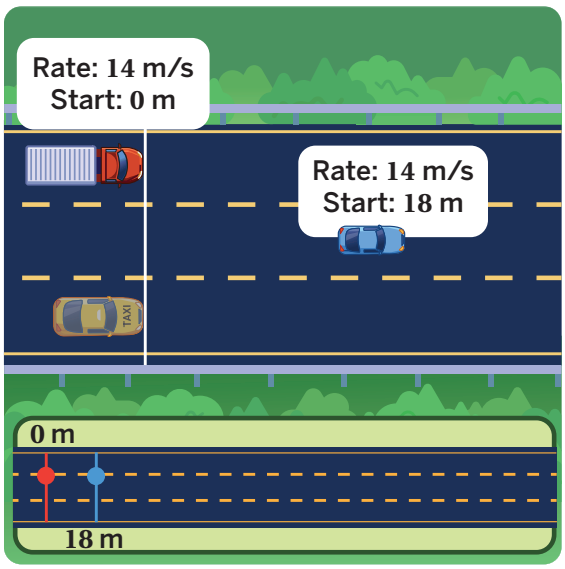


3 The truck and the car are moving at constant speeds.

Truck Position Expression	Car Position Expression
$14t$	$14t + 18$

a Let's see what happens at different times.

b **Discuss:** What do you know about when the truck and the car will be in the same position?



4 Here is Antwon's work on the previous problem.

What does his work say about the time, t, when the truck and the car will be in the same position?

Antwon

$$14t = 14t + 18$$
$$-14t \quad -14t$$
$$0 = 0 + 18$$

Name: _____ Date: _____ Period: _____

Same Position (continued)

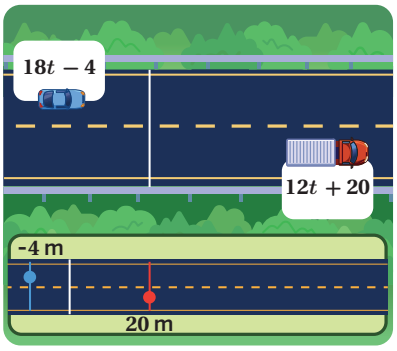
5 The following equations represent when these vehicles will be in the same position.

a $18t - 4 = 12t + 20$

How often will they be in the same position? Circle one.

Once Never Always

If once, then after how many seconds? _____

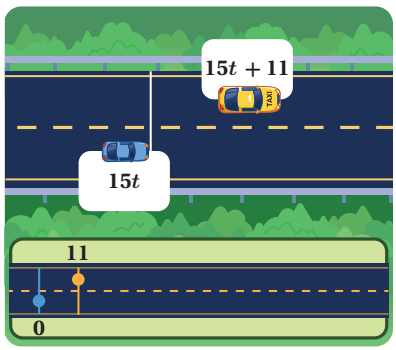


b $15t + 11 = 15t$

How often will they be in the same position? Circle one.

Once Never Always

If once, then after how many seconds? _____

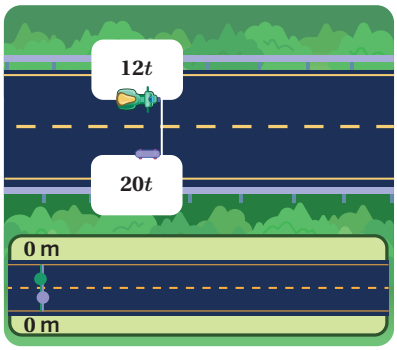


c $12t = 20t$

How often will they be in the same position? Circle one.

Once Never Always

If once, then after how many seconds? _____

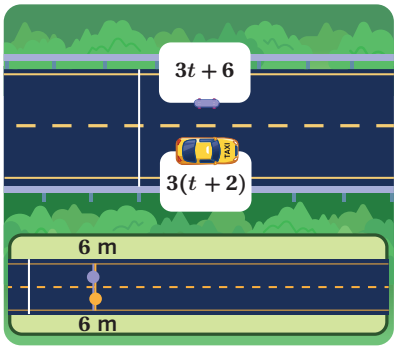


d $3t + 6 = 3(t + 2)$

How often will they be in the same position? Circle one.

Once Never Always

If once, then after how many seconds? _____



Activity 2

Name: _____ Date: _____ Period: _____

Once, Never, Always

6 Here are Jazz's and Nikhil's strategies for solving a challenge from the previous activity.

Jazz

$12t = 20t$

$-12t \quad -12t$

$0 = 8t$

$0 = t$

they will meet when $0 = t$

Nikhil

$\frac{12t}{t} = \frac{20t}{t}$

$12 = 20$

they will never meet

Discuss: Is each strategy correct?

7 Each equation represents the time, t, when two vehicles will meet.

$12 - t = t - 12$	$t + 1 = t + 1$	$t = t + 2$
$2t + 6 = 2(t + 3)$	$2t = 8t$	$8(t + 1) = 8t - 8$

Sort the six equations based on how often the vehicles will be in the same position.

Once	Never	Always

Name: _____ Date: _____ Period: _____

Once, Never, Always (continued)

8 Darryl and Jasmine solved $t + 1 = t + 1$ and got $0 = 0$.

- Darryl says the vehicles will never be in the same position.
- Jasmine says the vehicles will always be in the same position.

Who is correct? Explain your thinking.

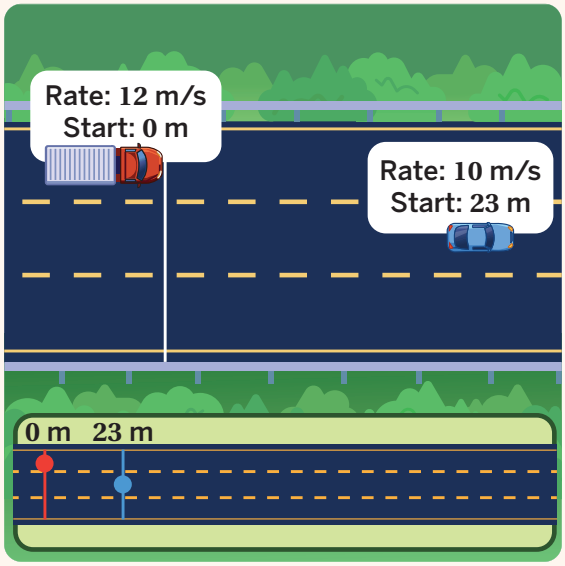
Explore More

9 The truck and the car are moving at constant speeds.

Write an expression, in terms of t, for the position of a taxi that makes *both* statements true:

- The taxi and the truck will *never* be in the same position.
- The taxi and the car will be in the same position when $t = 8$.

Vehicle	Position Expression
Truck	$12t$
Car	$10t + 23$
Taxi	

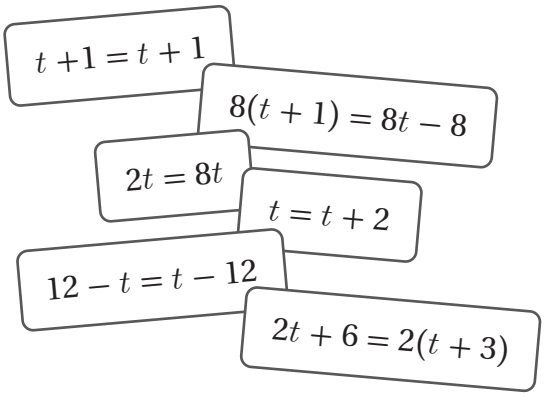


10 Synthesis

How can you tell whether an equation will have:

- No solution?

- Infinitely many solutions?



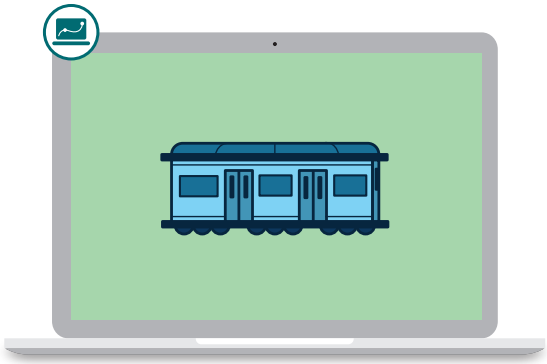
Things to Remember:

Unit 2
Lesson
4

Name: ..... Date: ..... Period: .....

Subway Seats

Let's explore what different forms of linear equations reveal about a situation.



Warm-Up

1 Which one doesn't belong?

Equation A	Equation B	Equation C	Equation D
$x + y = 5$	$x + y - 5 = 0$	$x = 5 - y$	$5 + x = y$

Explain your thinking.

Activity 1

Name: _____ Date: _____ Period: _____

Crowded Subways

2 Some subway cars can be crowded.

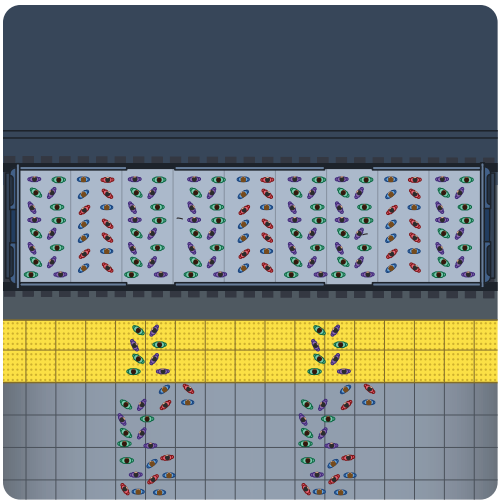
In order to fix this, the transit authority decided to remove seats to fit more people.

Some residents feel that this isn't fair. Why might they feel this way?

3 Some subways removed all of the seats to allow more room for people to stand.

- A subway car has about 600 square feet.
- A standing passenger requires 2 square feet.

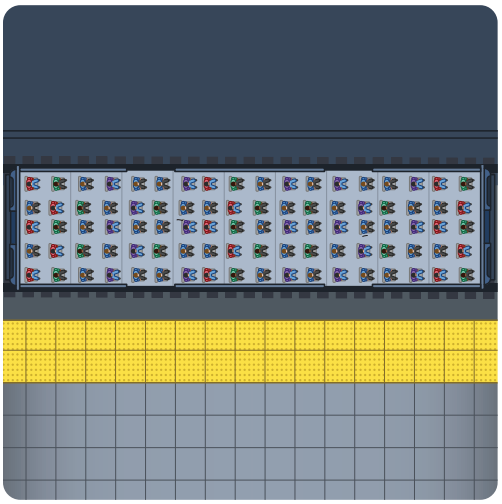
What is the *standing capacity* on this subway car with no seats?



4

- A subway car has 600 square feet of floor space.
- A seat requires 6 square feet.

What is the *seating capacity* on this subway car with no room to stand?



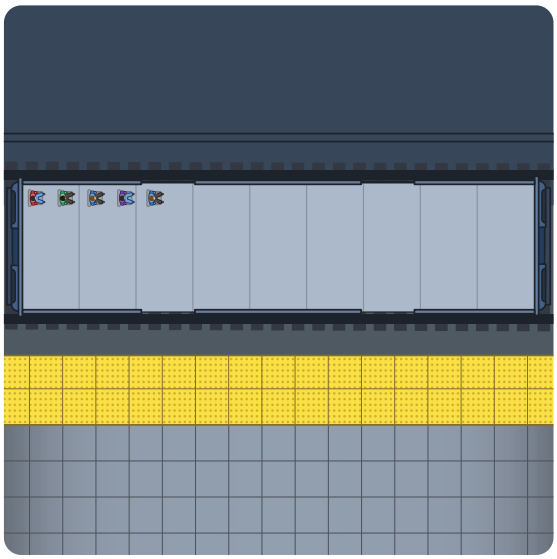
Name: _____ Date: _____ Period: _____

Crowded Subways (continued)

5 The capacity of this subway car is modeled by $6t + 2d = 600$, where t is the seating capacity and d is the standing capacity.

For each number of seats, determine how many standing passengers can fit.

Seating Capacity, t	Standing Capacity, d
5	
10	
15	



6 Here is Tiam's strategy for determining the number of standing passengers that can fit when you know the number of seats.

- t is the seating capacity.
- d is the standing capacity.

What do 300 and -3 mean in this situation?

Tiam

$$6t + 2d = 600$$
$$-6t \quad -6t$$
$$\frac{2d}{2} = \frac{600 - 6t}{2}$$
$$d = 300 - 3t$$

300:

-3:

Activity 2

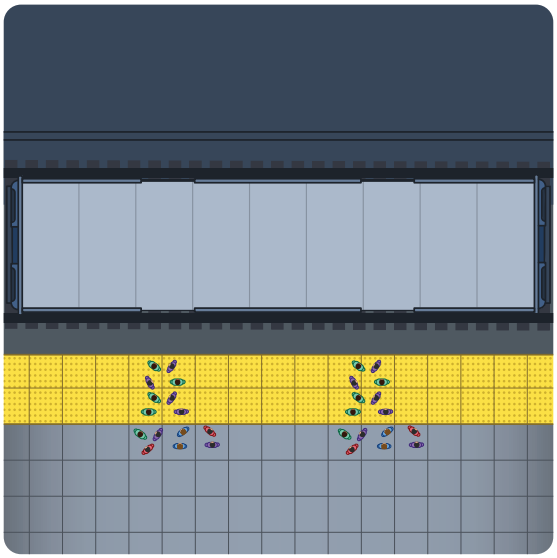
Name: _____ Date: _____ Period: _____

Standing and Sitting

7 The capacity of this subway car is modeled by $6t + 2d = 600$, where t is the seating capacity and d is the standing capacity.

For each number of standing passengers, determine the seating capacity.

Seating Capacity, t	Standing Capacity, d
	30
	150
	240



8 Solve for t so that the transit authority can calculate the seating capacity for any standing capacity.

9 Group the equations that represent the same relationship. One equation will have no match.

$x = \frac{50 - 3y}{2}$ $x = \frac{50 - 2y}{3}$ $y = \frac{50 - 3x}{2}$ $y = \frac{50 - 2x}{2}$

$2x + 3y = 50$	$3x + 2y = 50$

10 Synthesis

Here are two equations that we considered in this lesson about subway capacity.

$$6t + 2d = 600$$
$$t = 100 - \frac{1}{3}d$$

- t is the seating capacity.
- d is the standing capacity.

Pick *two* numbers and explain what they mean in this situation.

6:

2:

600:

100:

$-\frac{1}{3}$:

Things to Remember:

Unit 2
Lesson
6

Name: ..... Date: ..... Period: .....

Shelley the Snail

Let's connect graphs, tables, and equations to the situations they represent.

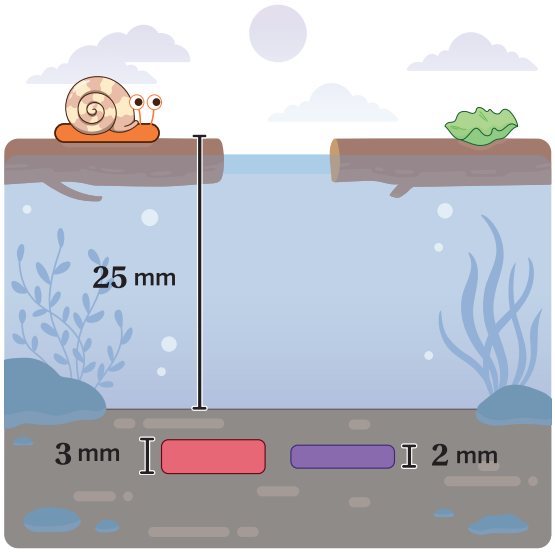
Warm-Up

1 Shelley the Snail loves eating lettuce leaves.

She needs to cross the gap to get them.

Write different combinations of blocks that fill the gap.

Number of 3 mm Blocks	Number of 2 mm Blocks

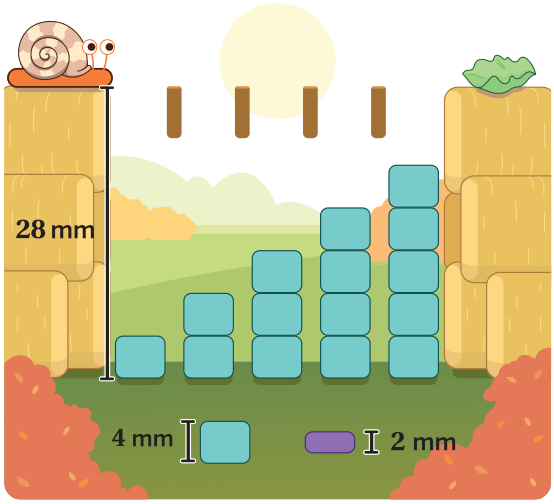


Activity 1

Name: _____ Date: _____ Period: _____

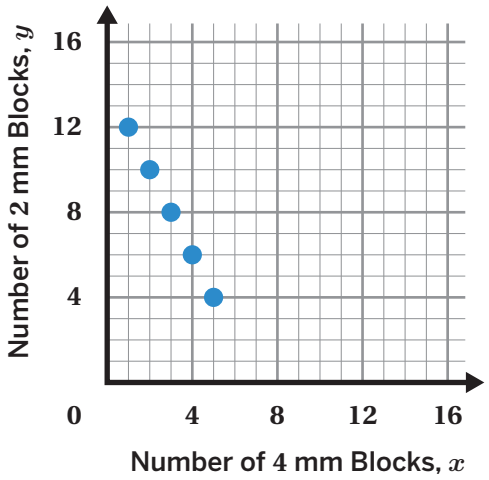
Mind the Gap

2 Complete the table to help Shelley get to the lettuce.



Number of 4 mm Blocks, x	Number of 2 mm Blocks, y
1	
2	
3	
4	
5	

3 Shelley has to cross a gap that is 28 mm deep using 4 mm and 2 mm blocks. This graph shows some of the possible combinations of blocks. Complete the table.



Number of 4 mm Blocks, x	Number of 2 mm Blocks, y
0	
6	
7	

4 Here is Tyrone's work from the previous problem. Write an equation for the number of 2 mm blocks, y, needed for any number of 4 mm blocks, x.

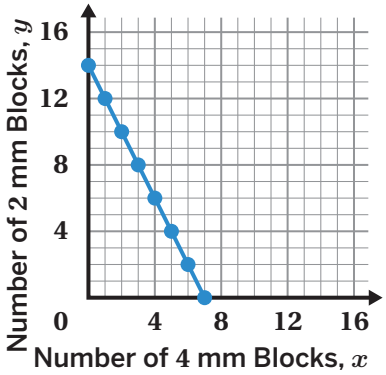
Number of 4 mm Blocks, x	Number of 2 mm Blocks, y
0	14 − 2(0)
6	14 − 2(6)
7	14 − 2(7)
x	?

Name: _____ Date: _____ Period: _____

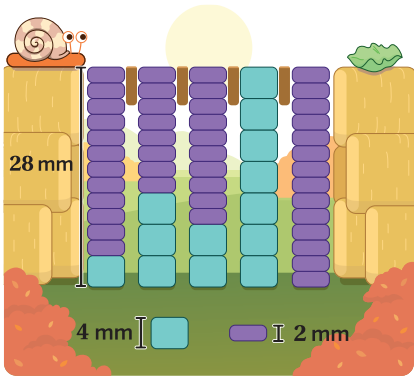
Mind the Gap (continued)

5 The x-*intercept* of the graph is $(7, 0)$. The y-*intercept* of the graph is $(0, 14)$.

Graph



Situation



Equation

$y = 14 - 2x$

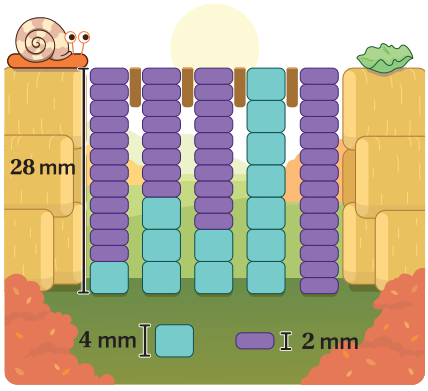
a Select *one* representation.

b Show or explain where you see the intercepts.

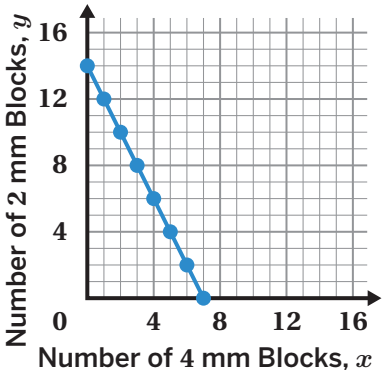
x-intercept: y-intercept:

6 Here is the same relationship represented in two different ways.

$4x + 2y = 28$



$y = 14 - 2x$



Discuss:

- How do you see the equation in each representation?
- Are these equations equivalent? Why or why not?

Activity 2

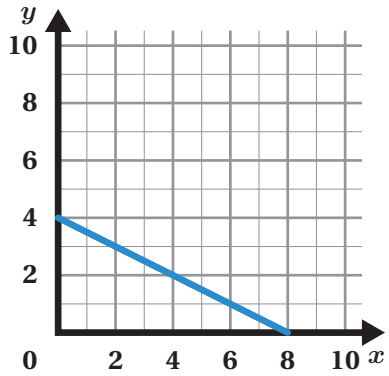
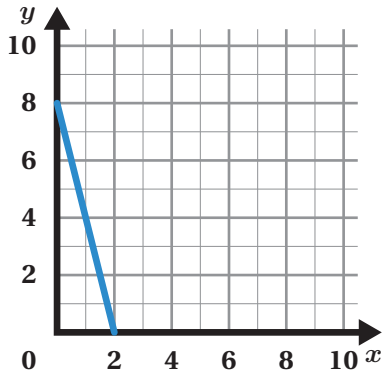
Name: _____ Date: _____ Period: _____

Rearrange It

7 Solve the equation $4x + 2y = 28$ for y to show that it is equivalent to $y = 14 - 2x$. Show or explain your thinking.

8 Match each graph with two equations. Two equations will have no match.

$2x + 8y = 24$	$y = 8 - 2x$	$2x + 4y = 16$
$y = 4 - \frac{1}{2}x$	$8x + 2y = 16$	$y = 8 - 4x$

Graph		
Equations		

9 Rearrange each equation to solve for y.

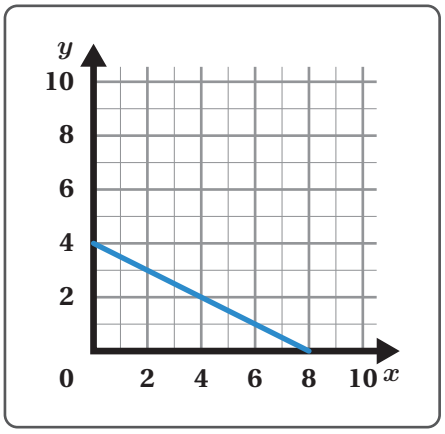
$6x + 2y = 34$	$5x + 2y = 46$
$y =$	$y =$
$2x + 4y = 26$	$3x + 4y = 40$
$y =$	$y =$

10 Synthesis

How would you convince a classmate that these three cards represent the same situation?

$2x + 4y = 16$

$y = 4 - \frac{1}{2}x$



Things to Remember:

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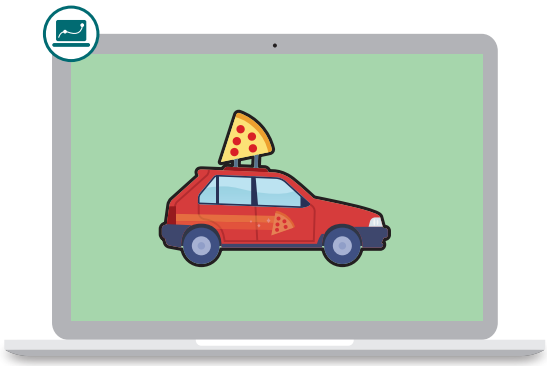


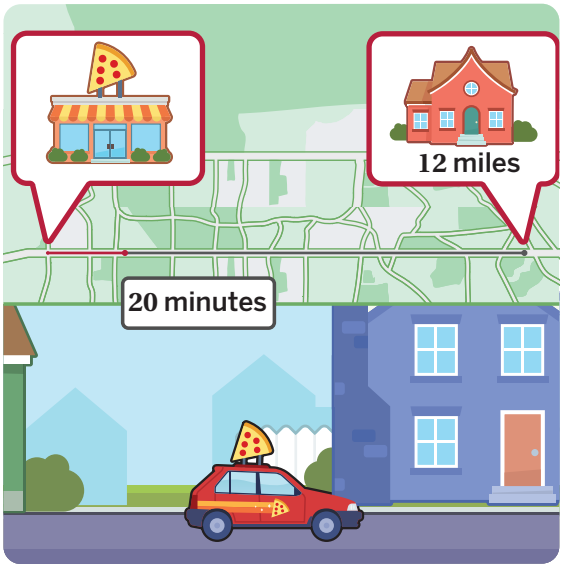
Pizza Delivery

Let's write inequalities to represent constraints.

Warm-Up

1 Let's watch an animation.

Write a story about what you see.



Activity 1

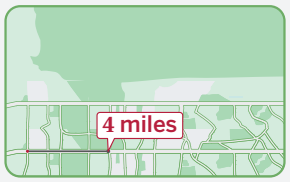
Name: _____ Date: _____ Period: _____

Order Up!

2 Desmos Pizza is now offering delivery!

It takes 15 minutes to prepare an order and 3 minutes to drive each mile.

How long would it take to deliver each order?

Distance (mi)			
Time (min)			

3 Here is Mariana's work from the previous problem.

Write an expression for the number of minutes it would take to deliver a pizza x miles away.

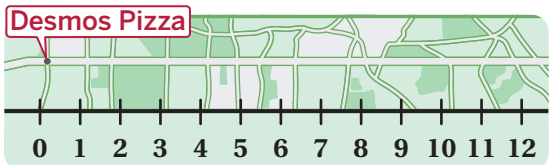
Distance (mi)	Time (min)
2	15 + 6
4	15 + 12
8	15 + 3(8)
x	?

4 Desmos Pizza wants to deliver orders in 30 minutes or less.

a Which inequality represents this situation?

A. $15 + 3x \leq 30$ **B.** $15 + 3x \geq 30$ **C.** $15 + 3x > 30$ **D.** $15 + 3x < 30$

b Show or describe all the distances they can deliver to in 30 minutes or less.



Name: _____ Date: _____ Period: _____

Order Up! (continued)

5 Here is how Mariana and Camila figured out all of the distances in which pizza could be delivered in 30 minutes or less.

Mariana

Miles, x	Minutes
1	18
2	21
3	24
4	27
5	30
6	33

$x \leq 5$

You have to live 5 miles away or less.

Camila

$15 + 3x = 30$

$-15 \quad -15$

$3x = 15$

$x = 5$

5 miles is the farthest you can live from Desmos Pizza.

Discuss: Why is each strategy helpful? Identify a weakness of each strategy.

6 A large pizza is \$12 and toppings are \$2 each. Mariana can spend as much as \$20 on a pizza.

a If x is the number of toppings, which of these inequalities represents the situation?

A. $12 + 2x \geq 20$ **B.** $12x + 2 \geq 20$ **C.** $12 + 2 \leq 20x$ **D.** $12 + 2x \leq 20$

b Show or describe all the numbers of toppings, x, that Mariana can get for \$20 or less.

7 A **constraint** is a limitation on possible values in a model. Here are some examples of constraints:

- Desmos Pizza wants to deliver in 30 minutes or less.
- Mariana can spend as much as \$20 on a pizza.
- Desmos Pizza wants to sell more than 50 pizzas per day.

What is a constraint in *your* life?

Activity 2

Name: _____ Date: _____ Period: _____

Trampoline World

8 Jamir is planning to host a party at Trampoline World.

Discuss: What constraints might Jamir think about when planning this party?

9 Hosting a party at Trampoline World costs a flat fee of $80, plus $30 per hour for the small room or $50 per hour for the large room.

Match each constraint to an inequality, where x represents the number of hours for the party. One inequality will have no match.

$30 + 80x \geq 140$	$80 + 30x \leq 140$	$80 + 50x \geq 140$	$80 + 80x \leq 140$

Mariana's party in the small room costs at most $140.	The owner wants to earn at least $140 for a party in the large room.	Amoli can spend up to $140 for a party that uses both the big and small rooms.

Name: _____ Date: _____ Period: _____

Trampoline World (continued)

10 Jamir can spend as much as $155 for a party in the large room.

Write an inequality to match this new constraint, where x represents the number of hours for the party.

11 Select *all* the possible numbers of hours, x, that could work with Jamir's constraints.

- ☐ **A.** 0.75 hours
- ☐ **B.** 1 hour
- ☐ **C.** 1.5 hours
- ☐ **D.** 2.25 hours
- ☐ **E.** 3 hours

10 Synthesis

What are some suggestions you have for writing a constraint as an inequality?

Things to Remember:

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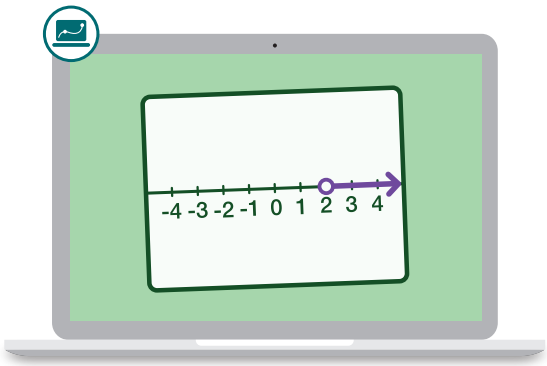
Name: ..... Date: ..... Period: .....

Graphing Inequalities

Let's represent solutions to inequalities on a number line.

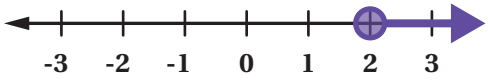


Warm-Up

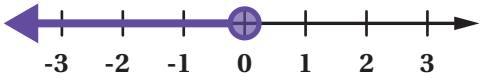
1 Here are four different inequalities.

Discuss: What do you notice?

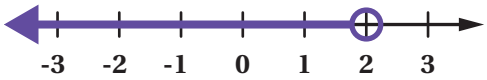
$x \geq 2$



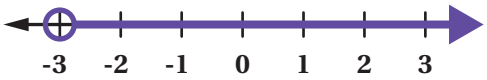
$x \leq 0$



$x < 2$

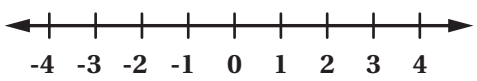


$x > -3$

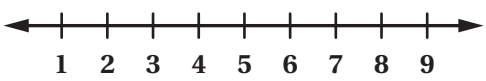


2 Make a graph of *all* the solutions to each inequality.

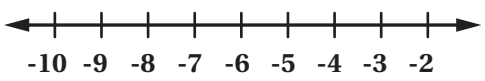
$x > -1.5$



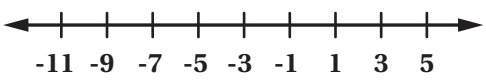
$x \leq 7$



$x < -5$



$x \leq -6$



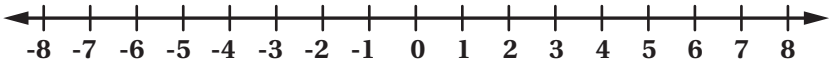
Activity 1

Name: ..... Date: ..... Period: .....

Show a Solution

3 **a** Plot a solution to this inequality.

$2x - 8 > 0$

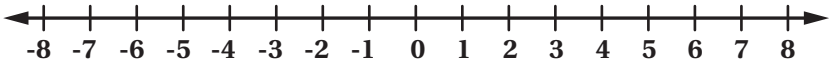


Share your response with your classmates.

b **Discuss:** Are any of the points incorrect? How do you know?

4 **a** Plot a solution to this inequality.

$6x \leq 3x$



b Explain how you know that your point is a solution (or why there is no solution).

5 Lan explains that $6x \leq 3x$ does not have a solution: *6 of something is always more than 3 of the same thing*.

Is Lan's statement correct? Circle one.

Yes No I'm not sure

Show or explain your thinking.

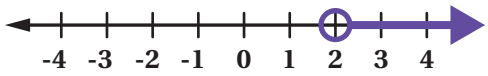
Activity 2

Name: ..... Date: ..... Period: .....

Solution Sets

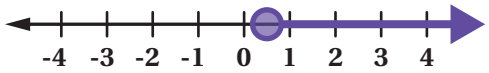
6 How can you check that this inequality and graph represent the solutions to $\frac{1}{4}x > \frac{1}{2}$?

$x > 2$

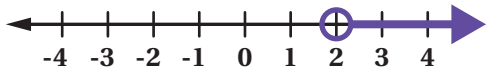


7 Match each inequality to a number line that represents its solutions.

a. $5x + 4 \geq 7x$ .....



b. $3 - x < 1$ .....



c. $2(x + 3) \geq 7$ .....

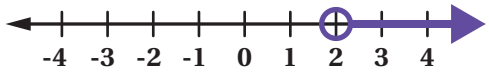
d. $8x - 2 < 4x$ .....

Name: _____ Date: _____ Period: _____

Solution Sets (continued)

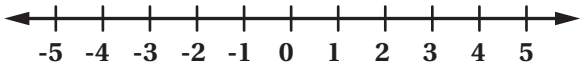
8 Lan matched this inequality and number line.

$$5x + 4 \geq 7x$$



How could you convince Lan that this inequality and number line don't match?

9 Create a graph of the solutions to the inequality $15 - x < 11$.

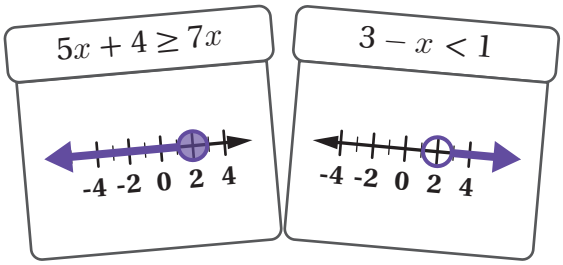


Explain your thinking.

10 Synthesis

Explain how you can determine whether a number line represents the solutions to an inequality.

Use these examples if they help with your thinking.



Things to Remember:

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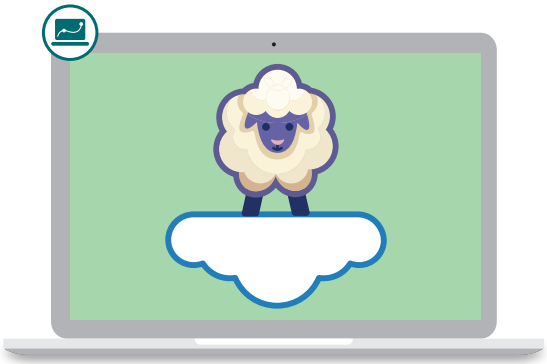
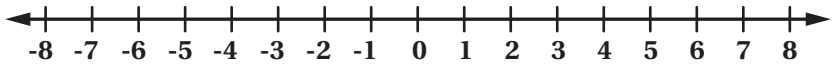
Solutions and Sheep

Let's make connections between solving one-variable equations and solving inequalities.



Warm-Up

1 **a** Plot *three* different solutions to the inequality $10 - 5x < 0$.



Share your response with your classmates.

b **Discuss:** Are any of the points incorrect? How do you know?

2 Let's look at all the correct solutions to $10 - 5x < 0$.

Kayleen and Leo are discussing how to write the solutions to this inequality.

Kayleen says the solutions are $x < 2$.
Leo says the solutions are $x > 2$.

Whose claim is correct? Circle one.

Kayleen's Leo's Both Neither

Explain your thinking.

Activity 1

Name: _____ Date: _____ Period: _____

Feed the Sheep

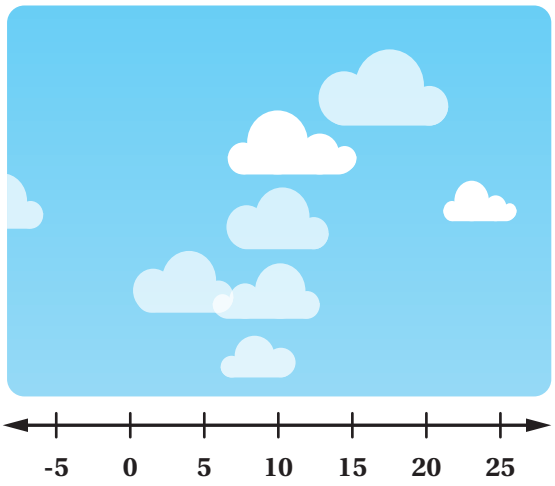
3 Shira the Sheep loves eating grass. She does *not* like water.

a. Let's watch what happens when we try out different inequalities.

b. **Discuss:** What do you notice?

4 Here is an inequality: $\frac{x}{3} \geq 5$.

Solve the inequality to help Shira eat all the grass.



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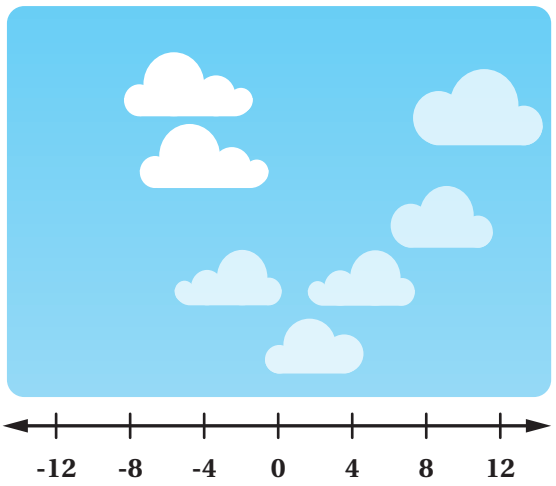
Feed the Sheep (continued)

5 Here is a new inequality: $9 > 2x - 4$.

Solve the inequality to help Shira eat all the grass.



6 Here is some of Kayleen's work from the previous problem.

$9 > 2x - 4$

$9 = 2x - 4$

$13 = 2x$

$6.5 = x$

Test

$x = 6$

$9 > 2(6) - 4$

$9 > 8$ ✓

$x = 7$

$9 > 2(7) - 4$

$9 > 10$

False!

a **Discuss:** What do you notice and wonder about Kayleen's strategy?

b Describe how Kayleen's work can help her decide where to shade the solutions on the number line.

Activity 2

Name: Date: Period:

Solving for Sheep

7 Here is a new inequality: $8x - 4 < 10x + 2$.

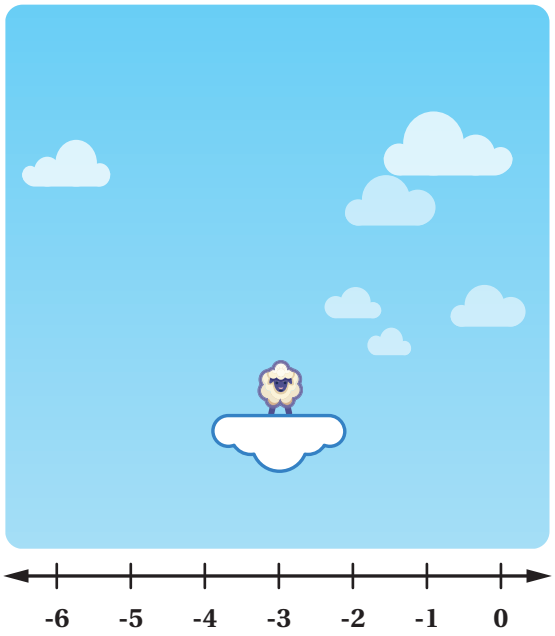
Solve the inequality to help Shira eat all the grass.

8 Leo was trying to solve the previous inequality: $8x - 4 < 10x + 2$.

He knew the sheep needed to land at -3, but didn't know if the grass was to the *right* or the *left*.

He wrote $8(0) - 4 < 10(0) + 2$.

How might that help Leo decide where the grass is?



Name: Date: Period:

Solving for Sheep (continued)

9 Here is a new inequality: $-2(x + 2) \leq -23$

Solve the inequality to help Shira eat all the grass.

10 For each of the challenges:

- Decide with your partner who will complete Column A and who will complete Column B.
- Solve as many inequalities as you have time for.
- The solutions in each row should be the same. Compare your solutions, then discuss and resolve any differences.

Column A	Column B
$2x + 10 > 20$	$\frac{x}{5} + 9 > 10$
$\frac{x}{5} + 10 \leq 12$	$5x - 10 \leq 40$
$-10 \geq 4x + 4$	$14 \leq -2x + 7$
$3 - 2x > 3$	$8 > 4x + 8$
$\frac{x + 3}{5} < 2$	$\frac{x + 8}{5} < 3$
$5x + 10 \geq 3x + 12$	$2x - 4 \geq x - 3$

11 Synthesis

Describe a strategy for solving any inequality.

Use the examples if they help with your thinking.

$10 - 5x < 0$

$9 > 2x - 4$

$8x - 4 < 10x + 2$

Things to Remember:

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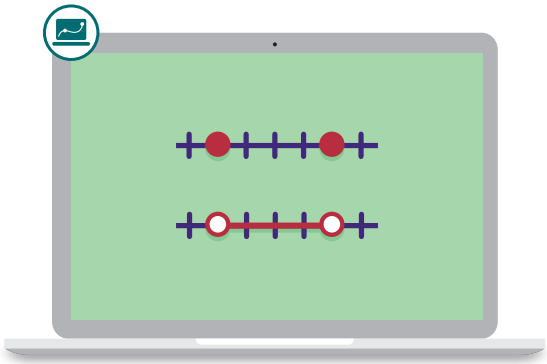
Absolute Value Solutions

Let's solve absolute value equations and inequalities.

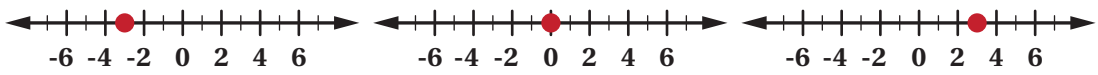


Warm-Up

1 $|x|$ is pronounced "the *absolute value* of x."

a Here are three different values of $|x|$.

$|\text{-}3| = 3$ $\quad$ $|0| = 0$ $\quad$ $|3| = 3$



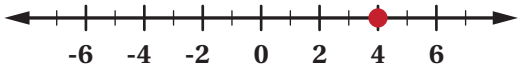
b How would you explain to someone else what $|x|$ means?

Activity 1

Name: Date: Period:

Showing Solutions

2 Here is one solution to $|x| = 4$.



Show another solution to $|x| = 4$.

Explain your thinking.

3 **a** Show a solution to $|x| < 4$.

Share your response with your classmates.

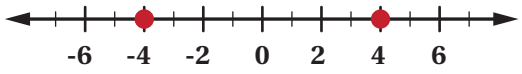
b **Discuss:**

- Are any points incorrect?
- Are any solutions missing?

4 Here are all the solutions to $|x| = 4$ and $|x| < 4$.

How are the solutions alike? How are they different?

$|x| = 4$



$|x| < 4$

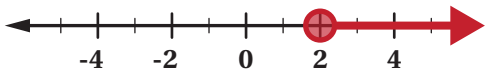
Alike:

Different:

Name: _____ Date: _____ Period: _____

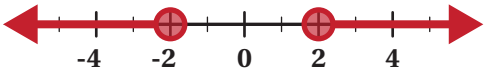
Showing Solutions (continued)

5 Which number line represents the solutions to $|x| \geq 2$?

A. 

B. 

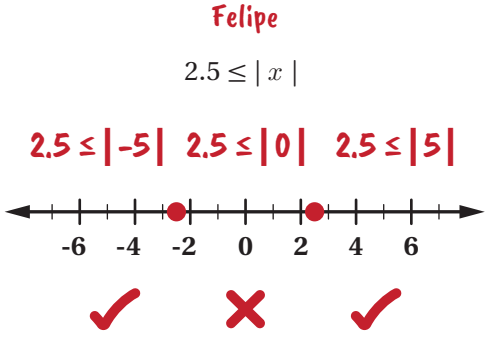
C. 

D. 

6 Felipe is graphing the solutions to $2.5 \leq |x|$.

Here is some of his work.

Describe how Felipe's work can help him decide where to shade the solution set on the number line.



Activity 2

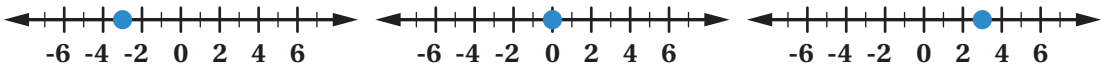
Name: _____ Date: _____ Period: _____

Solving Strategies

7 Polina says the value of $|x - 2|$ is the distance between any number and 2.

a Here are three different values of $|x - 2|$.

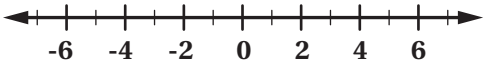
$|-3 - 2| = 5$ $|0 - 2| = 2$ $|3 - 2| = 1$



b **Discuss:** Do you agree with Polina?

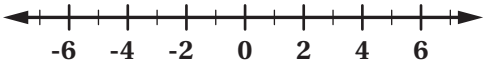
8 **a** Show or describe all the solutions to $|x - 2| = 4$.

$|x - 2| = 4$



b Show or describe all the solutions to $|x - 2| \leq 4$.

$|x - 2| \leq 4$



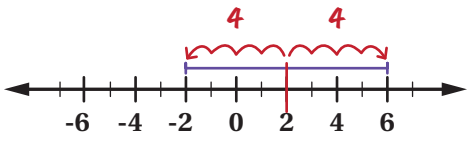
Name: _____ Date: _____ Period: _____

Solving Strategies (continued)

9 Two students graphed the solutions to $|x - 2| \leq 4$.

Esi

$|x - 2| \leq 4$

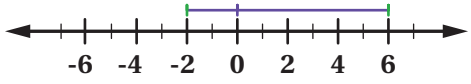


Deja

$|x - 2| \leq 4$

$(x - 2) = 4$ $-(x - 2) = 4$

$x = 6$ $x = -2$



$|0 - 2| \leq 4$ ✓

Discuss: How would you describe each students' strategy?

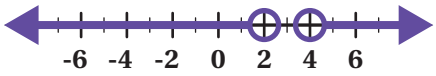
10 Match each inequality to the number line that represents its solutions.

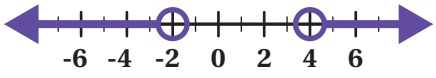
a. $|x - 3| > 1$ _____

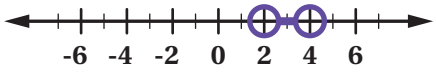
b. $|x - 3| < 1$ _____

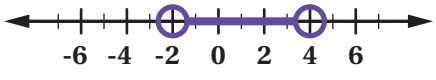
c. $|x - 1| > 3$ _____

d. $|x - 1| < 3$ _____









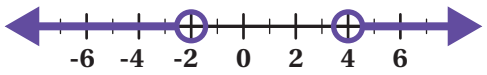
Activity 3

Name: _____ Date: _____ Period: _____

Repeated Challenges

11 Raven wants to use the graph of $|x - 1| > 3$ to help her make a graph of $|x + 1| > 3$.

$| x - 1 | > 3$



a **Discuss:**

- How would the solutions be alike?
- How would they be different?

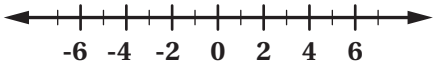
b Graph all the solutions to $|x + 1| > 3$.

$| x + 1 | > 3$

12 Graph all the solutions to each inequality or equation.

$| x | \leq 2$

$| x | \geq 5$

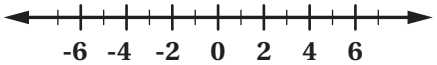


$| x - 3 | = 1$

$| x - 2 | > 4$

$4 \geq | x |$

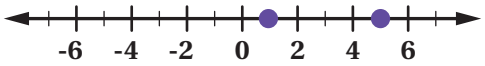
$| x - 2 | > 2$



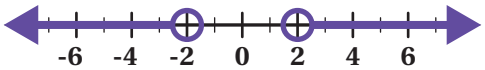
13 Synthesis

Describe a strategy you would use to graph the solutions to absolute value equations and inequalities.

$| x - 3 | = 2$



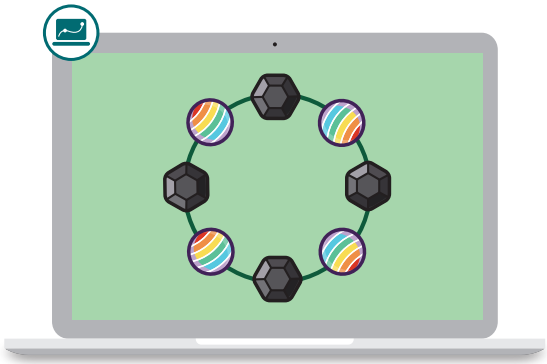
$| x | > 2$



Things to Remember:

Unit 2
Lesson
13

Name: ..... Date: ..... Period: .....



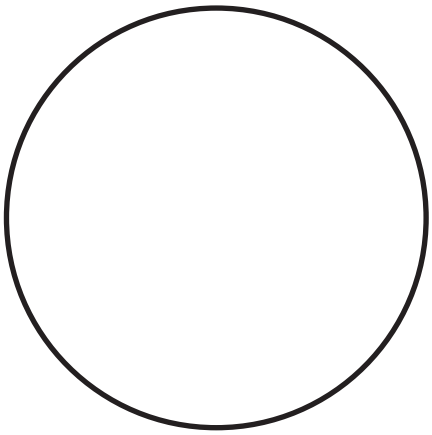
Bracelet Budgets

Let's explore solutions to two-variable inequalities graphically and symbolically.

Warm-Up

1 Draw or describe a bracelet. You can use any combination of the beads shown.

Tell us about your bracelet design.



Activity 1

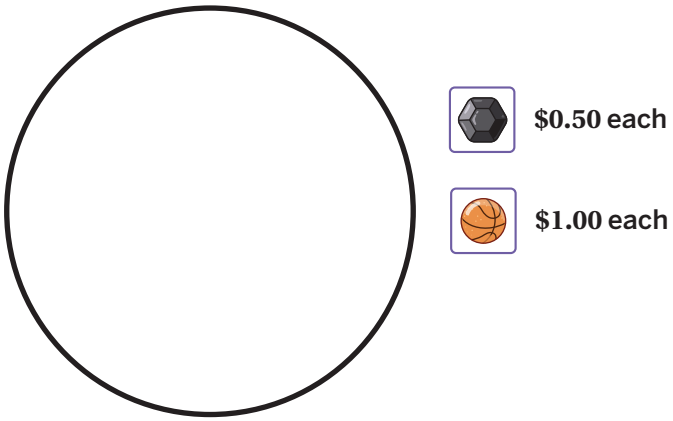
Name: _____ Date: _____ Period: _____

Modeling with Inequalities

2 Here are two types of beads:

- Black beads are $0.50 each.
- Basketball beads are $1 each.

Draw or describe a $5 bracelet.



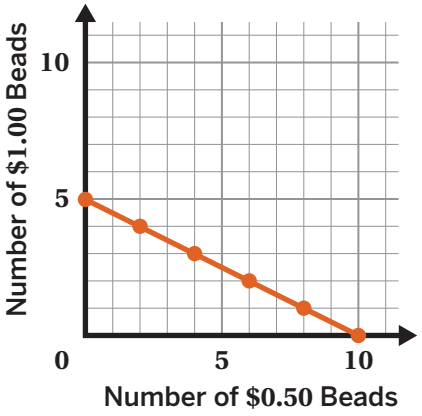
3 Each of these points represents a $5 bracelet.

- x is the number of $0.50 beads.
- y is the number of $1.00 beads.

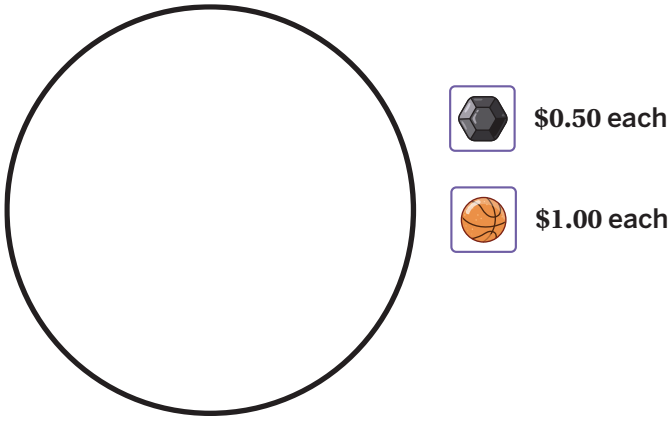
Which equation represents all the $5 bracelets? Circle one.

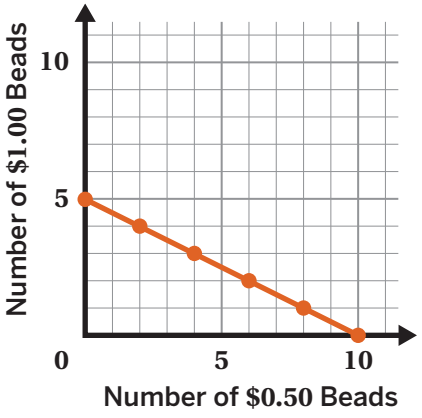
$x + y = 5$ $0.5x + y = 5$ $y = 0.5x + 5$

Explain your thinking.



4 Binta can spend $5 or less on a bracelet. Graph some bracelets that Binta could buy. Draw or describe them if it helps with your thinking.

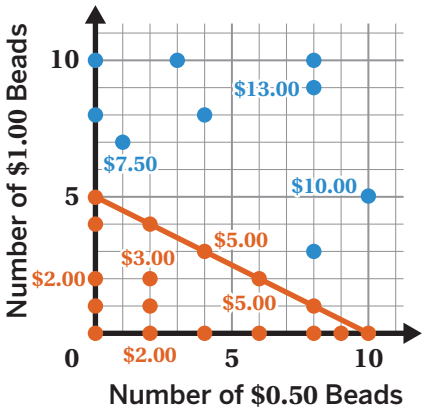




Name: _____ Date: _____ Period: _____

Modeling with Inequalities (continued)

5 Here are some bracelets that Binta could buy and some she could not buy for $5 or less.



What do you notice? What do you wonder?

I notice:

I wonder:

6 Binta can spend $5 or less on a bracelet.

- x is the number of $0.50 beads.
- y is the number of $1.00 beads.

Which statement describes all the bracelets that Binta can buy? Circle one.

$0.5x + y \leq 5$ $0.5x + y \geq 5$ $0.5x + y = 5$

Explain your thinking.

Activity 2

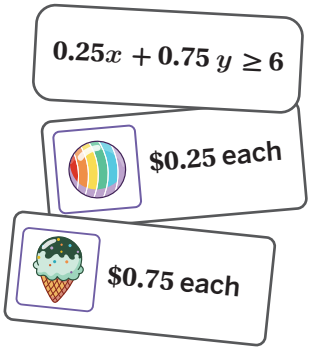
Name: _____ Date: _____ Period: _____

Solutions to Inequalities

7 Caleb loves Binta's bracelet and wants to make his own.

The inequality represents all the bracelets he can make.

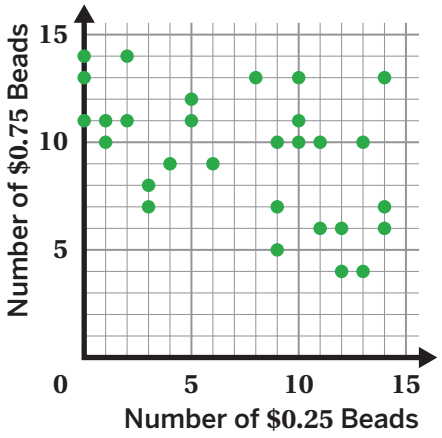
Discuss: What does each number and variable in the inequality represent about Caleb's bracelet?



8 Each of these points is a solution to $0.25x + 0.75y \geq 6$.

a Let's look at several points to see what this means.

b In your own words, what is a solution to an inequality with two variables?



9 Is $(2, 10)$ also a solution to $0.25x + 0.75y \geq 6$? Circle one.

Solution Not a solution I'm not sure

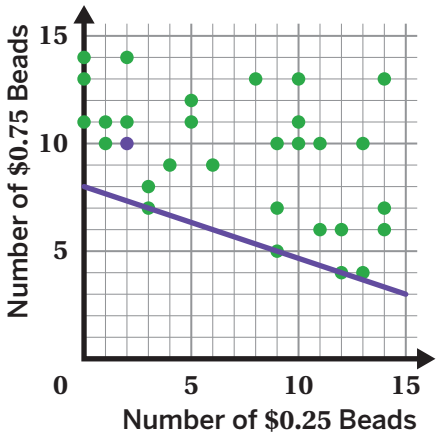
Explain your thinking.

Name: _____ Date: _____ Period: _____

Solutions to Inequalities (continued)

10 Here is how two students determined that $(2, 10)$ is a solution to $0.25x + 0.75y \geq 6$.

Marco



Jada

$0.25x + 0.75y \geq 6$

$0.25(2) + 0.75(10) \geq 6$

$0.50 + 7.50 \geq 6$

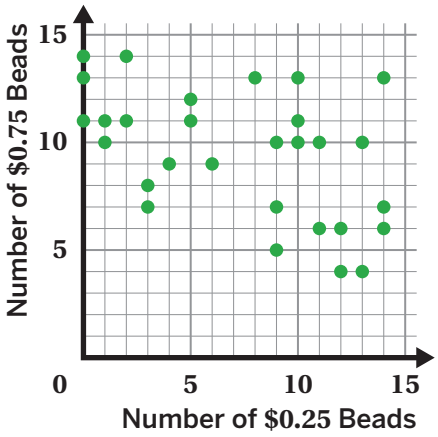
$8 \geq 6$

Explain each person's strategy to a partner.

11 The graph shows some solutions to $0.25x + 0.75y \geq 6$.

Select *all* of the other points that are also solutions.

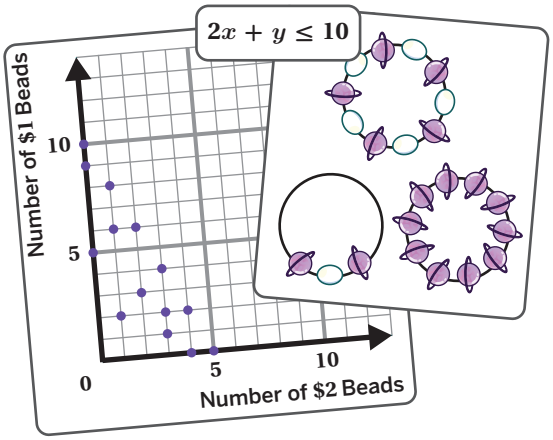
- ☐ **A.** $(14, 8)$
- ☐ **B.** $(13, 1)$
- ☐ **C.** $(7, 5)$
- ☐ **D.** $(1, 13)$
- ☐ **E.** $(0, 8)$



13 Synthesis

How can you tell if a point is a solution to a two-variable inequality?

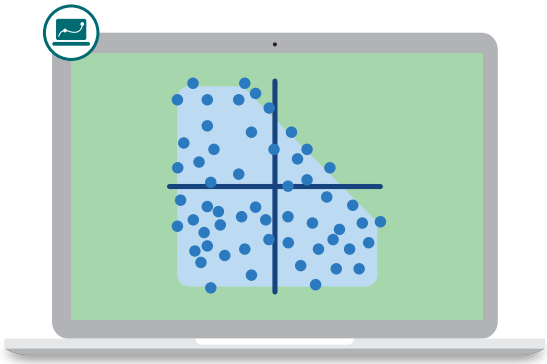
Use the example if it helps with your explanation.



Things to Remember:

Unit 2
Lesson
14

Name: ..... Date: ..... Period: .....



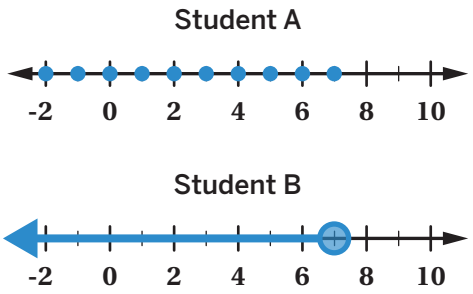
All of the Solutions

Let's represent all of the solutions to two-variable inequalities graphically.

Warm-Up

1 Two students created graphs of the *solutions* to $x \leq 7$.

How are their graphs alike? How are they different?



Alike:

Different:

Activity 1

Name: _____ Date: _____ Period: _____

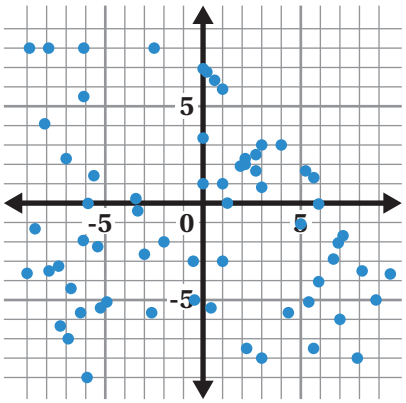
Some to All

2 Write three solutions to $x + y \leq 7$. Try thinking of x- and y-values that no one else will!

Solution 1 (x, y)	Solution 2 (x, y)	Solution 3 (x, y)

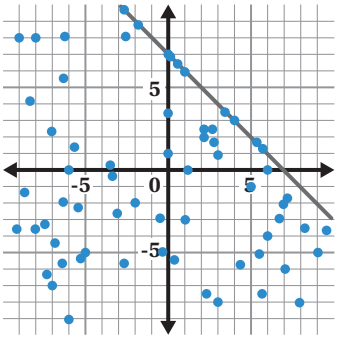
3 Here is a graph of some of the solutions to $x + y \leq 7$.

Sketch what you think the graph of *all* the solutions to $x + y \leq 7$ looks like.

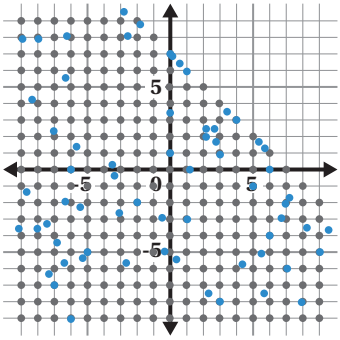


4 Here are Pablo's, Deven's, and Adhira's sketches of *all* the solutions to $x + y \leq 7$.

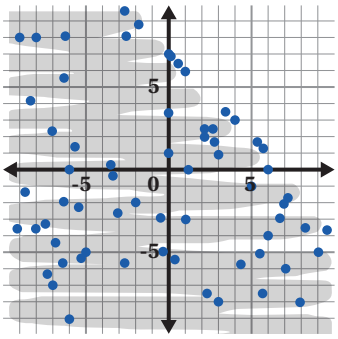
Pablo



Deven



Adhira



a Select *one* sketch.

b **Discuss:** What do you like about this sketch? What would you change?

Activity 2

Name: _____ Date: _____ Period: _____

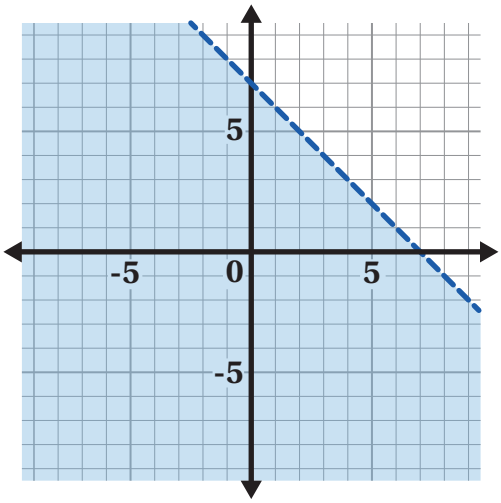
Shading the Solutions

5 **a** Let's watch an animation to see what a graph of all solutions looks like.

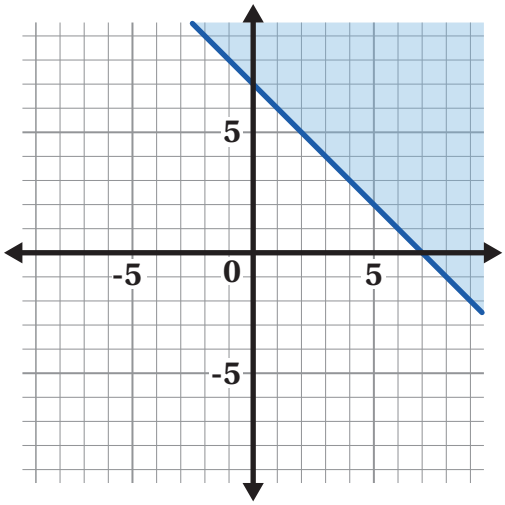
b How does this graph represent all the solutions to $x + y \leq 7$?

6 Here are the graphs of two other inequalities.

$x + y < 7$



$x + y \geq 7$



Discuss:

- How are the graphs of the two inequalities different?
- What does the **boundary line** represent?

Activity 3

Name: _____ Date: _____ Period: _____

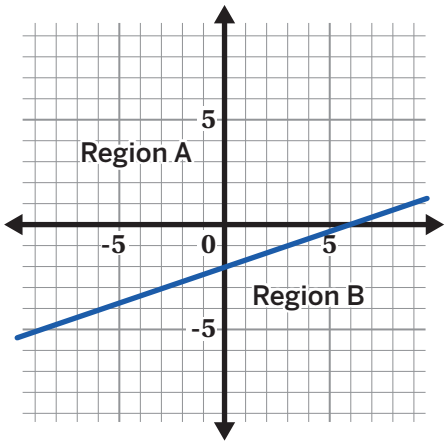
Which Region?

7 Here is the graph of $x - 3y = 6$.

Where will the solutions to $x - 3y \geq 6$ be? Circle one.

Region A Region B I'm not sure

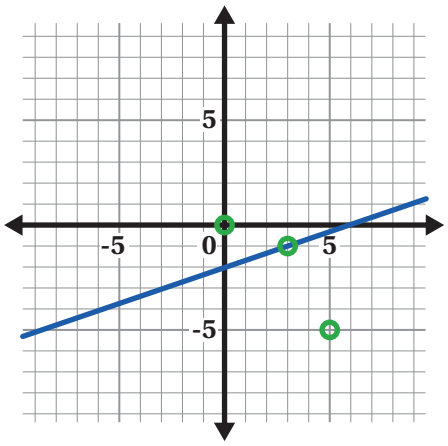
Explain your thinking.



8 Rebecca thought that checking points would help her decide how to graph the solutions to $x - 3y \geq 6$.

She chose the points $(0, 0)$, $(3, -1)$, and $(5, -5)$.

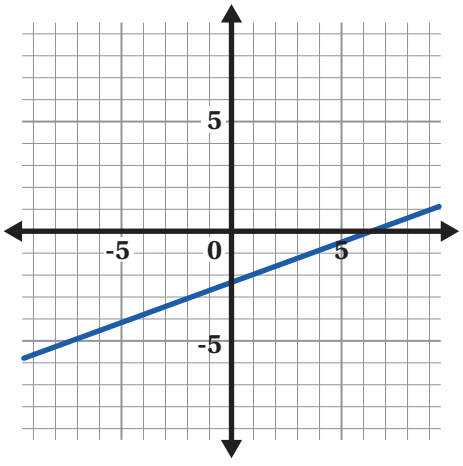
a **Discuss:** Why do you think Rebecca chose these points?



b Select *all* the points that are solutions to $x - 3y \geq 6$.

☐ **A.** $(0, 0)$ ☐ **B.** $(3, -1)$ ☐ **C.** $(5, -5)$

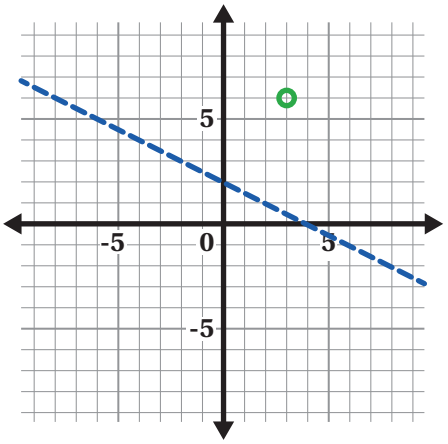
9 Graph *all* the solutions to $x - 3y \geq 6$.



Name: _____ Date: _____ Period: _____

Which Region? (continued)

10 Nathan is graphing the solutions to $x + 2y < 4$.



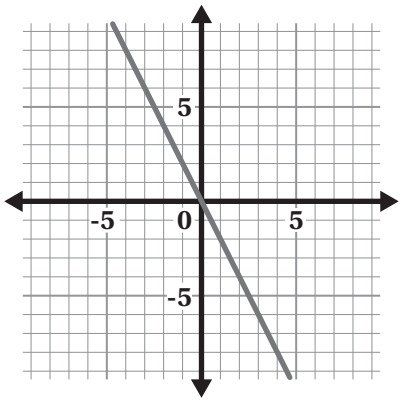
a **Discuss:** Why is his line dashed?

b Nathan determined that $(3, 6)$ is *not* a solution.

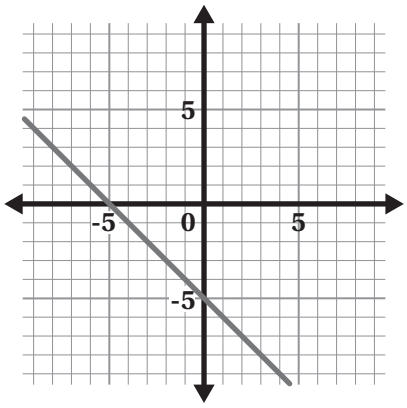
Does he have enough information to graph all the solutions?

11 Graph all the solutions to the following inequalities. The graph of each corresponding equation has been given to you.

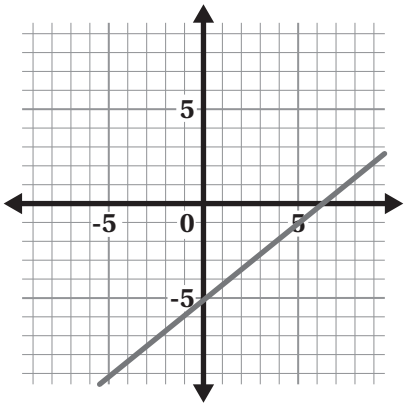
$2x + y \leq 0$



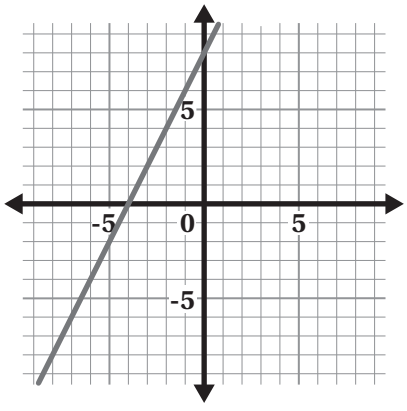
$x + y > -5$



$5x - 6y < 30$



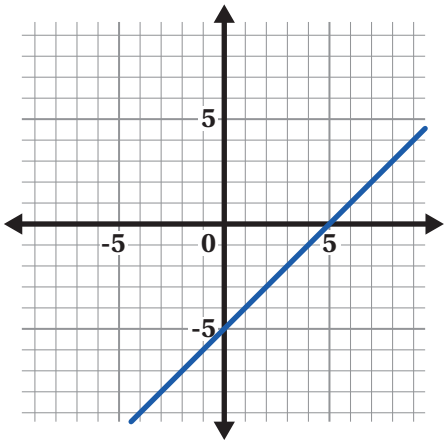
$-2x + y \geq 8$



13 Synthesis

Here is the graph of $x - y = 5$

Describe a strategy for graphing all the solutions to $x - y < 5$, also called the **solution region**.



Things to Remember:

Unit 3
Lesson
1

Name: ..... Date: ..... Period: .....

Survey Says

Let's make sense of the kinds of data that can be collected and write questions to get to know each other better.

Warm-Up

1 Every year, the U.S. government sends the American Community Survey (A.C.S) to thousands of people.

The goal is to gather "vital information about our nation and its people."

What would you like to know about our nation and its people? List at least three questions.

Activity 1

Name: _____ Date: _____ Period: _____

Types of Data

2 A local group conducted a survey similar to the ACS to gather more information about their community.

a Take a look at this list that includes the initials of seven survey participants and their responses.

Anonymous People	What is your age?	Do you have health insurance?	What is your main occupation?	How did you get to work or school last week?	What is the monthly rent where you live?
M.A.	19	No	Cook	Bicycle	$1,200
P.Y.	25	Yes	Construction worker	Car	$900
Z.M.	53	Yes	Nurse	Walked	$1,700
B.H.	62	Yes	I don't work	–	–
J.S.	44	Yes	Accountant	Worked from home	$1,250
O.L.	12	I'm not sure	Student	Car	–
A.B.	29	No	Carpenter	Bus	$1,050

b What do you notice about the responses? What do you wonder?

I notice: I wonder:

Name: _____ Date: _____ Period: _____

Types of Data (continued)

3 Asking different types of questions can produce different types of data.

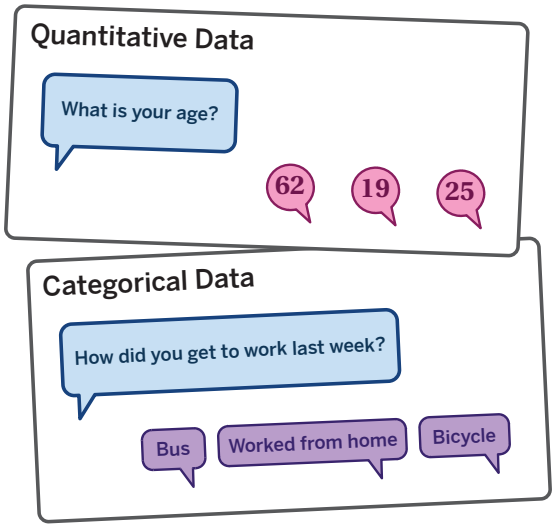
What is your age? produces **quantitative data** (also called *numerical data*).

How did you get to work last week? produces *categorical data*.

Describe what you think these terms mean.

Quantitative data:

Categorical data:



4 Which type of data would each of these questions produce?

Question	Categorical or Quantitative?
Do you live within two miles of a grocery store?	
What type of fuel is used to heat your home?	
What is the monthly rent where you live?	
How old is the youngest person in your family?	
How many minutes does it take you to get to work or school?	
What type of building do you live in?	

5 Convince someone about which type of data this question would produce: *Do you live within two miles of a grocery store?*

Activity 2

Name: _____ Date: _____ Period: _____

Crafting and Asking Questions

6 Jalen is making a survey that includes a question about people's television-watching habits.

He wrote three different ways to ask the question.

- Do you watch a lot of television?
- How many hours of T.V. did you watch last night?
- How much T.V. did you watch last month?

a **Discuss:** What are some advantages and disadvantages of each question?

b If you were making the survey, how would you write the question?

7 Think of something you want to learn about your classmates.

a Write a question for your classmates to answer. Will the data you collect be quantitative or categorical?

Question:

Circle one: Categorical Quantitative

b Collect responses to your question.

Initials	Response

Initials	Response

Name: _____ Date: _____ Period: _____

Crafting and Asking Questions (continued)

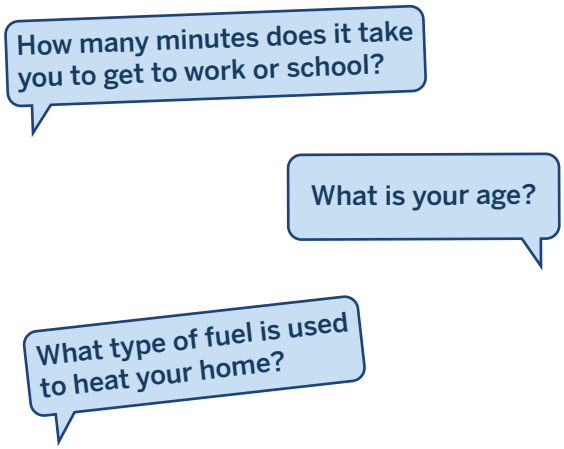
8 Look at your classmates' responses to your question.

a Summarize the data in some way.

b Now that you've seen the responses, how would you change your survey question?

9 Synthesis

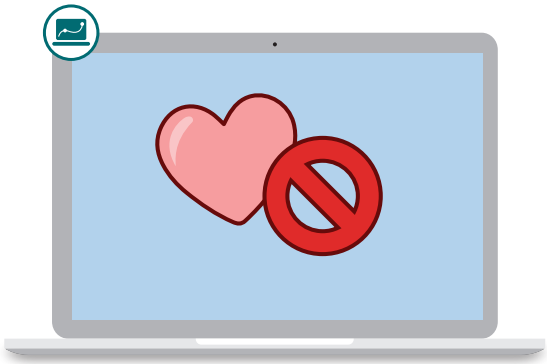
How can you decide whether a question will produce quantitative or categorical data?



Things to Remember:

Unit 3
Lesson
4

Name: ..... Date: ..... Period: .....



Love It or Leave It

Let's make sense of dot plots and histograms as ways to visualize one-variable data.

Warm-Up

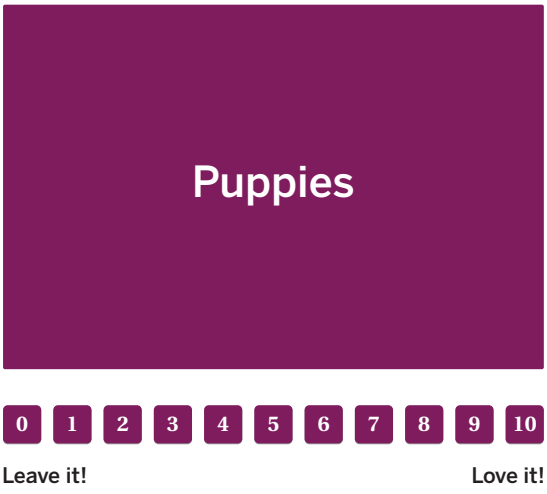
1 This is a game called *Love It or Leave It*.

A word or phrase will appear on the screen.

Rate it on a scale of 0-10.

10 = Love it!
0 = Leave it!

Use the digital activity to play several rounds of this game. rounds of this game.



Activity
1

Name: _____ Date: _____ Period: _____

Dot Plots

You'll use the digital activity for Problems 2–4.

2 Look at some ratings from *Love It or Leave It* in a *dot plot.*

What surprises you?

3 Look at another dot plot showing ratings from your class.

Which of these topics could the dot plot represent? Circle one.

Hiking The zoo Traffic Saturday

Explain your thinking.

4 **a** Replace P with C or H in dotplot(P) to see the dot plot for those data sets.

b **Discuss:** What ratings do you think data set H could represent?

Activity 2

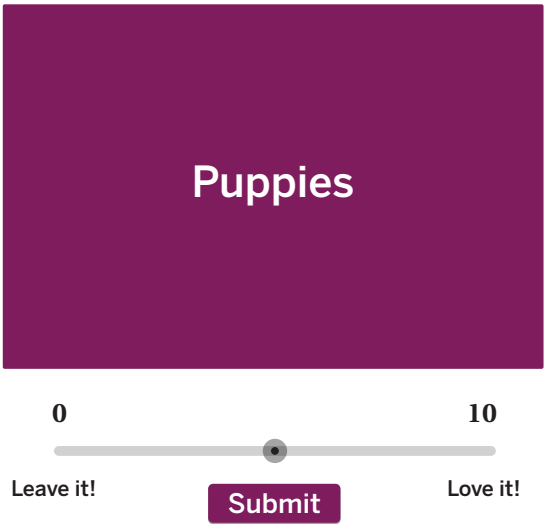
Name: ..... Date: ..... Period: .....

Histograms

5 The makers of *Love It or Leave It* are considering changing the way people rate words.

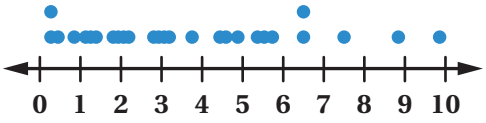
a Use the digital activity to play several rounds of the new version.

b How would the new version change the data that gets collected?



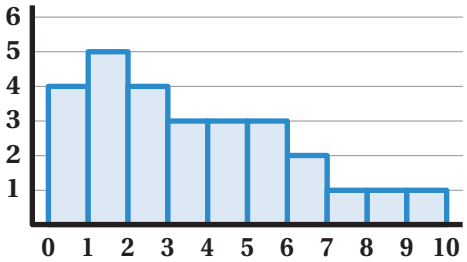
6 This dot plot and *histogram* show many students' ratings from the new version.

Dot Plot



Ratings for Snakes

Histogram



Ratings for Snakes

a Take a look at how a histogram is made in the digital activity.

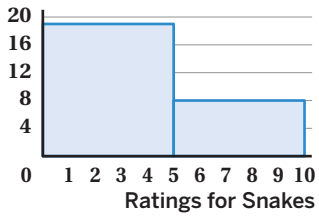
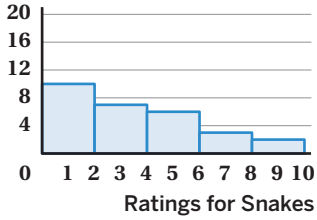
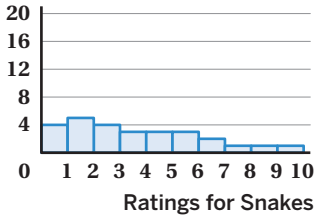
b How many students gave snakes a rating between 2 and 3?

Name: _____ Date: _____ Period: _____

Histograms (continued)

7 Here is the data for snakes from the previous problem.

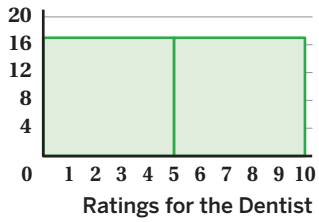
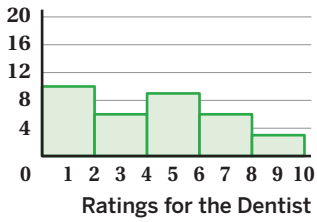
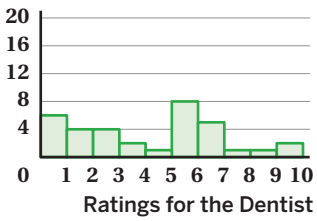
a Take a look at how the bin width changes the histogram.

Bin Width: 5	Bin Width: 2	Bin Width: 1
		

b **Discuss:**

- What do you think *bin width* means?
- What stays the same when the bin width changes? What does not?

8 Here are ratings for the dentist from Kwame's class.

Bin Width: 5	Bin Width: 2	Bin Width: 1
		

Kwame says: *Based on the data, people seem to like and dislike the dentist equally.*

What is true about Kwame's claim? What is misleading?

It's true that . . .

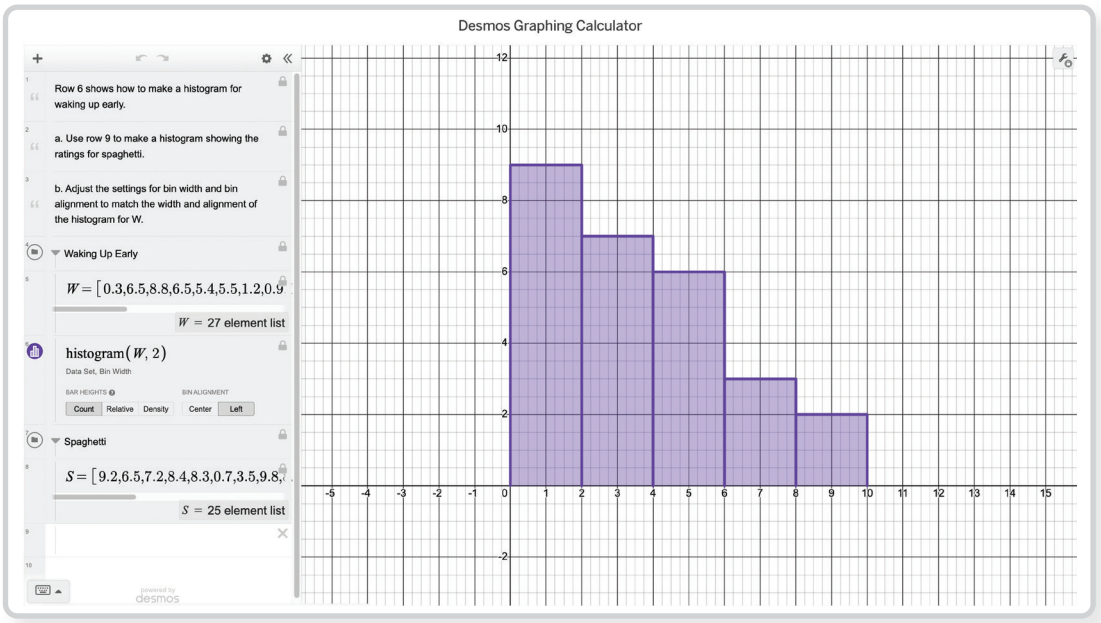
It's misleading that . . .

Activity 3

Name: _____ Date: _____ Period: _____

Making Histograms

9 **a** Use the digital activity to make a histogram showing the ratings for spaghetti.



b Adjust the settings for bin width and bin alignment to match the width and alignment of the histogram for W.

Explore More

10 **a** Ask people to rate something on a $0-10$ scale. Collect your responses in the table.

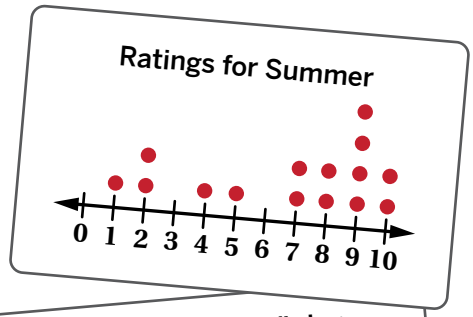
Initials	Rating	Initials	Rating	Initials	Rating

b Use the digital activity to make a histogram and/or dot plot of the data.

11 Synthesis

In this lesson, we examined two ways of visualizing data.

What are the advantages of a dot plot?



What are the advantages of a histogram?

Things to Remember:

Unit 3
Lesson
5

Name: ..... Date: ..... Period: .....

Better Weather?

Let's use box plots to visualize and compare weather data.

Warm-Up

1 Mia lives in Seattle, Washington. Her best friend Bao lives in Charleston, South Carolina.

Mia and Bao are debating whose city has better weather.

How could they compare the weather in Seattle and Charleston?

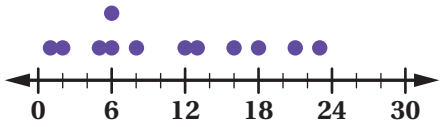
Activity 1

Name: _____ Date: _____ Period: _____

Rainy Days

2 Mia and Bao wondered: *Which city has rainier weather?*

Mia made a dot plot of the number of rainy days for each month in Seattle in 2021.



Number of Rainy Days Each Month (Seattle 2021)

Source: *National Weather Service*

a What was the *greatest* number of rainy days in a month?

b What was the *fewest* number of rainy days in a month?

3 In the digital activity, look at some *statistics* of rainy days for each month in Seattle in 2021. A statistic is a single number that measures something about a data set.

Minimum	Q1	Median	Q3	Maximum
1	5.5	10	17	23

Discuss: What do you think each of these statistics tells us about the data?

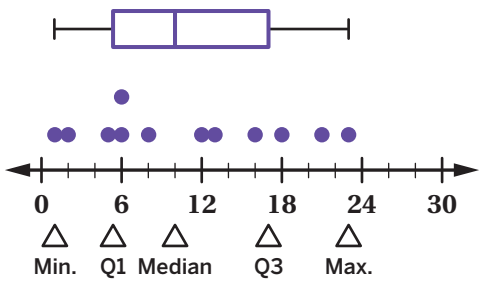
4 The five statistics on the previous problem split the data into quarters, which are called *quartiles*.

A *box plot* is one way to represent quartiles.

a Use the digital activity to see how a box plot for the Seattle data gets made.

b **Discuss:**

- Why do you think this is called a box plot?
- What percentage of the data is inside the box? How do you know?



Number of Rainy Days Each Month (Seattle 2021)

Activity 2

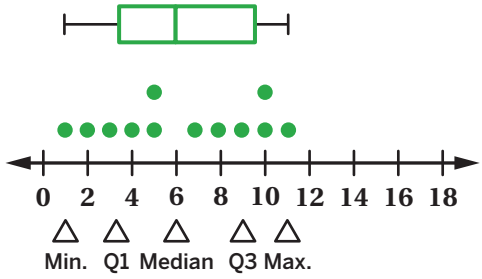
Name: _____ Date: _____ Period: _____

Comparing

5 Here is a dot plot and a box plot of the number of rainy days for each month in Charleston in 2021.

Bao says: *In half of the months of the year, Charleston had at least 6 rainy days.*

a **Discuss:** How can the box plot help you know that Bao's statement is true?

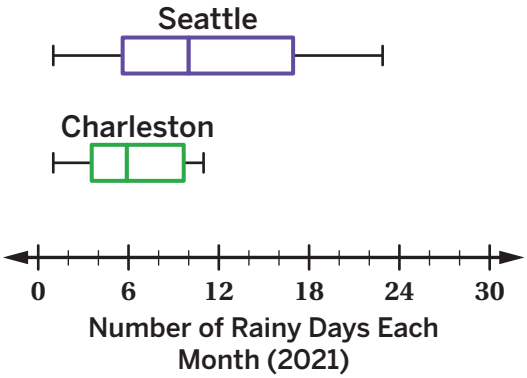


Number of Rainy Days Each Month (Charleston 2021)

b Write *two* more true statements that can be determined from the dot plot or box plot.

6 Here are two box plots showing the number of rainy days for each month in Seattle and Charleston in 2021.

Use the box plots to help Bao convince Mia that Seattle is the rainier city.



Number of Rainy Days Each Month (2021)

Activity 3

Name: ..... Date: ..... Period: .....

Temperature

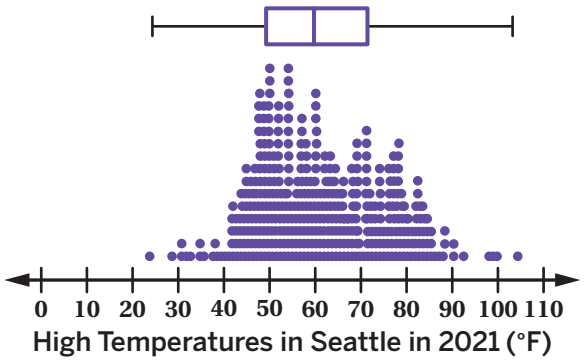
7 Mia and Bao also wondered: *Which city has hotter weather?*

Here are the high temperatures in Seattle for each day of 2021.

Which representation do you prefer for making sense of the data? Circle one.

Dot plot Box plot

Explain your thinking.



8 Use the digital activity to create a box plot for the Charleston data.

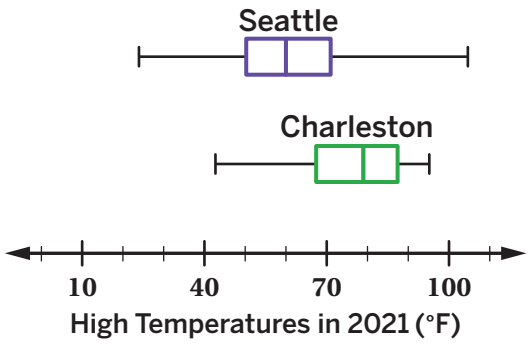
Discuss: How does the temperature in Seattle compare to the temperature in Charleston?

9 Here are two box plots showing the high temperatures in Seattle and Charleston.

What is the *median* of each data set?

Seattle: .....

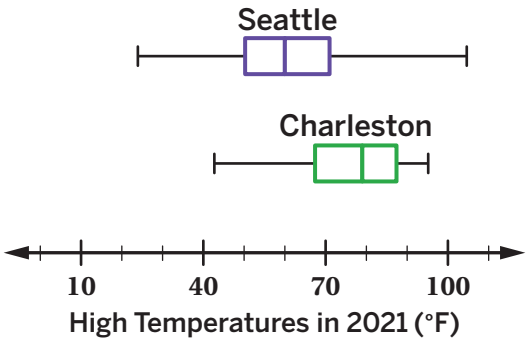
Charleston: .....



Name: _____ Date: _____ Period: _____

Temperature (continued)

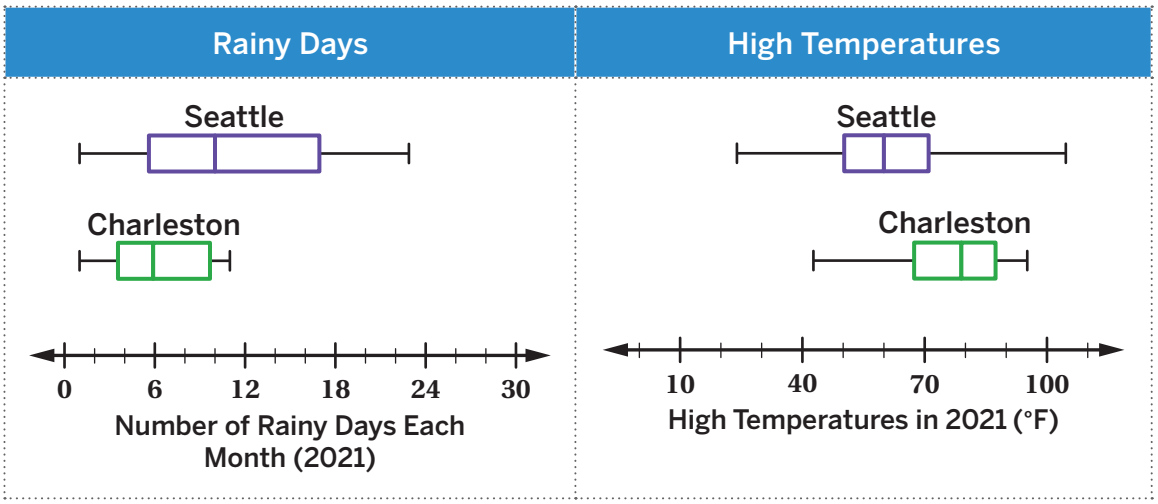
10 Here are the box plots for Seattle and Charleston.



Select *all* the statements that are true.

- ☐ **A.** Seattle had at least one day over 100°.
- ☐ **B.** Charleston had exactly 25 days with a high temperature over 87°.
- ☐ **C.** For about half of the days, Seattle's high temperature was between 50° and 72°.

11 Here are two ways that we compared the weather in Seattle and Charleston in this lesson.



Which city's weather do you prefer? Circle one.

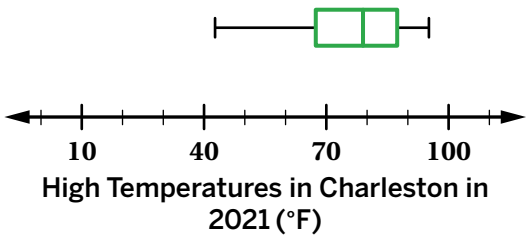
Seattle Charleston Neither

Use information from the box plots to support your reasoning.

12 Synthesis

Describe what each part of a box plot tells you about a data set.

Use the box plot if it helps with your thinking.



High Temperatures in Charleston in 2021 (°F)

Things to Remember:

Unit 3
Lesson
6

Name: ..... Date: ..... Period: .....

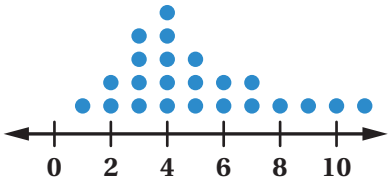
Shapes of Data

Let's describe different shapes of data.

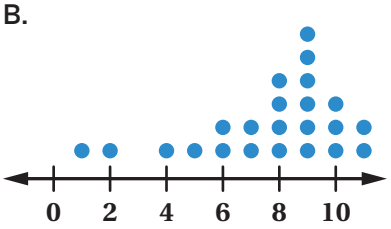
Warm-Up

1 Which one doesn't belong? Explain your thinking.

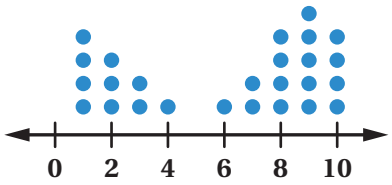
A.



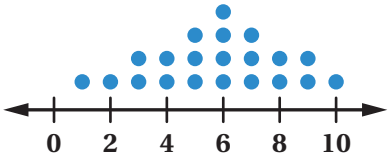
B.



C.



D.



Activity 1

Name: _____ Date: _____ Period: _____

Polygraph

2 Play a few rounds of Polygraph with your classmates!

You will use the Activity 1 Sheet. For each round:

- You and your partner will take turns being the Picker and Guesser.
- Picker: Select a histogram from the Activity 1 Sheet. Keep it a secret!
- Guesser: Ask the Picker yes-or-no questions, eliminating histogram until you're ready to guess which histogram the Picker chose.

Record helpful questions from each round in this workspace:

Activity 2

Name: _____ Date: _____ Period: _____

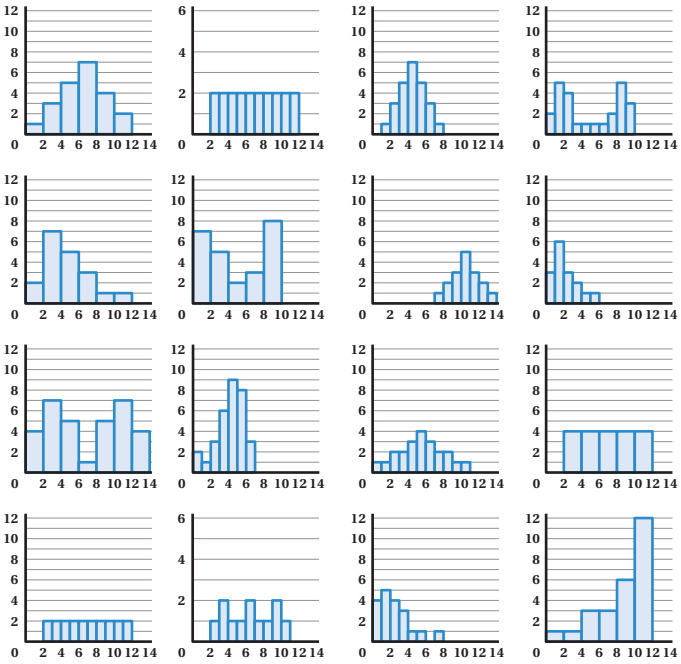
Shapes of Data Sets

3 Here are some terms that describe the *shape* of a data set:

- **Skewed**
- **Uniform**
- **Symmetric**
- **Bimodal**
- **Bell-shaped**

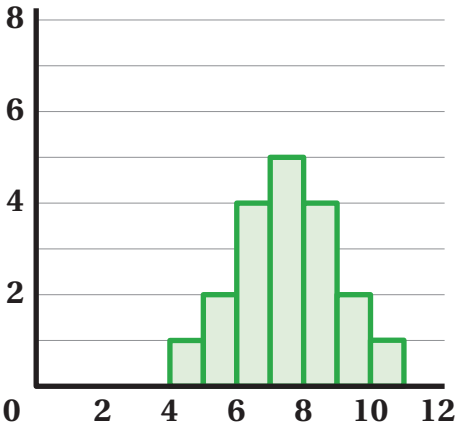
a Let's see which histograms each term describes.

b **Discuss:** What does each term mean?



4 Here is a histogram.

Select *all* the terms that describe the shape of this histogram.



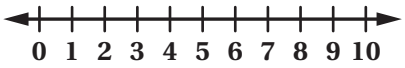
- ☐ **A.** Skewed
- ☐ **B.** Uniform
- ☐ **C.** Symmetric
- ☐ **D.** Bimodal
- ☐ **E.** Bell-shaped

Name: _____ Date: _____ Period: _____

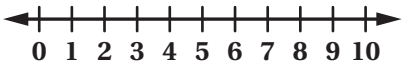
Shapes of Data Sets (continued)

5 Make a dot plot that matches each shape.

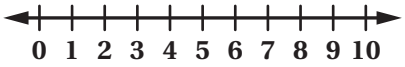
Skewed



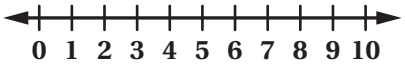
Uniform



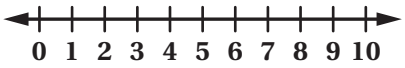
Symmetric



Bimodal



Bell-shaped



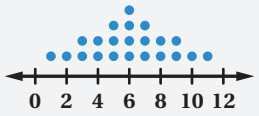
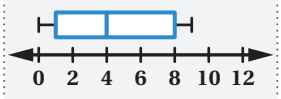
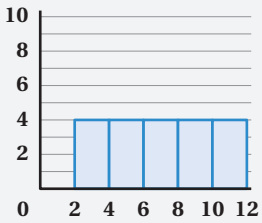
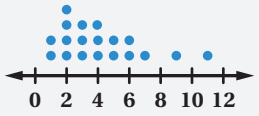
Activity 3

Name: _____ Date: _____ Period: _____

All the Representations

6 You will use a set of cards to complete this activity.

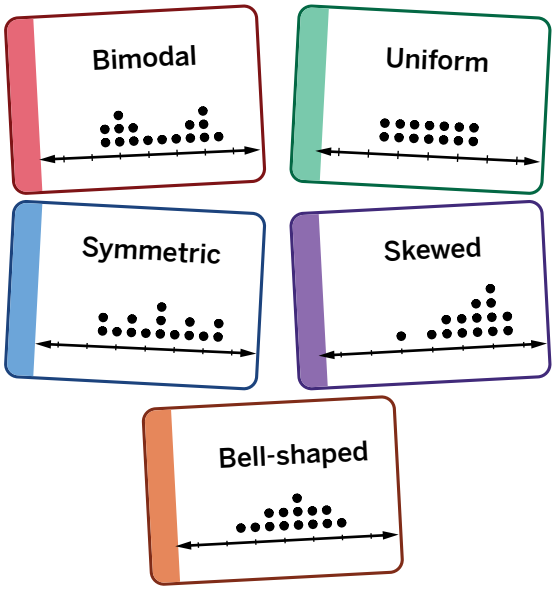
- Match each card with one of the data sets.
- Select the terms that describe the shape of each data set. Share your reasoning with a partner.

	Dot Plot	Histogram	Box Plot	Description
Data Set 1				☐ Skewed ☐ Symmetric ☐ Bimodal ☐ Bell-shaped ☐ Uniform
Data Set 2				☐ Skewed ☐ Symmetric ☐ Bimodal ☐ Bell-shaped ☐ Uniform
Data Set 3				☐ Skewed ☐ Symmetric ☐ Bimodal ☐ Bell-shaped ☐ Uniform
Data Set 4				☐ Skewed ☐ Symmetric ☐ Bimodal ☐ Bell-shaped ☐ Uniform

7 Synthesis

Here are some of the terms we used to describe data sets today.

Choose *three* terms. Describe what each term means in your own words.

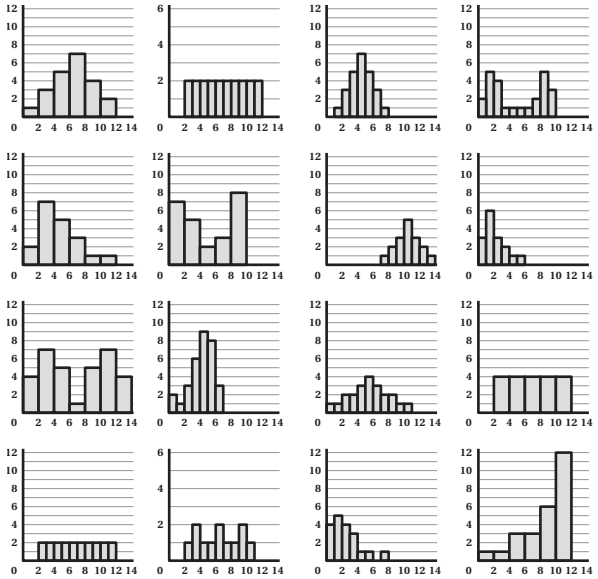


Things to Remember:

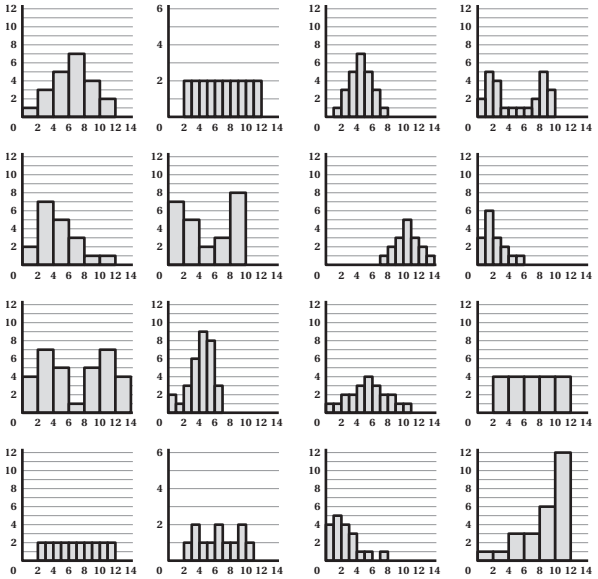
Name: ..... Date: ..... Period: .....

Polygraph Set A

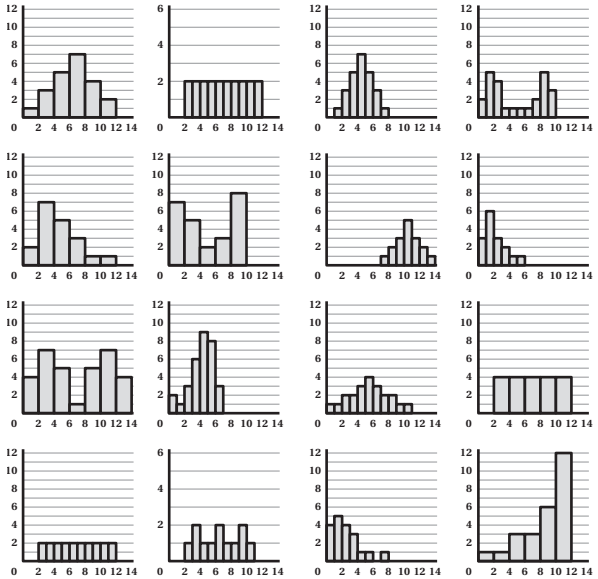
Round 1



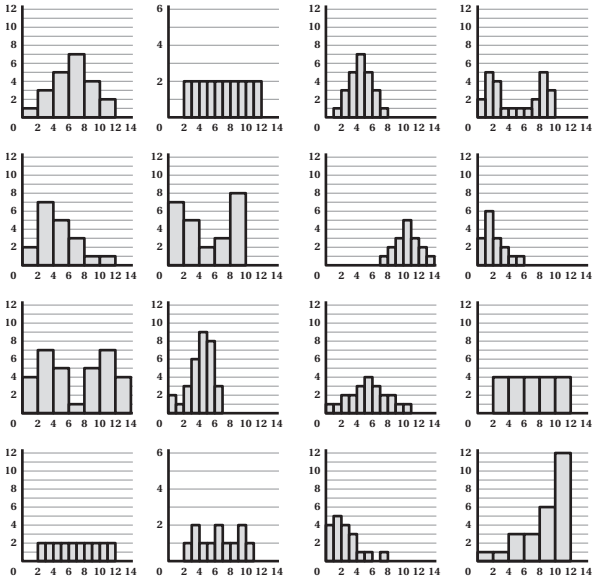
Round 2



Round 3



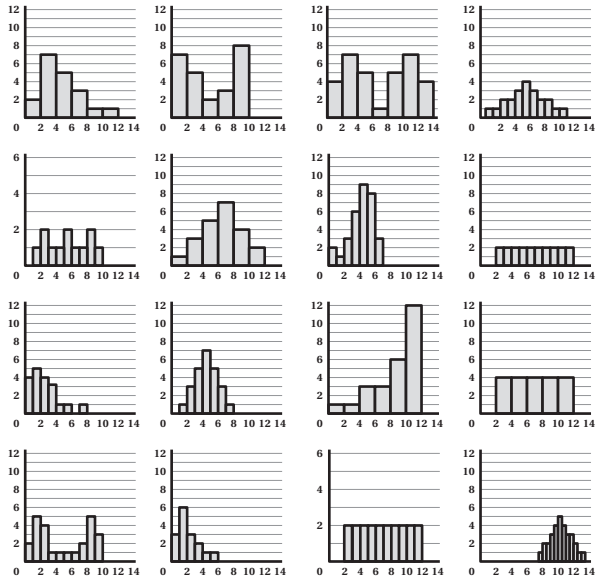
Round 4



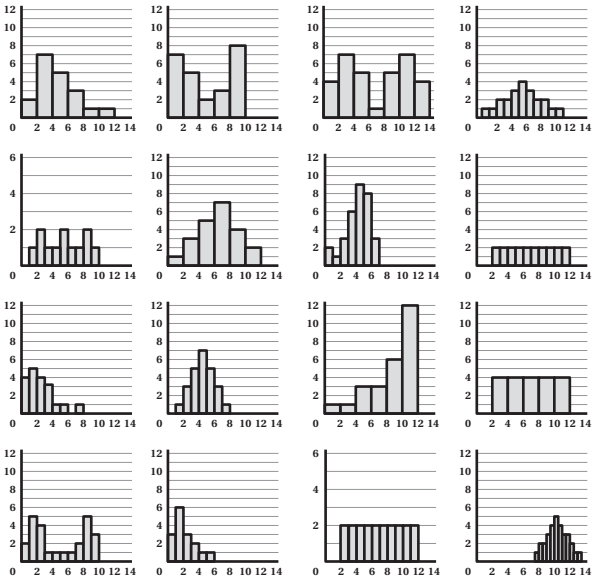
Name: ..... Date: ..... Period: .....

Polygraph Set B

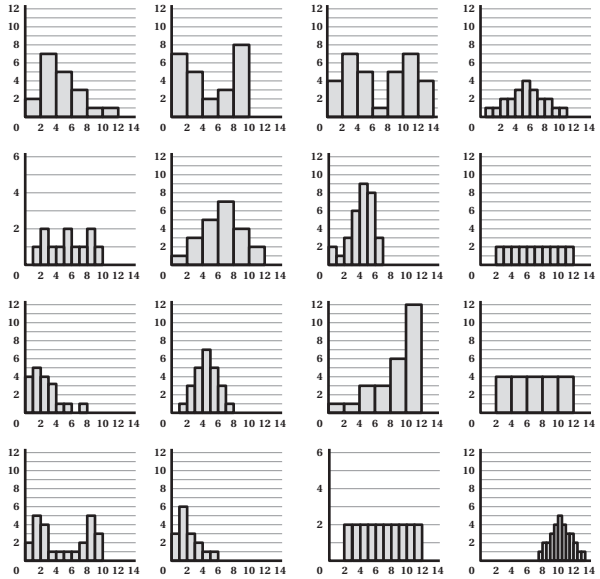
Round 1



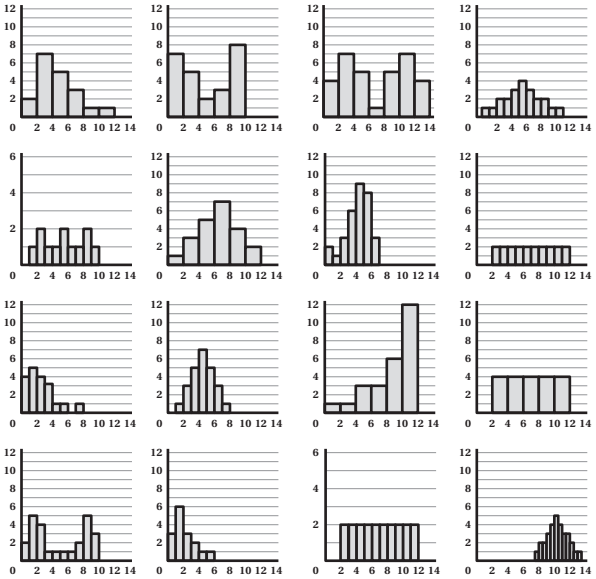
Round 2



Round 3



Round 4

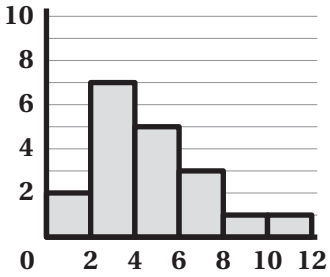


Lesson 3.06
Activity 3

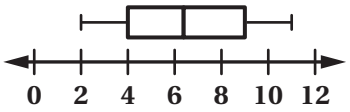
All the Representations

Directions: Make one copy per pair of students. Then pre-cut the cards and give each pair of students one set.

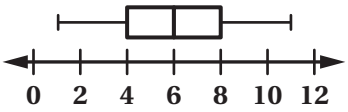
Card A



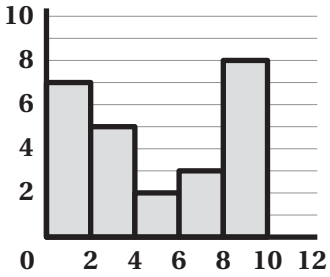
Card B



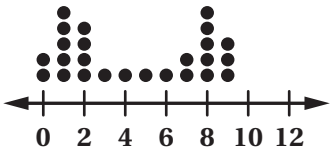
Card C



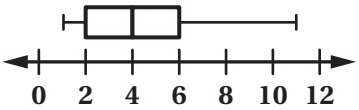
Card D



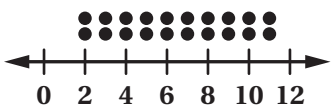
Card E



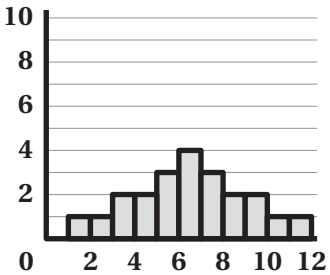
Card F



Card G

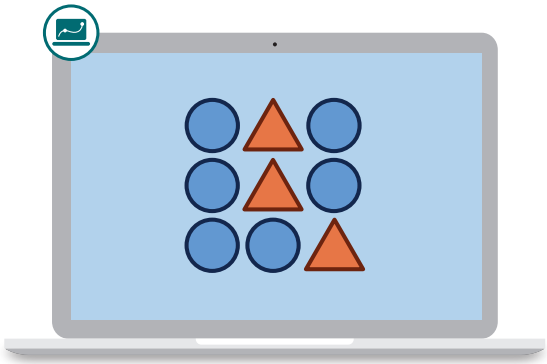


Card H



Unit 3
Lesson
7

Name: ..... Date: ..... Period: .....



Quick Pick

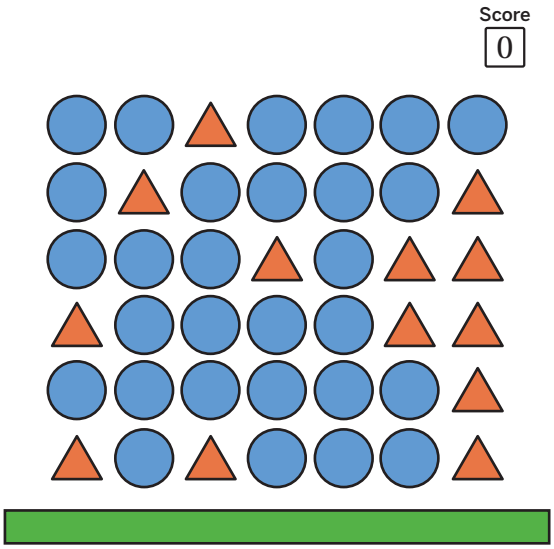
Let's explore how extreme values impact mean and median.

Warm-Up

1 This game is called Quick Pick. Here's how it works:

- You pick different shapes to get points.
- Triangles are 3 points. Circles are 2 points.
- A five-second timer starts after your first pick.

Use the digital activity to play up to 15 times.



Activity 1

Name: _____ Date: _____ Period: _____

Measures of Center

2 In the digital activity, look at your scores and the scores of three students from another class.

Choose a student and describe their scores.

3 A **measure of center** is a single number that represents a central value in a data set.

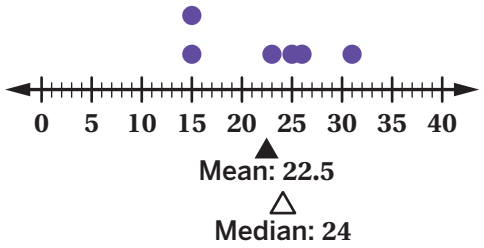
Mean and *median* are two measures of center.

Here are the mean and median for Oscar's scores.

What are some things you know about calculating the mean and median?

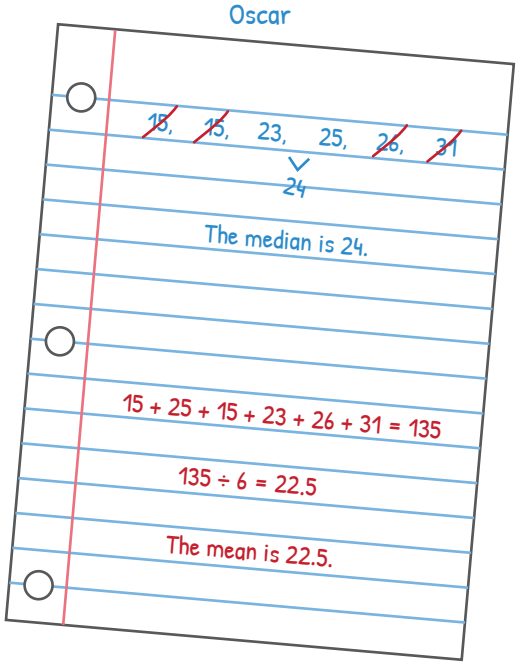
Oscar's Scores

15	25	15	23	26	31



4 Here is the work Oscar did to calculate the median and the mean.

Discuss: How did Oscar calculate each measure of center?



Activity 2

Name: ..... Date: ..... Period: .....

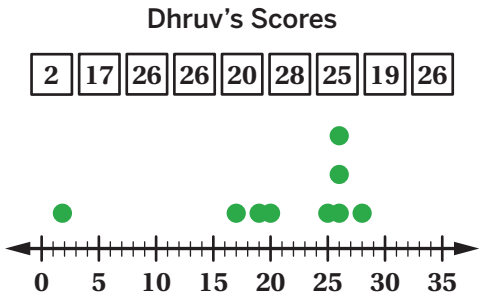
Shape and Center

5 Here are Dhruv's scores from the game. Determine the median and mean.

Use the Desmos Graphing Calculator if it helps with your thinking.

Median: .....

Mean: .....



6 In the digital activity, look at the mean and median for Dhruv's scores.

a **Discuss:** Why do you think the mean and median are far apart?

b Which measure of center would you use to represent Dhruv's typical score? Circle one.

Mean Median

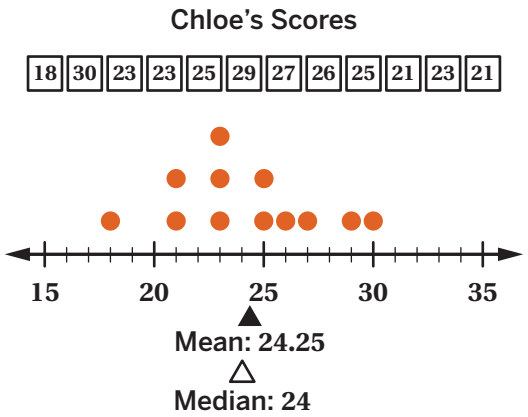
Explain your thinking.

7 Here are Chloe's scores from the game.

Imagine that her highest score was 300 instead of 30.

Which measure(s) of center would increase? Circle one.

Mean Median Both Neither

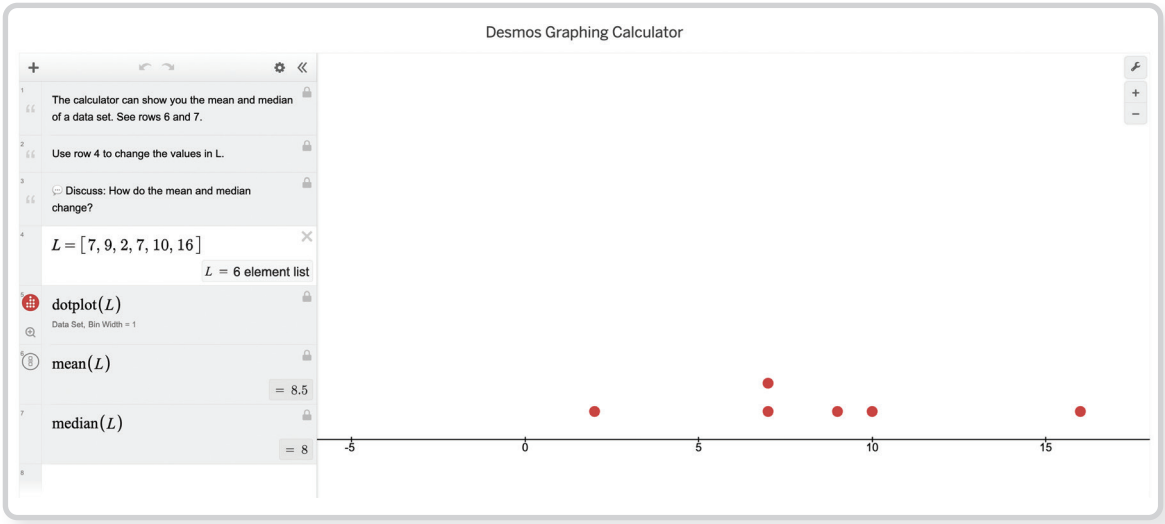


Activity 3

Name: _____ Date: _____ Period: _____

Calculate and Create

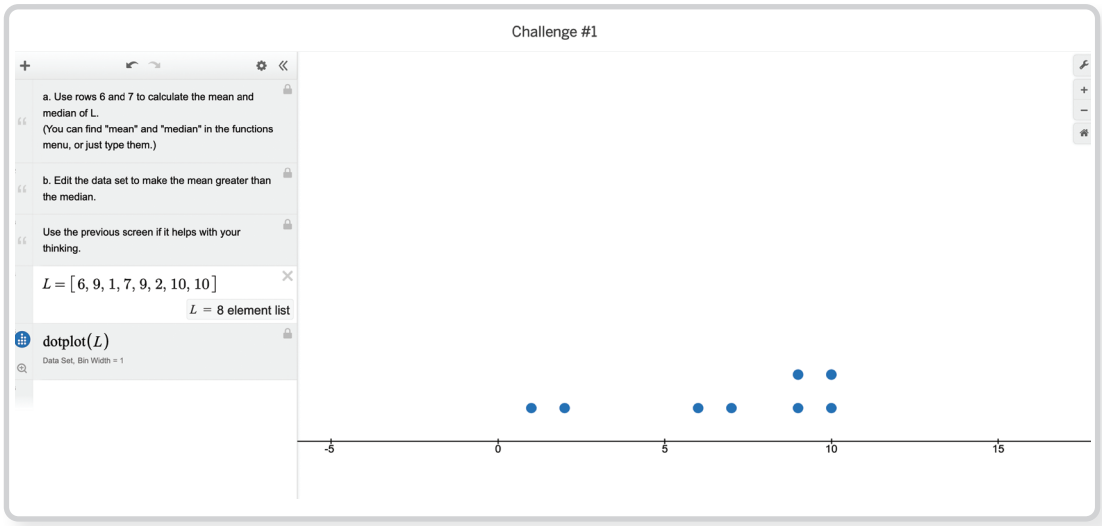
8 The Desmos Graphing Calculator can show you the mean and median of a data set.



Use the digital activity to change the values in L.

Discuss: How do the mean and median change?

9 **a** In the digital activity, calculate the mean and median of L. (You can find "mean" and "median" in the functions menu, or just type them.)

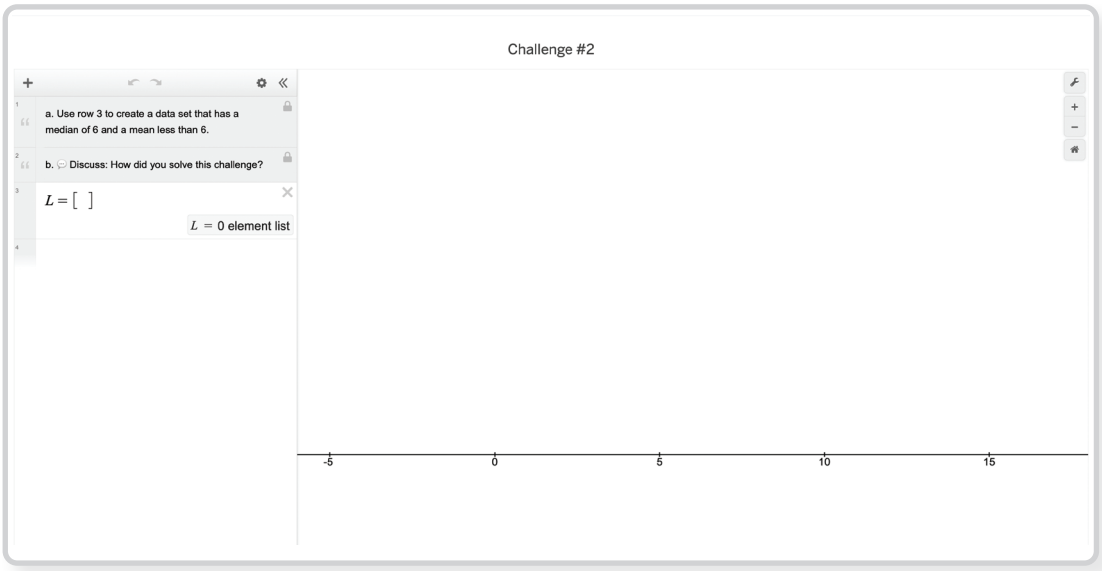


b Edit the data set to make the mean greater than the median.

Name: _____ Date: _____ Period: _____

Calculate and Create (continued)

10 **a** In the digital activity, create a data set that has a median of 6 and a mean less than 6.



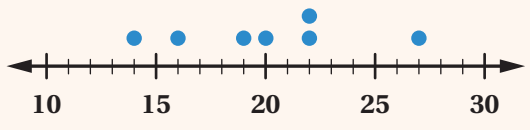
b **Discuss:** How did you solve this challenge?

Explore More

11 Here is a dot plot.

The mean and median are currently equal.

Add data points to make the mean and median as far apart as you can.



12 Synthesis

How are median and mean *alike*? How are they *different*?

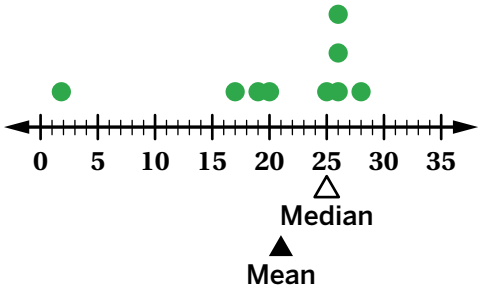
Use the data set if it helps with your thinking.

Alike:

Different:

Dhruv's Scores

2	17	26	26	20	28	25	19	26



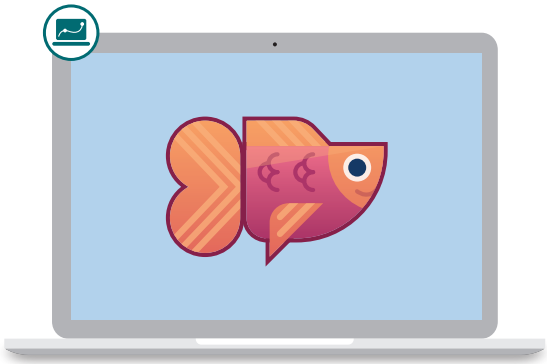
Things to Remember:

Unit 3
Lesson
8

Name: ..... Date: ..... Period: .....

Finding Desmo

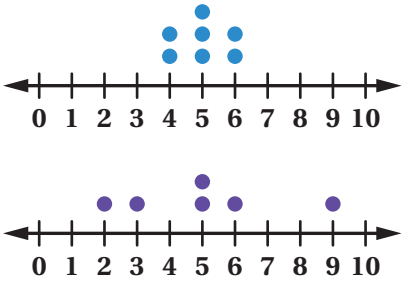
Let's explore what standard deviation describes about a data set.



Warm-Up

1 Here are two data sets.

How are they alike? How are they different?



Activity
1

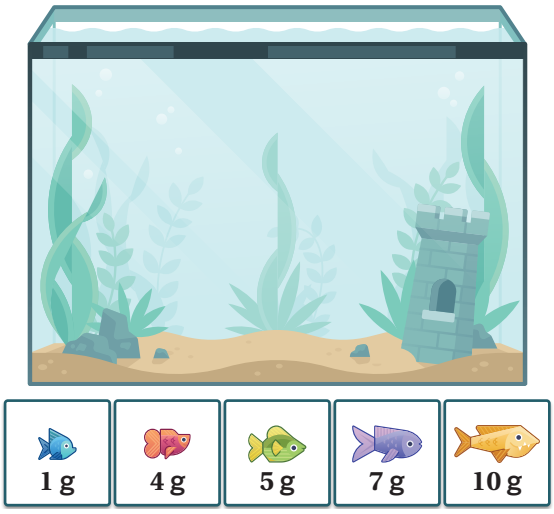
Name: _____ Date: _____ Period: _____

Introduction to Standard Deviation

2 You are getting a new fish tank.

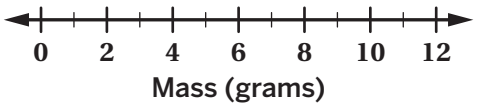
a In the digital activity, add up to 10 fish to your tank.

b Tell a partner how you decided which fish you wanted and how many.



3 A customer at the pet store says: *I would like fish that are close to the same size.*

a In the digital activity, build them a fish tank you think they would like.



b **Discuss:** What do you notice and wonder about the dot plot?

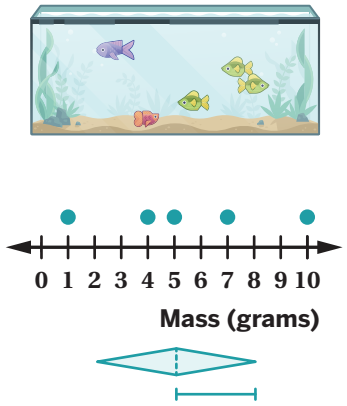
Name: _____ Date: _____ Period: _____

Introduction to Standard Deviation (continued)

4 One way to determine the consistency of data is to calculate the **standard deviation**, which is a **measure of spread**.

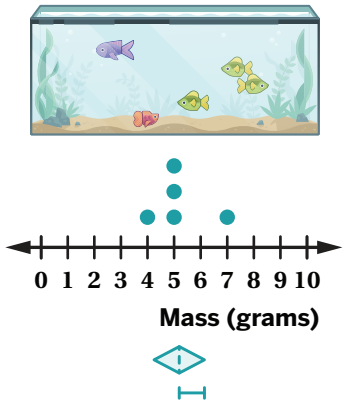
Here are three different fish tanks.

Tank A



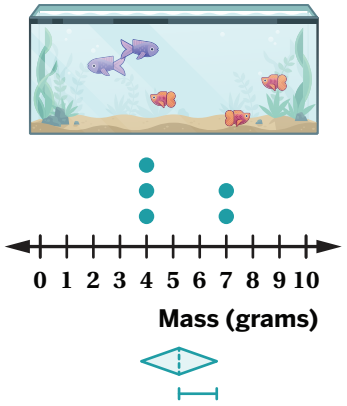
Standard deviation: 3.01 grams

Tank B



Standard deviation: 0.98 grams

Tank C



Standard deviation: 1.47 grams

Explain what you think standard deviation measures.

Activity 2

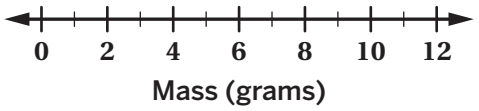
Name: _____ Date: _____ Period: _____

Standard Deviation Challenges

Use the digital activity for Problems 5–7.

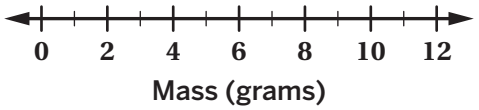
5 Make a fish tank with a large standard deviation. Discuss your strategy with a partner.

Note: You can add up to 10 fish to your tank.



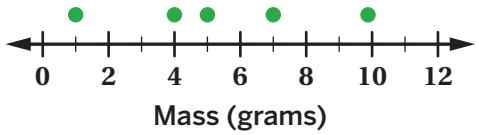
6 Make a fish tank with a small standard deviation that is greater than 0. Discuss your strategy with a partner.

Note: You can add up to 10 fish to your tank.



7 Add fish to this tank to increase the standard deviation.

Discuss: Is there anything you find surprising?



Original standard deviation: 3.01 g

Activity 3

Name: _____ Date: _____ Period: _____

Calculating Standard Deviation

8 Here are the masses, in grams, of the fish in three new tanks.

Tank D	13, 13, 13, 13, 14, 15
Tank E	3, 4, 6, 7, 7, 9
Tank F	7, 7, 7, 7, 7, 9

Which tank do you think has the largest standard deviation? Circle one.

Tank D Tank E Tank F I'm not sure

Explain your thinking.

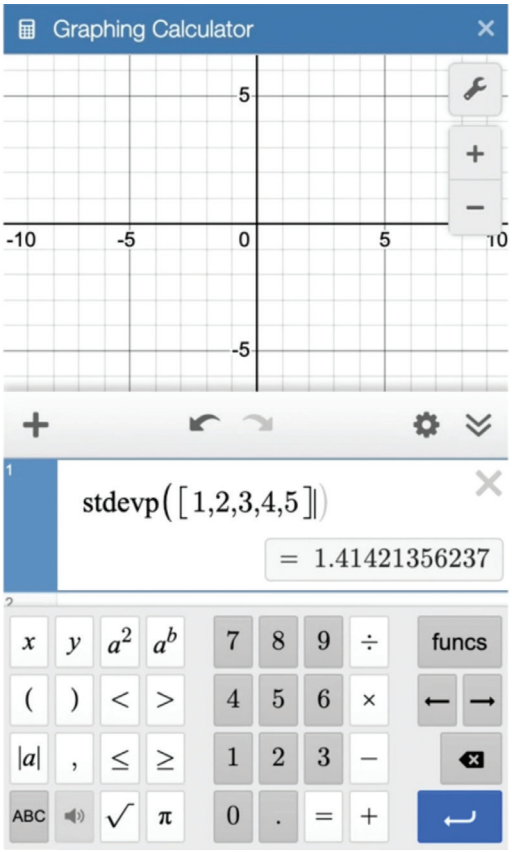
9 Calculators can help calculate standard deviation.

a In the digital activity, watch an animation to see how to calculate the standard deviation of [1, 2, 3, 4, 5].

b Use the Desmos Graphing Calculator to answer:

What is the standard deviation of [3, 4, 6, 7, 7, 9]? Circle one.

1.75 2 3.14 6



Name: _____ Date: _____ Period: _____

Calculating Standard Deviation (continued)

10 Order the fish tanks from smallest to largest standard deviation.

Use the Desmos Graphing Calculator to help with your thinking.

Tank A	13, 13, 13, 13, 14, 15
Tank B	3, 4, 6, 7, 7, 9
Tank C	7, 7, 7, 7, 7, 9

Smallest Standard Deviation		**Largest Standard Deviation**

11 Here is a data set: 1, 2, 3, 4, 5, 6, 7.

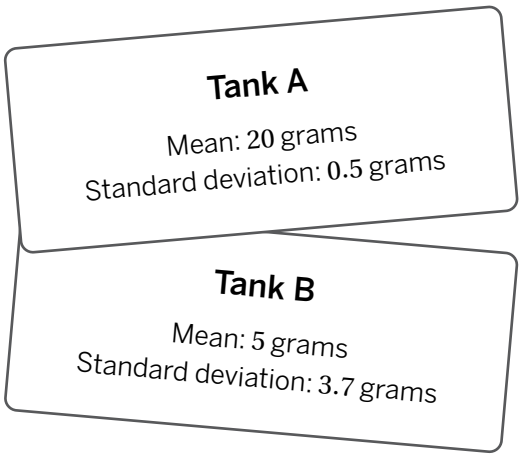
Select *all* the moves that would make the standard deviation *larger*.

Use the calculator to help you with your thinking.

- ☐ **A.** Removing the 7 from the data set.
- ☐ **B.** Adding a 0 to the data set.
- ☐ **C.** Removing the 3 from the data set.
- ☐ **D.** Adding a 4 to the data set.
- ☐ **E.** Increasing each value by 1.

7 Synthesis

Describe what the mean and standard deviation tell us about the fish in Tank A and Tank B.



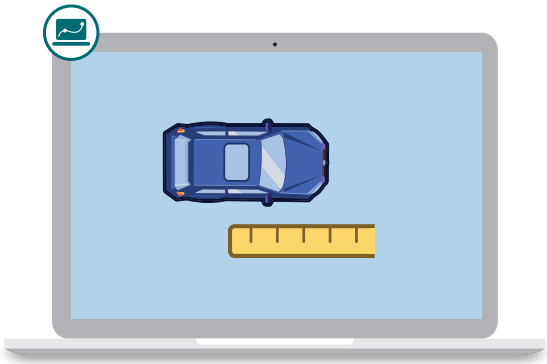
Things to Remember:

Unit 3
Lesson
10

Name: ..... Date: ..... Period: .....

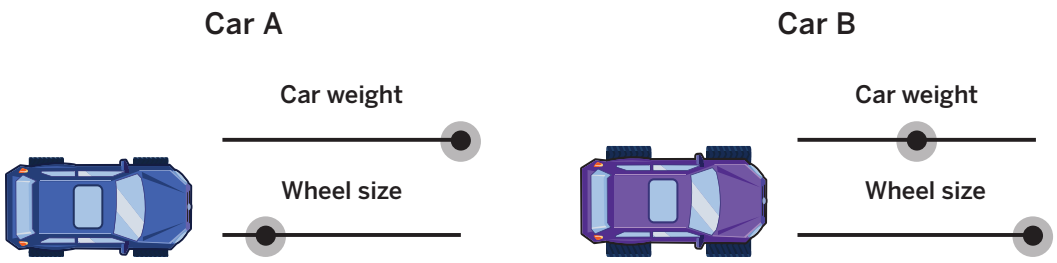
Race Car

Let's compare measures of spread in skewed data sets.

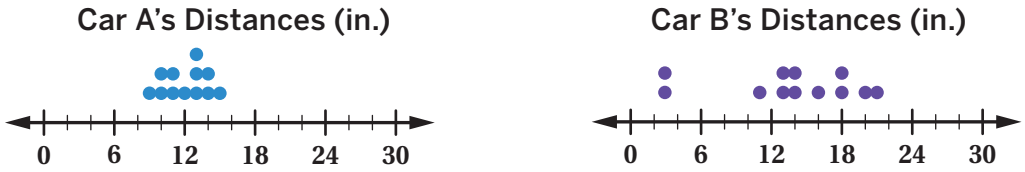


Warm-Up

1 Take a look at two toy race cars with different colors, weights, and wheel sizes.



2 **a** Each car was launched 12 times. Compare their results.



b **Discuss:** Which car generally travels farther? How do you know?

Activity 1

Name: _____ Date: _____ Period: _____

Box Plots and Races

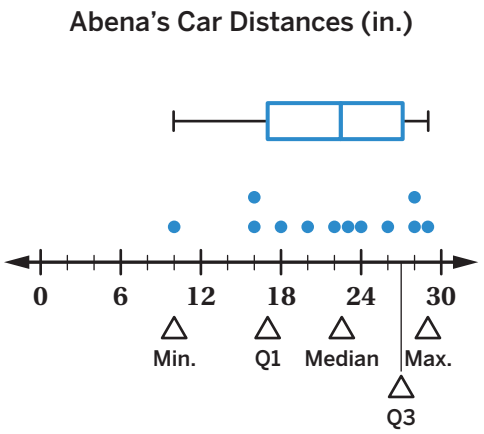
3 Abena launched her car 12 times.

a. Let's watch how the dot plot of Abena's data is made into a box plot.

b. What do you think are the advantages of each representation?

Dot plot:

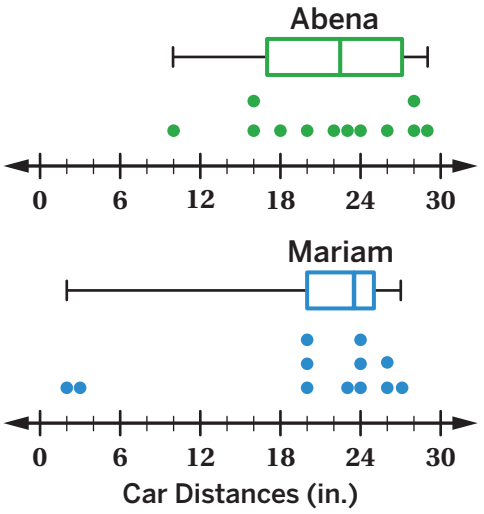
Box plot:



4 Professor Cho is giving out some racing awards.

Discuss:

- What awards do you think Professor Cho should give out?
- For each award you come up with, who would win between Abena and Mariam?



Name: _____ Date: _____ Period: _____

Box Plots and Races (continued)

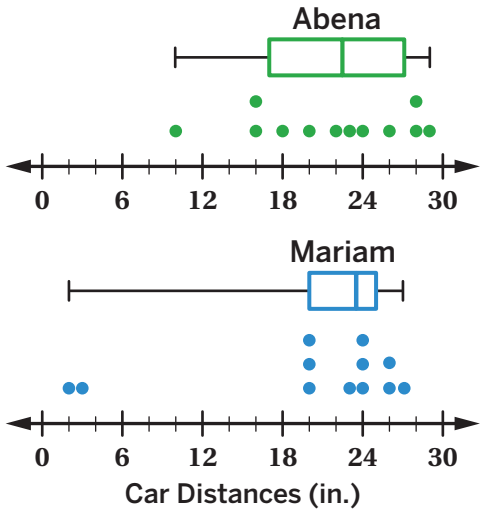
5 Professor Cho wants to give an award for Most Consistent Race Car using the middle half of each car's data.

Select one question to answer:

A. Do you think Abena's or Mariam's race car is more consistent? Why?

B. Why might Professor Cho look at only the middle half of the data?

C. How might Professor Cho measure the middle half of the data?



Name: _____ Date: _____ Period: _____

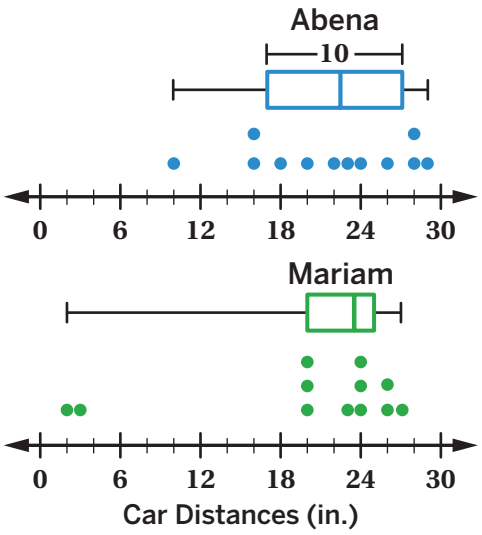
Interquartile Range

6 One way to measure the middle half of a data set is to use the *interquartile range* (or *IQR*).

IQR is a statistic that measures the spread of a data set.

The middle half of Abena's distances are between 17 and 27 inches, so the IQR of her data is 10 inches.

What is the interquartile range (IQR) of Mariam's data?

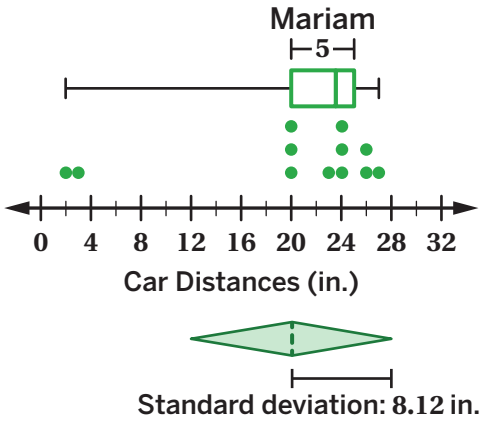


7 Standard deviation and IQR are statistics that measure the spread of a data set.

Which statistic is a more appropriate measure of the spread of Mariam's data? Circle one.

IQR Standard deviation

Explain your thinking.

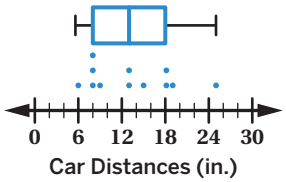


Name: _____ Date: _____ Period: _____

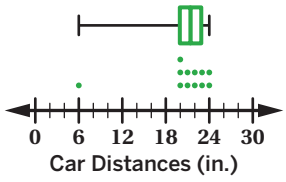
Interquartile Range (continued)

8 Help Professor Cho give out the award for Most Consistent Race Car.

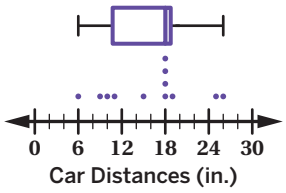
DeAndre



Marcela



Habib



a Determine the IQR for each student's data.

Student	IQR (in.)
Abena	10
Mariam	5
DeAndre	
Marcela	
Habib	

b Who do you think Professor Cho will give the award to? Circle one.

Abena Mariam DeAndre Marcela Habib

Explain your thinking.

Activity 3

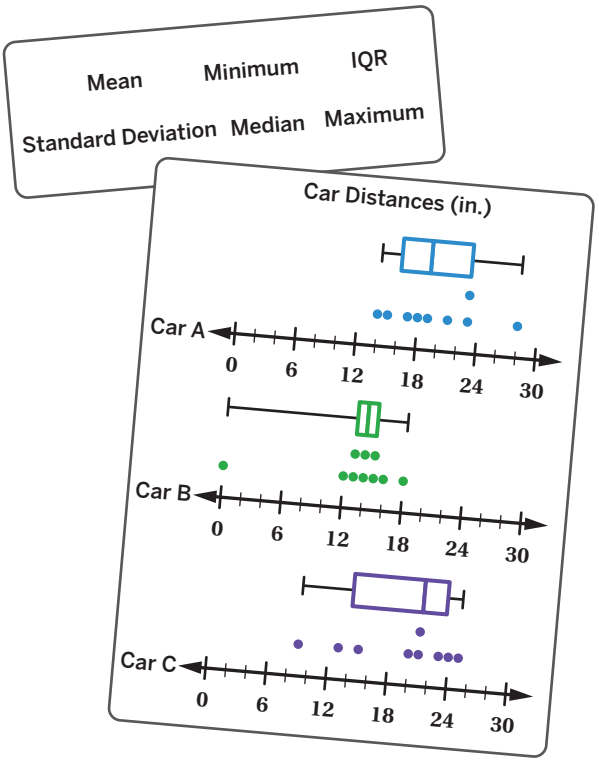
Name: _____ Date: _____ Period: _____

Which Statistic?

9 Here is the data for three new cars.

Which statistic would you use to answer each of the following questions?

Question	Statistic
Which car had the best individual launch?	
Which car was the most consistent?	
Which car typically traveled the farthest distance?	



10 Which car would you give the Best Overall Car award to? Circle one.

Car A Car B Car C

Use vocabulary from this unit to justify your thinking.

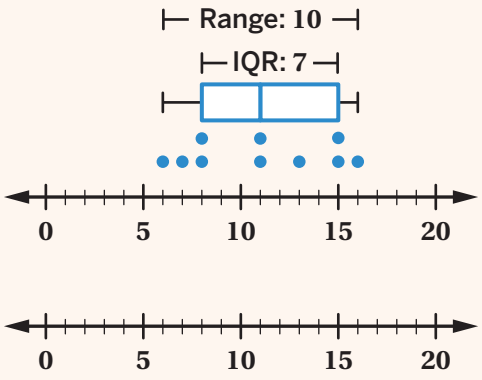
Explore More

11 Range and IQR are both measures of spread.

Here is a box plot with an IQR of 7 and a range of 10.

Can you make a second box plot with the following features? Select *all* the box plots that are possible to create.

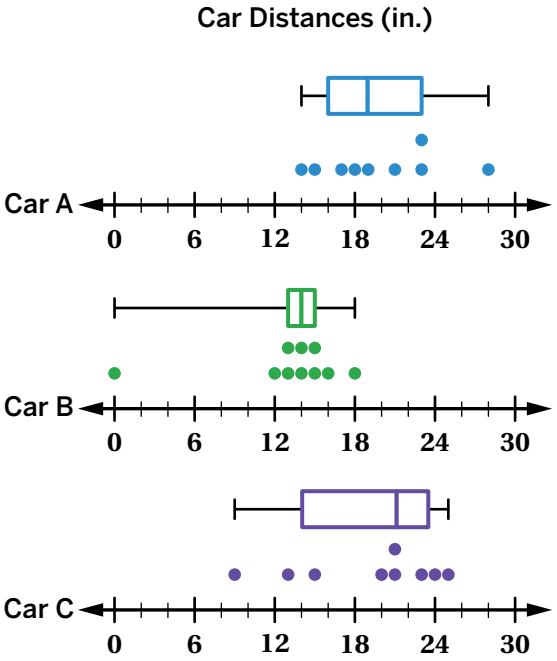
- ☐ **A.** A smaller IQR and larger range
- ☐ **B.** A larger IQR and smaller range
- ☐ **C.** An IQR of 10 and a range of 7
- ☐ **D.** An IQR of 0 and a large range



12 Synthesis

How can the interquartile range help you compare data sets?

Use the example data if it helps with your thinking.



Things to Remember:

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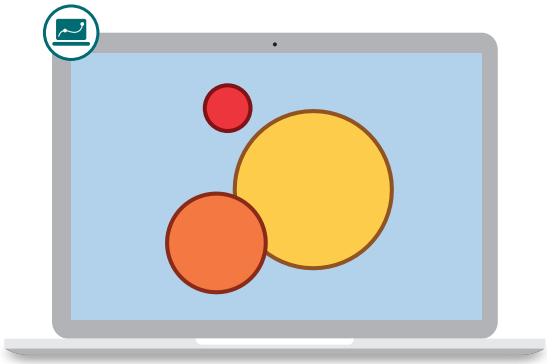
Name: ..... Date: ..... Period: .....

Far Out

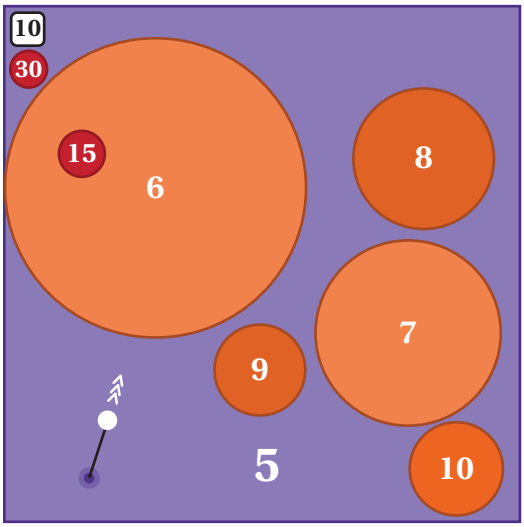
Let's determine whether a data point is an outlier and consider its effect on the mean and median.



Warm-Up

1 Use the digital activity to play a game.

Play the game as many times as you want.



Activity 1

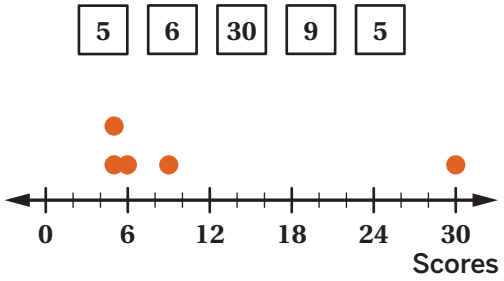
Name: _____ Date: _____ Period: _____

Outliers and Their Effects

2 Koharu played the game 5 times.

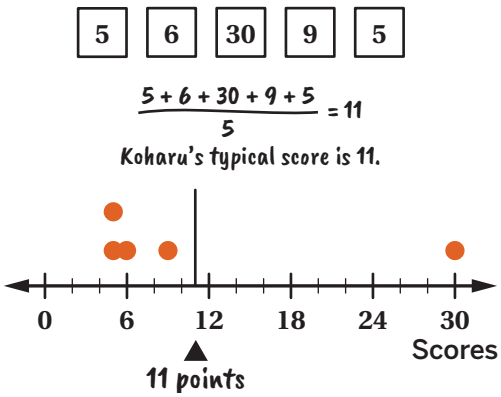
Here are her scores.

What do you think is her typical score?

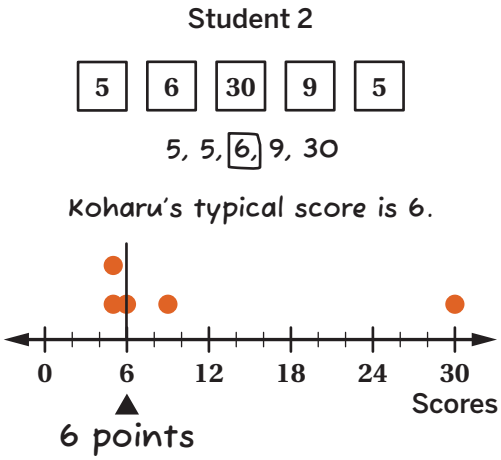


3 Here are two different strategies for determining Koharu's typical score.

Student 1



Student 2



Discuss: How are Student 1's and Student 2's strategies alike? How are they different?

Name: _____ Date: _____ Period: _____

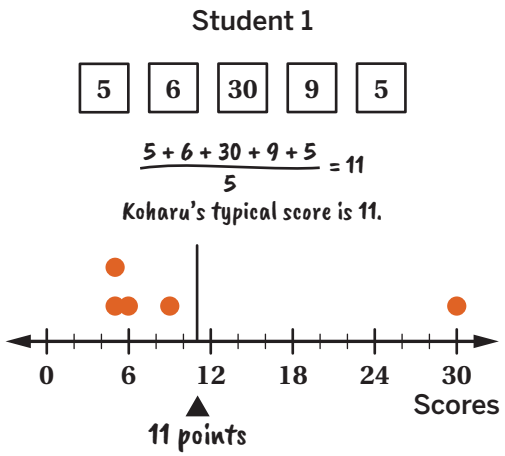
Outliers and Their Effects (continued)

4 Koharu says: *You shouldn't use the mean for the typical score because the 30 messes it up.*

Do you agree? Circle one.

Yes No I'm not sure

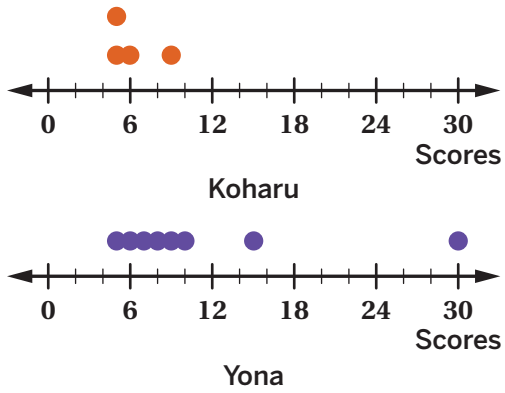
Explain your thinking.



5 In Koharu's data, 30 is an *outlier* because it's far from other values in the data set.

Outliers can have a big impact on the mean of a data set.

Circle the point(s) in Yona's data that you think are outliers.



6 Let's look at a dot plot and a box plot for Yona's data.

a Watch what happens when we click and unclick the "Exclude outliers" checkbox.

b **Discuss:** How does the checkbox change the box plot?

Activity 2

Name: _____ Date: _____ Period: _____

Outliers and Box Plots

7 You can make a box plot in the Desmos Graphing Calculator to determine whether there are any outliers in a data set.

a Look at the rows in the folder "Yona's Data" in the digital activity to see how to make a box plot.

b Turn on and open the folder labeled "Brandon's Data".

c Use the next available row to make a box plot for Brandon's data.

d **Discuss:** Are there any outliers?

8 Making a box plot can help us determine the outliers in any data set.

a Let's watch an animation.

b Use the Desmos Graphing Calculator to answer:

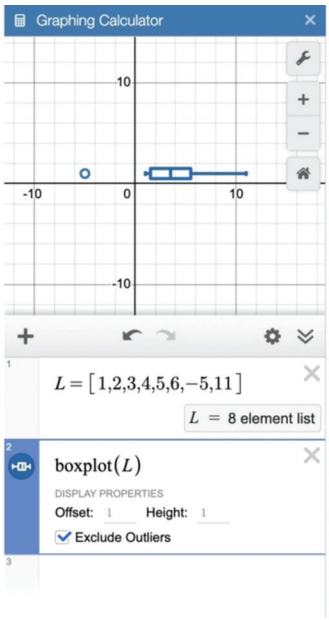
Which value is an outlier in this data set?
[1, -1, 0, 0, 1, -7, 2, 3, 7]

A. -7

B. -1

C. 3

D. 7



Name: _____ Date: _____ Period: _____

Outliers and Box Plots (continued)

9 **a** In the digital activity, drag the point to see when a value becomes an outlier.

b How do you think the calculator determines if a value in a data set is an outlier?

10 One way the Desmos Graphing Calculator decides if a value is an outlier is by looking at its distance from *quartiles* 1 or 3.

a Let's watch an animation to see what we mean.

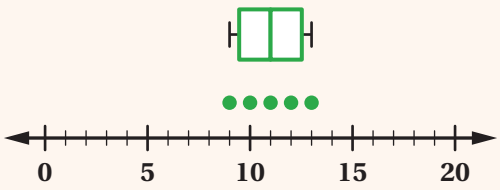
b **Discuss:** How would you describe this strategy for determining whether a point is an outlier?

Explore More

11 Use the digital activity to add points to the dot plot.

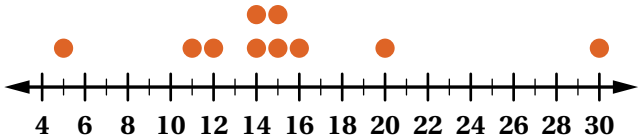
Can you make a dot plot that has:

- One outlier?
- Two outliers?
- Three outliers?
- Four outliers?
- More than four outliers?



12 Synthesis

Describe how to use the Desmos Graphing Calculator to determine which values in a data set are outliers.



Use the Desmos Graphing Calculator if it helps with your thinking.

Things to Remember:

Unit 3
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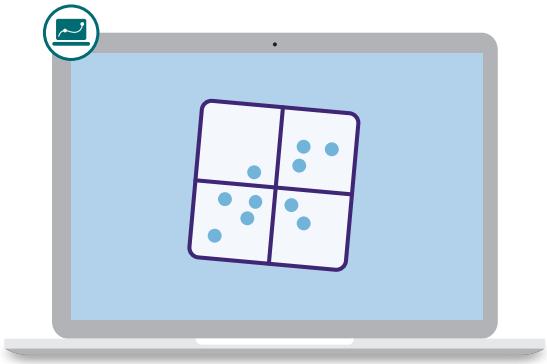
Name: ..... Date: ..... Period: .....

Correlation Coefficient

Let's learn about the correlation coefficient (r-value) as a way to measure the strength and direction of a linear relationship.



Warm-Up

1 Play a few rounds of Polygraph with your classmates!

You will use a Warm-Up Sheet. For each round:

- You and your partner will take turns being the Picker and the Guesser.
- Picker: Select a scatter plot from the Warm-Up Sheet. Keep it a secret!
- Guesser: Ask the Picker yes-or-no questions, eliminating scatter plots until you're ready to guess which scatter plot the Picker chose.

Record helpful questions from each round in the space below.

Activity 1

Name: _____ Date: _____ Period: _____

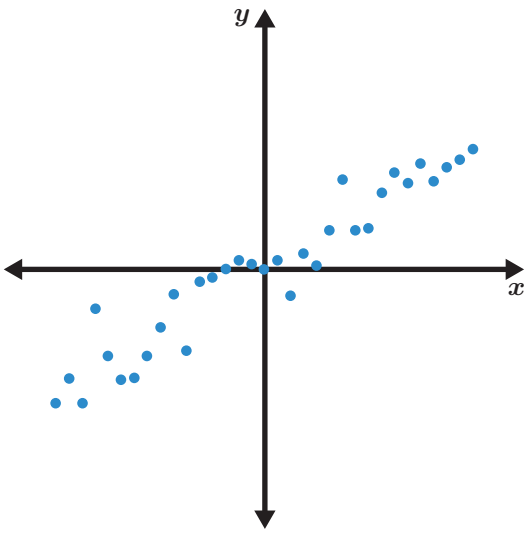
Linear Associations and Scatter Plots

2 In Polygraph, you looked at different *scatter plots*.

Now it's your turn to make your own!

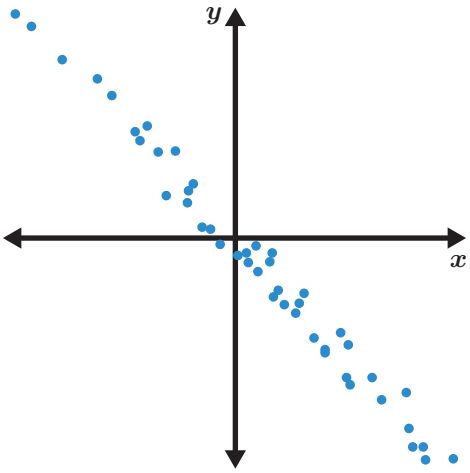
a Drag the sliders in the digital activity to make a scatter plot you like.

b Describe your scatter plot.

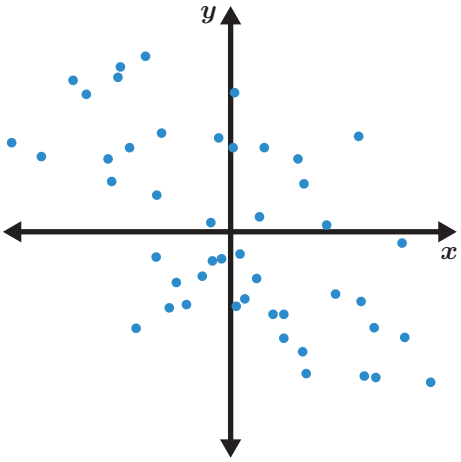


3 Here are the scatter plots Mio and Ayaan made.

Mio



Ayaan



Discuss: How are they alike? How are they different?

Name: _____ Date: _____ Period: _____

Linear Associations and Scatter Plots (continued)

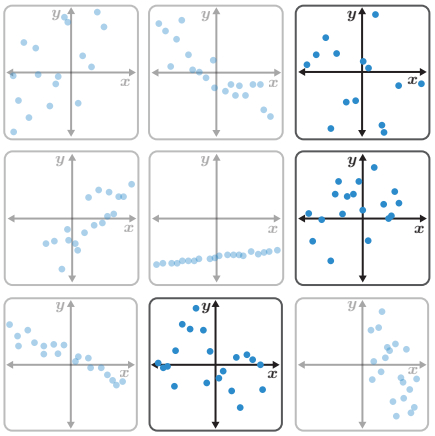
4 When the points on a scatter plot follow a line, we say there is a *linear association* between x and y.

a Here are some terms that describe linear associations.

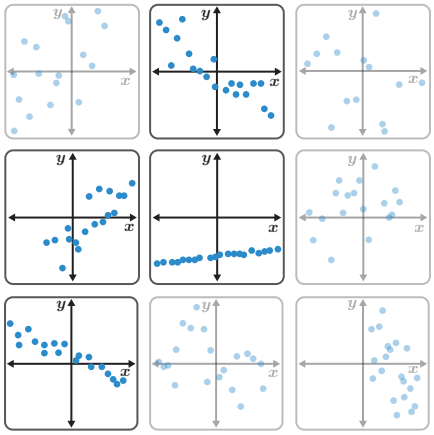
b **Discuss:** What does each term mean?

Strength

Weak

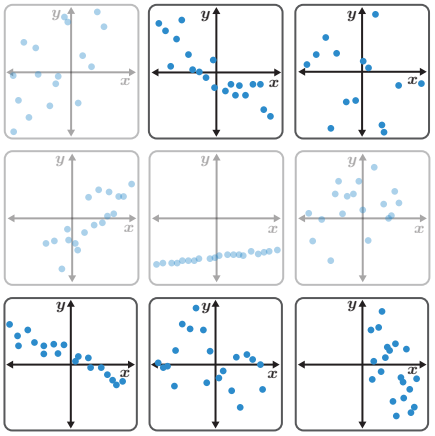


Strong

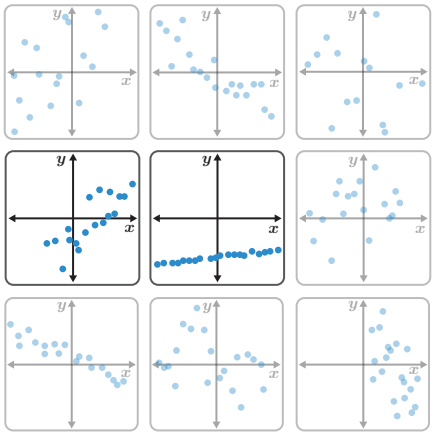


Direction

Negative



Positive



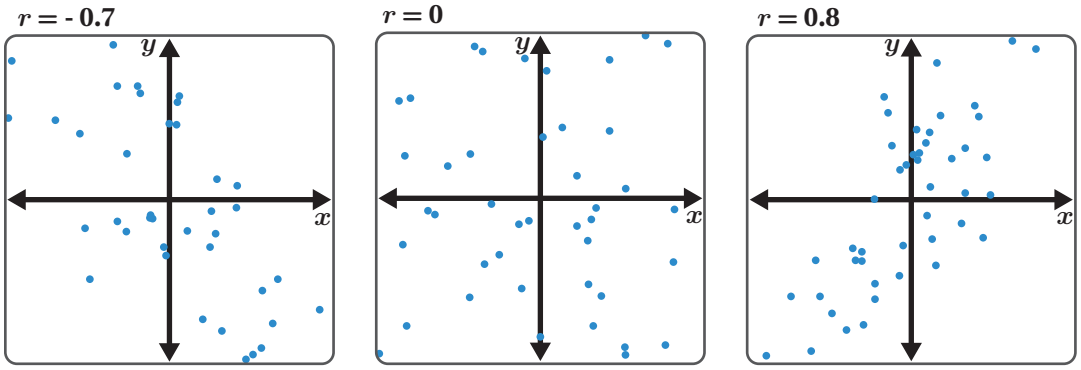
Activity 2

Name: ..... Date: ..... Period: .....

The r-value

5 The **r-value** is a number that measures the strength and direction of a linear association.

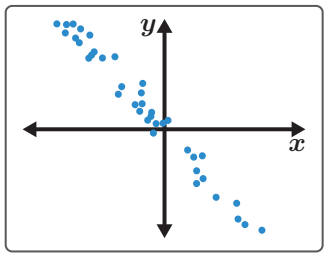
a Take a look at three scatter plots with different r-values:

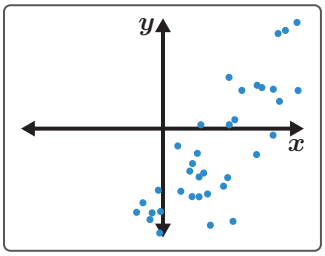


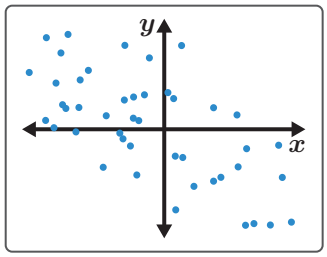
b What do you notice and wonder about the r-value?

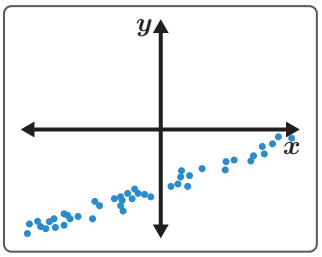
6 Match each scatter plot to its r-value.

-0.99	-0.65	0.86	0.99









Name: _____ Date: _____ Period: _____

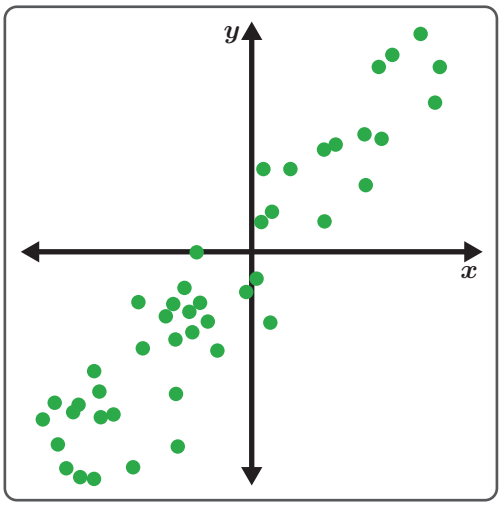
Correlation Coefficient

7 The r-value is also known as the **correlation coefficient**.

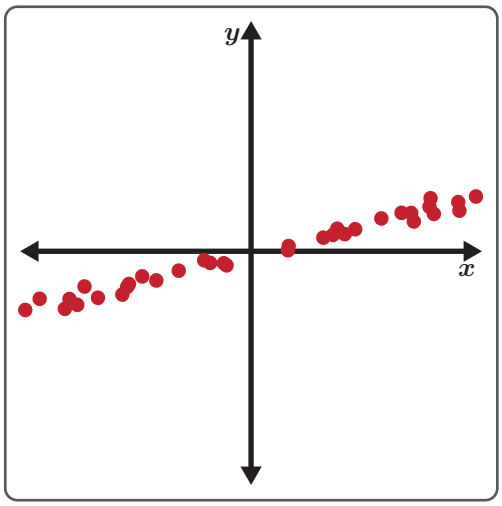
a **Discuss:** What do you think *correlation* means?

b Circle the scatter plot that has a greater correlation coefficient. Revisit Problem 5 if it helps with your thinking.

A



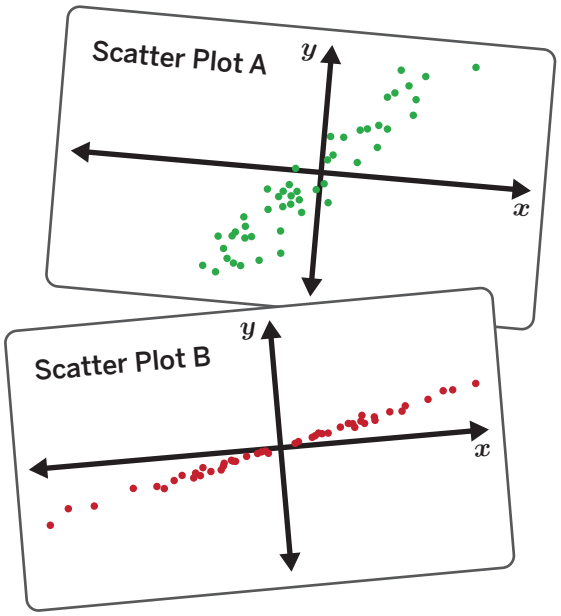
B



Explain your thinking.

8 Mio says Scatter Plot A has a greater correlation coefficient because the slope of its line is larger.

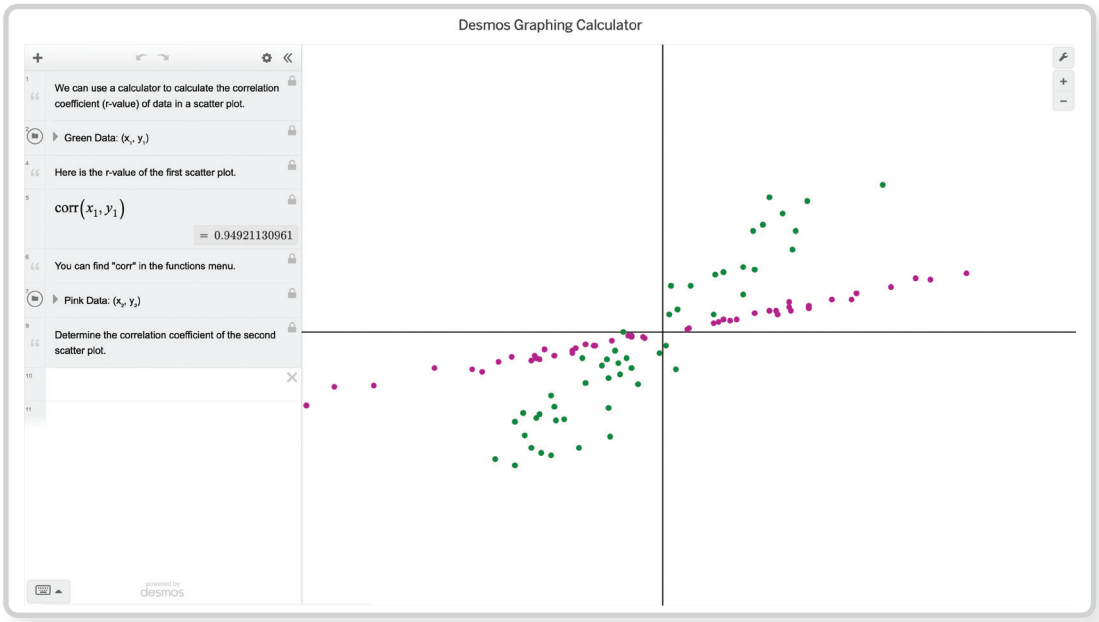
What would you say to Mio to help her understand her mistake?



Name: _____ Date: _____ Period: _____

Correlation Coefficient (continued)

9 We can use the Desmos Graphing Calculator to calculate the correlation coefficient (r-value) of data in a scatter plot.



Use the digital activity to determine the correlation coefficient of the second scatter plot.

Explore More

10 Use the digital activity to drag the sliders and see how the scatter plot changes.

a What does the top slider control?

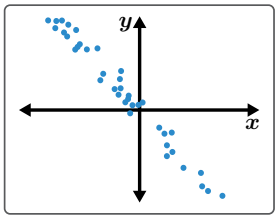
b What does the bottom slider control?

11 Synthesis

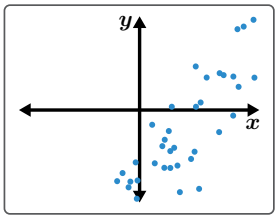
What does the correlation coefficient tell us about the data in a scatter plot?

Use these scatter plots if they help with your thinking.

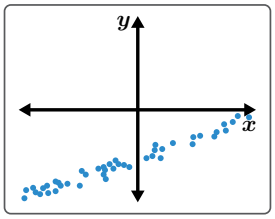
$r = -0.99$



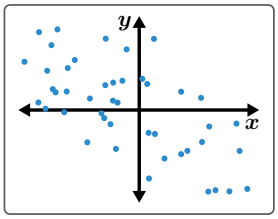
$r = 0.86$



$r = 0.99$



$r = -0.65$

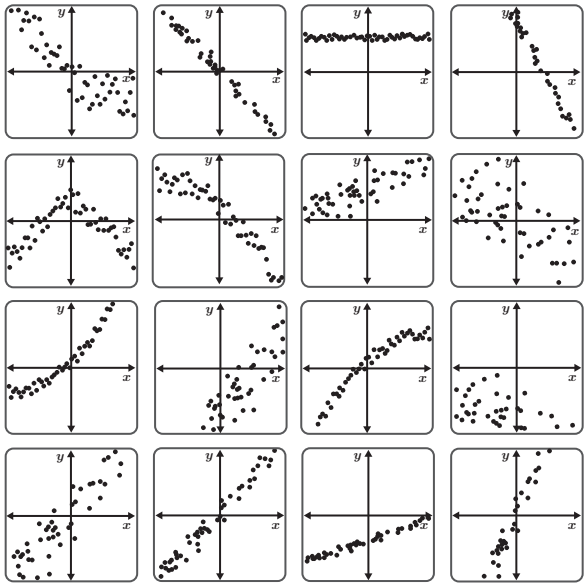


Things to Remember:

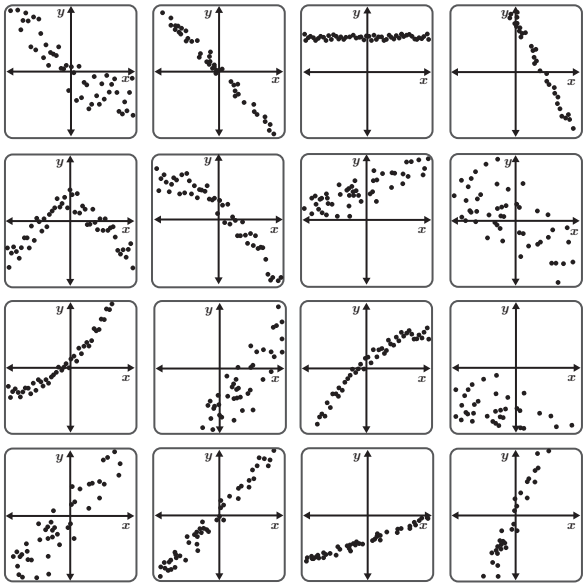
Name: ..... Date: ..... Period: .....

Polygraph Set A

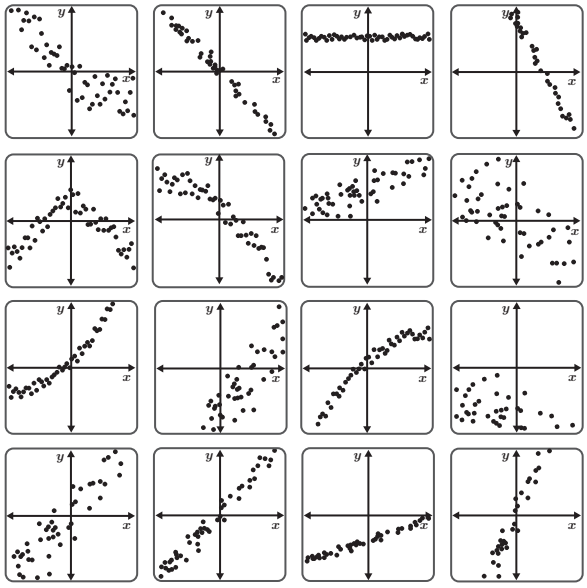
Round 1



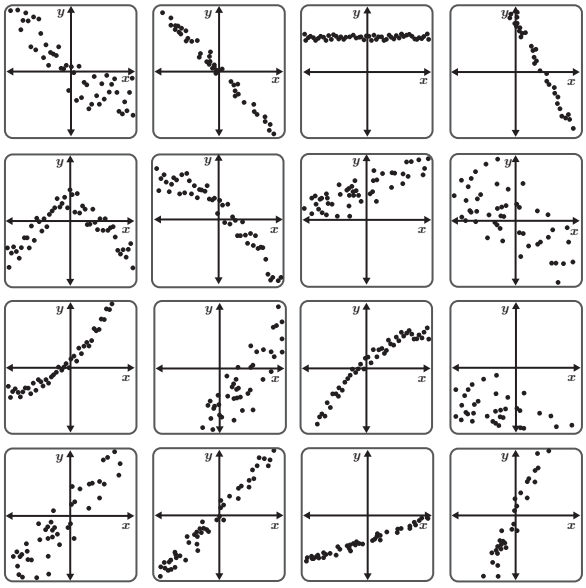
Round 2



Round 3



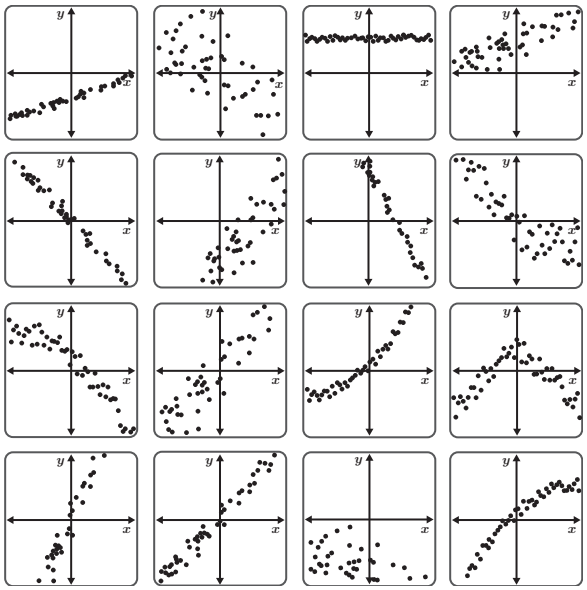
Round 4



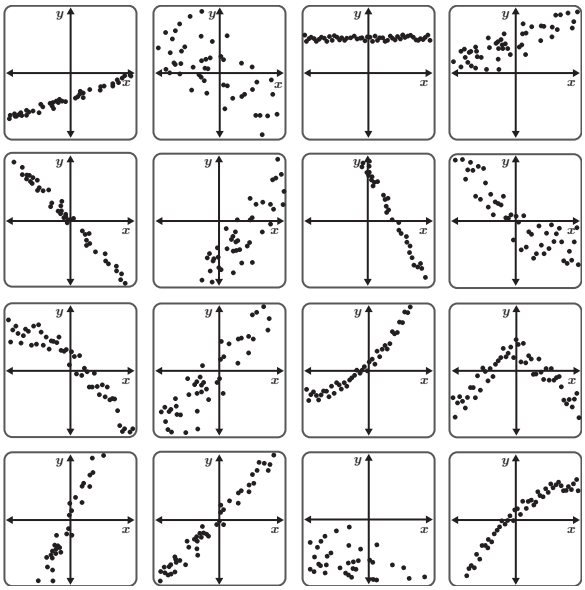
Name: ..... Date: ..... Period: .....

Polygraph Set B

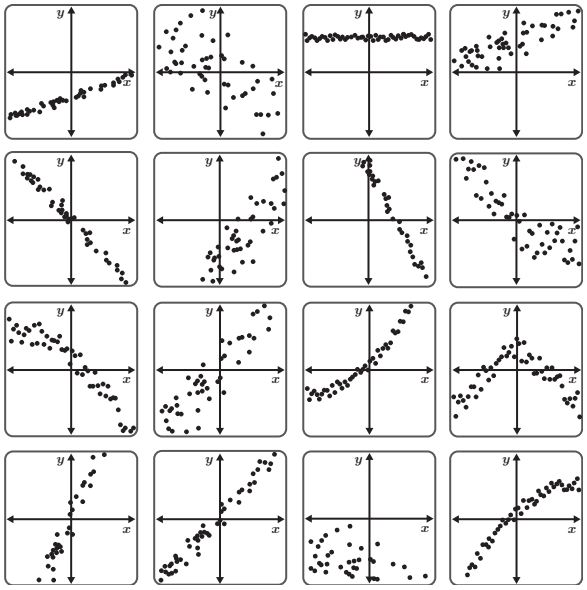
Round 1



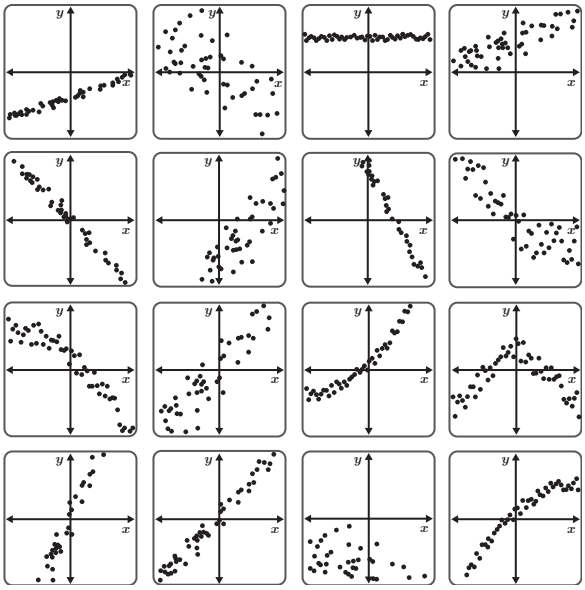
Round 2



Round 3



Round 4



Unit 3
Lesson
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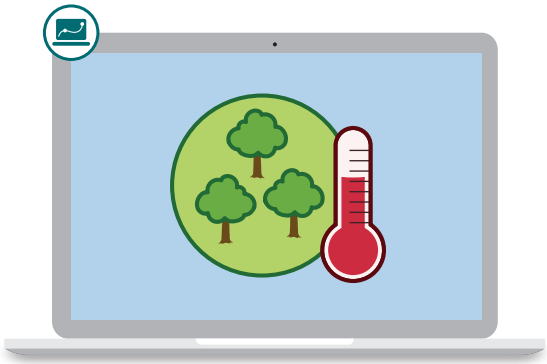
Name: ..... Date: ..... Period: .....



How Hot Is It?

Let's use correlation coefficients to analyze relationships between income, tree cover, and average temperature.

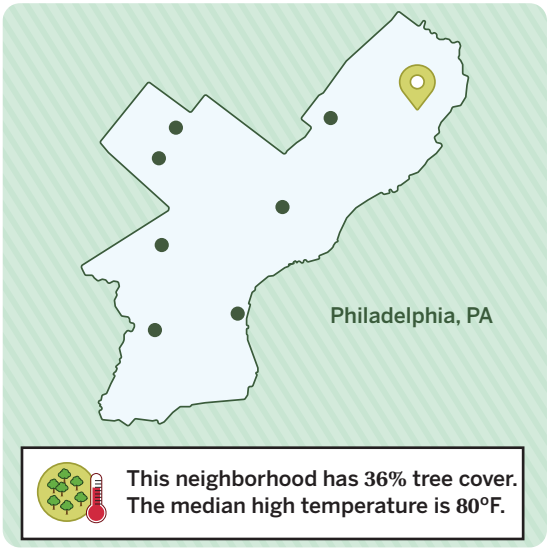
Warm-Up

1 Here is a map of Philadelphia, PA.

Let's look at a few neighborhoods.

What do you notice? What do you wonder?

I notice:



Source: *PLoS One*

I wonder:

Activity 1

Name: _____ Date: _____ Period: _____

Tree Cover vs. Temperature

2 Laila talked to people in different parts of Philadelphia to learn more about tree cover and temperature.

a Let's watch the data get collected.

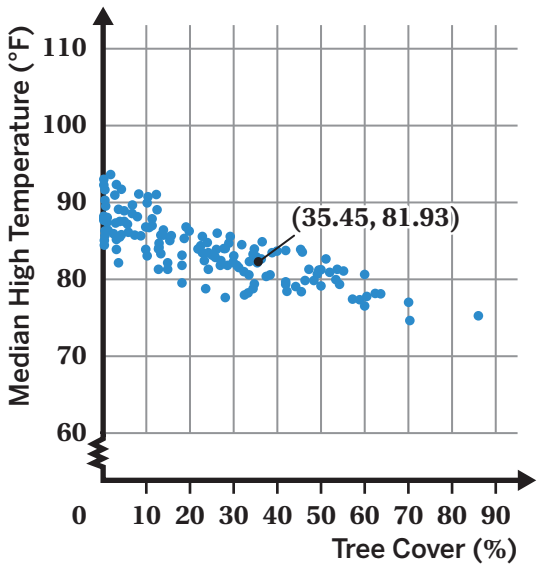
b **Discuss:** Based on this data, do you think there is a relationship between tree cover and temperature?

Tree Cover (%)	Median High Temperature (°F)
43	78
88	74
20	88
27	81
36	80
78	75
4	90
8	85

3 Here is some data from 150 blocks in Philadelphia.

One of the coordinates is shown.

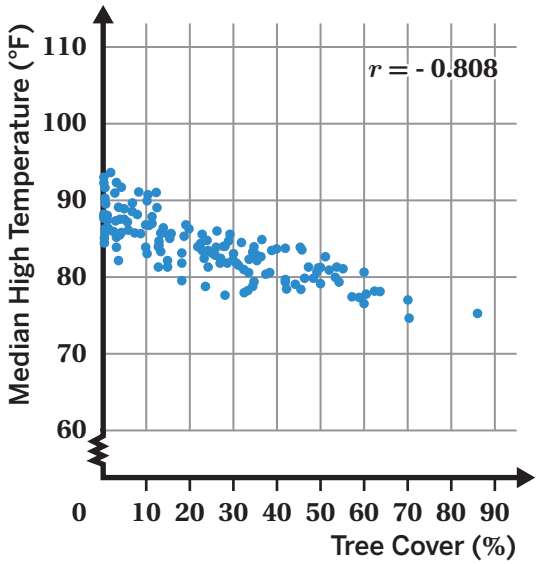
Describe what the coordinates tell you about that block.



Name: _____ Date: _____ Period: _____

Tree Cover vs. Temperature (continued)

4 Here is the correlation coefficient (the r-value) of the data.



a Based on the r-value, what kind of association is there between tree cover and temperature? Circle one.

Positive Negative No association

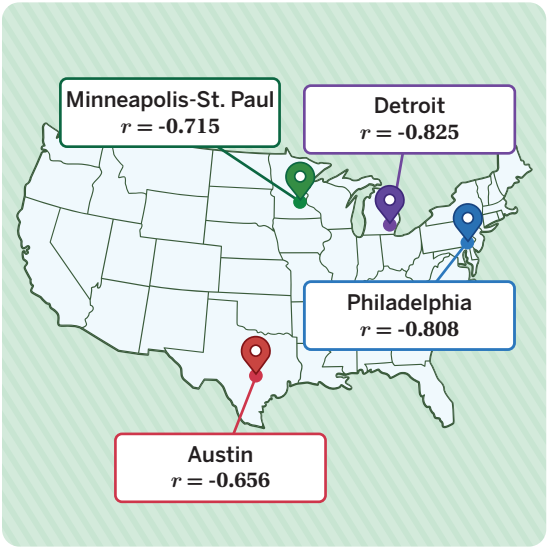
b What is the strength of the association? Circle one.

Weak Strong

5 Laila wonders how other cities compare to Philadelphia.

Let's look at other city data on the map.

Discuss: What do you notice? What do you wonder?



Activity 2

Name: _____ Date: _____ Period: _____

Including Income

6 Temperature is one of many variables associated with tree cover.

What other variables do you think could be associated with tree cover?

7 Laila wonders: *Is there an association between income and tree cover?*

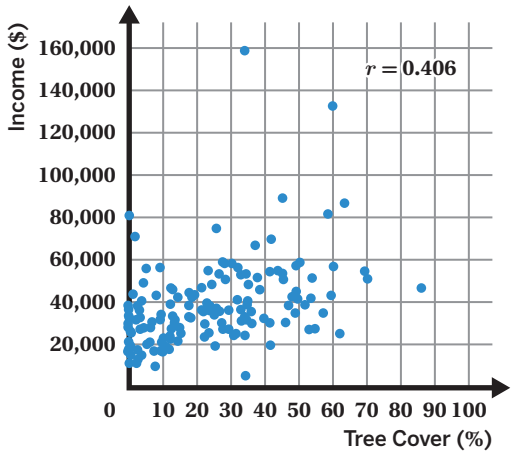
a Make a prediction: What kind of association do you expect between these variables? (E.g., weak positive or strong negative.)

b Let's look at the data.

Discuss: Was your prediction correct?

8 This graph shows the average income and the percentage of tree cover for 150 blocks in Philadelphia.

What does the r-value say about the association between income and tree cover?



Name: _____ Date: _____ Period: _____

Including Income (continued)

9 The Philadelphia data has a correlation coefficient of 0.406.

Here is some data about income and tree cover in Detroit.

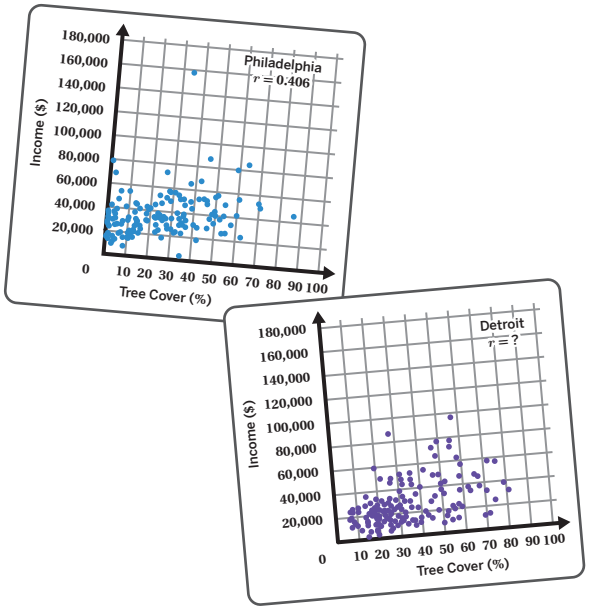
Which could the r-value for Detroit be?

A. $r = 0.35$

B. $r = 0.85$

C. $r = -0.35$

D. $r = -0.85$



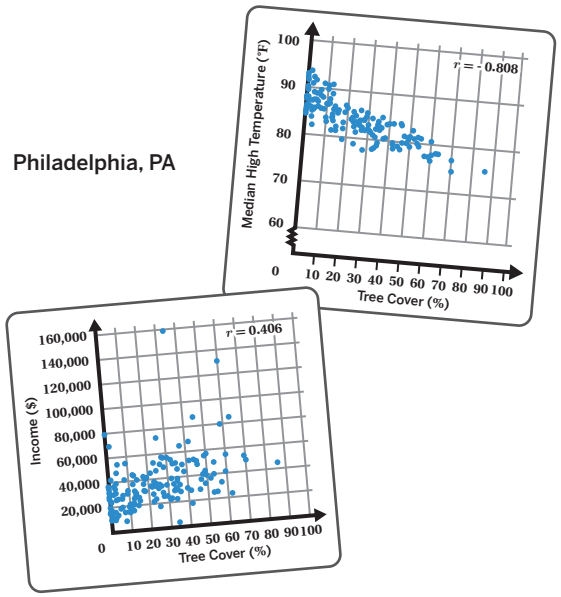
10 At a neighborhood meeting, someone said:

It is unfair that lower income neighborhoods are hotter in Philadelphia.

a How does the data support this statement?

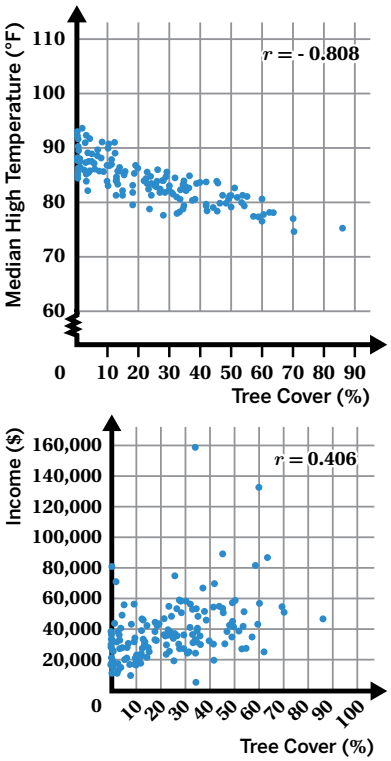
b What changes do you think should be made?

Philadelphia, PA



11 Synthesis

How can correlation coefficients help us describe the relationship between two variables in the real world?



Things to Remember:

Unit 3
Lesson
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Name: ..... Date: ..... Period: .....

City Slopes

Let's use a line of fit to describe the relationship between two variables and make predictions.

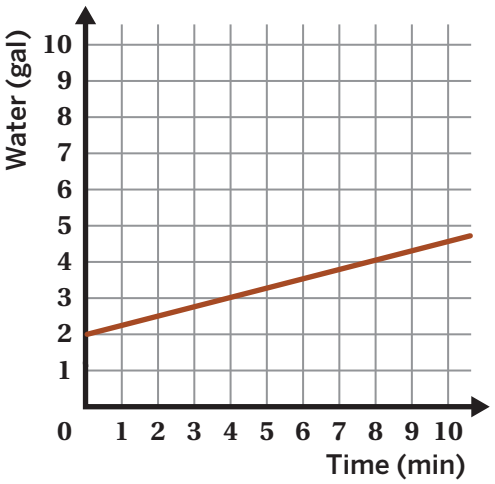
Warm-Up

1 An equation for this line is $y = \frac{1}{4}x + 2$.

Show or explain where you see $\frac{1}{4}$ and 2 in the graph.

$\frac{1}{4}$:

2:



Source: *PLoS One*

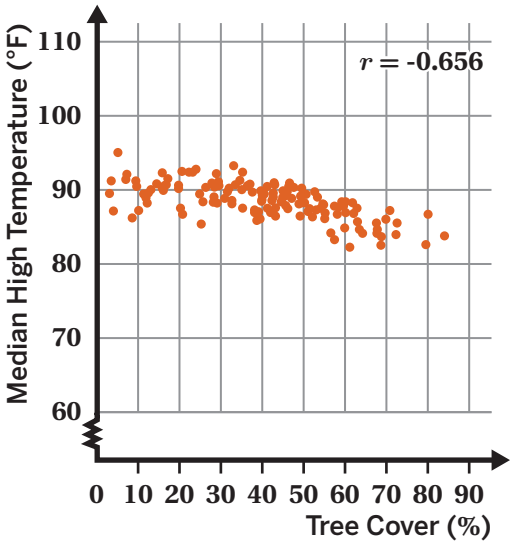
Activity 1

Name: ..... Date: ..... Period: .....

Lines of Fit

2 Here's the graph of temperature and tree cover for Austin, Texas.

Mathematicians use a *line of fit* to describe relationships and make predictions.



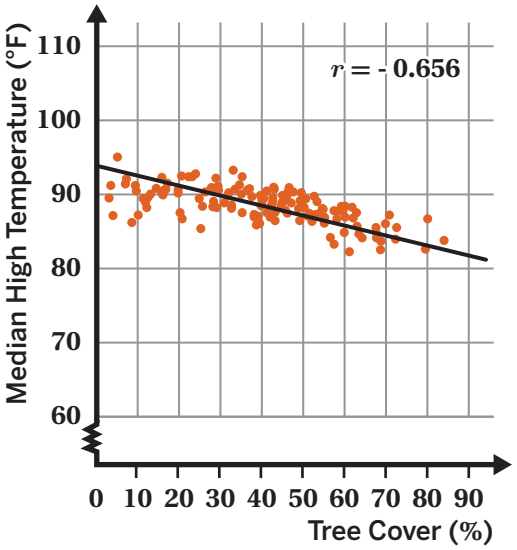
a. **Discuss:** Why is a line a good fit for this data?

b. Draw a line of fit for the data.

3 Here is the line of fit a student drew.

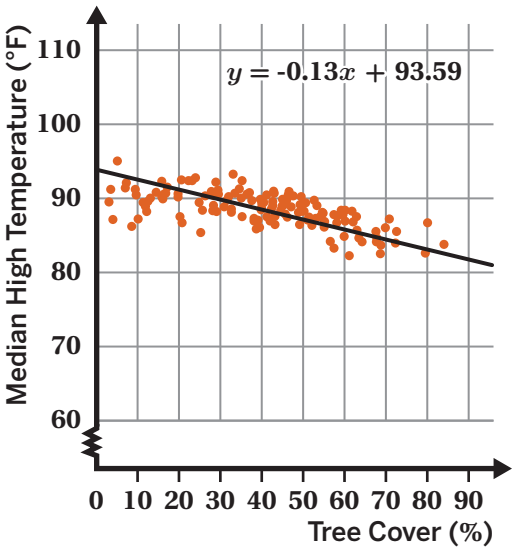
Jamal lives in Austin, on a block that has 75% tree cover.

What might the median high temperature be on his block?



4 Here is an equation for this line of fit.

What does the *slope* tell us the relationship between temperature and tree cover? Circle one.



When the tree cover increases by 1%, the temperature decreases by 0.13°F.

When the tree cover increases by 1%, the temperature decreases by 93.59°F.

Activity 2

Name: _____ Date: _____ Period: _____

Interpreting in Context

5 Let's compare some cities.

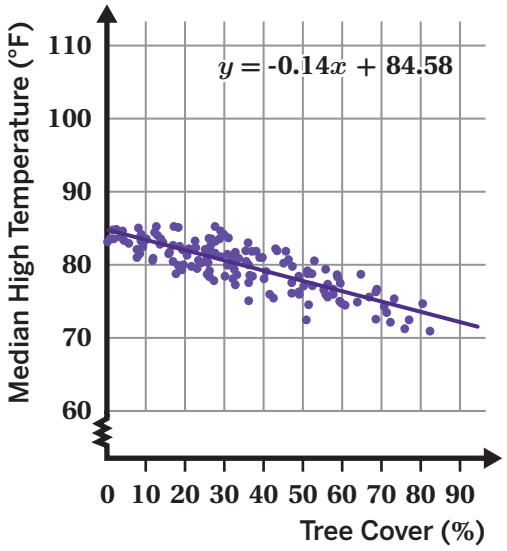
Discuss: What do you notice? What do you wonder?

6 Here is an equation for Detroit's line.

What do the -0.14 and 84.58 tell us about the relationship between temperature and tree cover?

-0.14:

84.58:

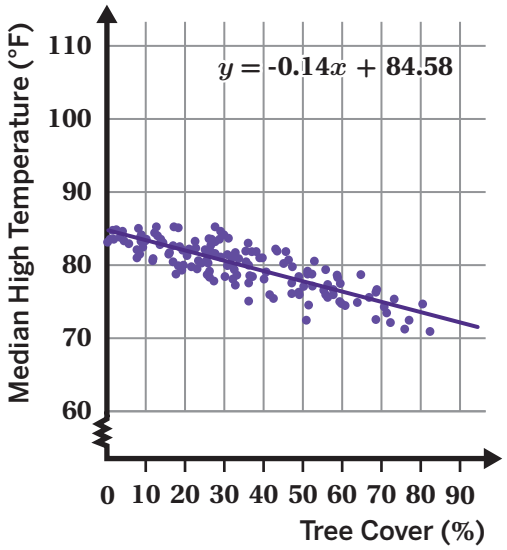


Name: _____ Date: _____ Period: _____

Interpreting in Context (continued)

7 A community in Detroit wants to build a park.

If the park has 80% tree cover, what might its median high temperature be?

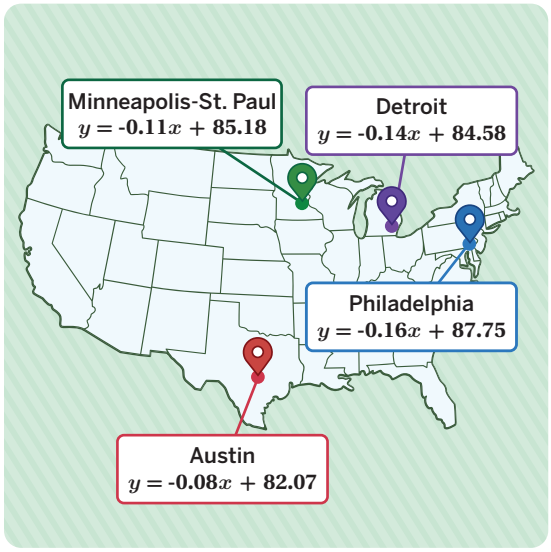


8 **a** Order the cities according to where tree cover has the greatest impact on temperature.

Greatest

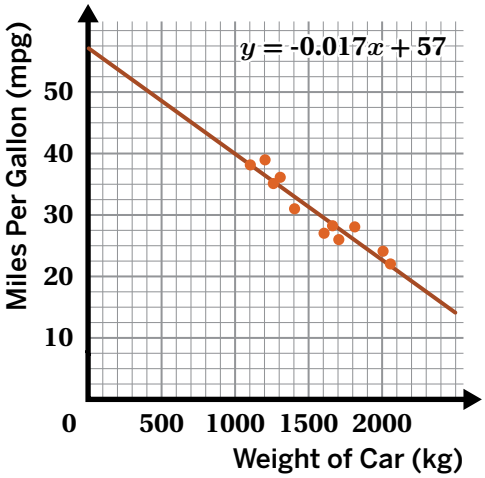
Least

b **Discuss:** How did you order each city?



9 Synthesis

How can a line help us make predictions about data?



Things to Remember:

Unit 3
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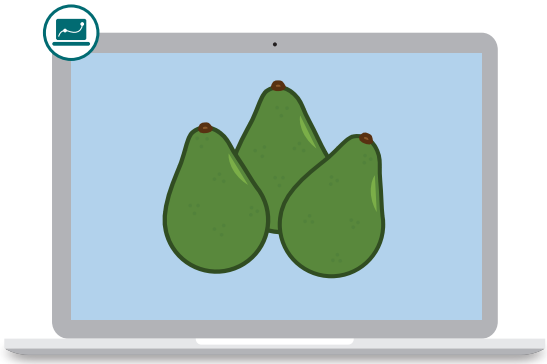
Name: Date: Period:

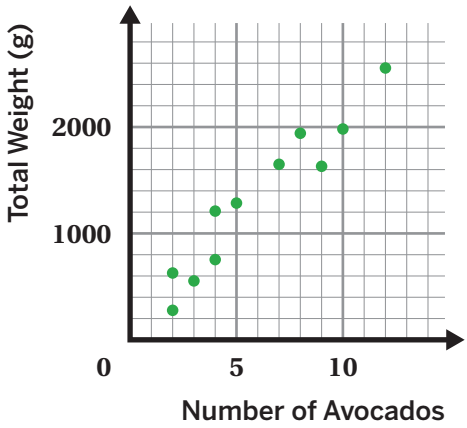


Residual Fruit

Let's use residual plots to determine how well a line fits data.

Warm-Up

1 Brianna has a business that ships different kinds of fruit.



a. Let's watch orders being weighed.

b. **Discuss:** How much do you think 11 avocados will weigh?

Activity 1

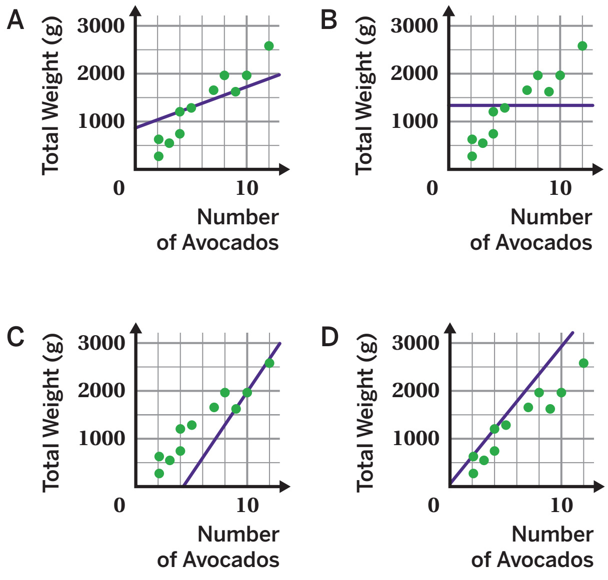
Name: .... Date: .... Period: ....

Predicting With Lines

2 Lines that fit the data well can help us make predictions.

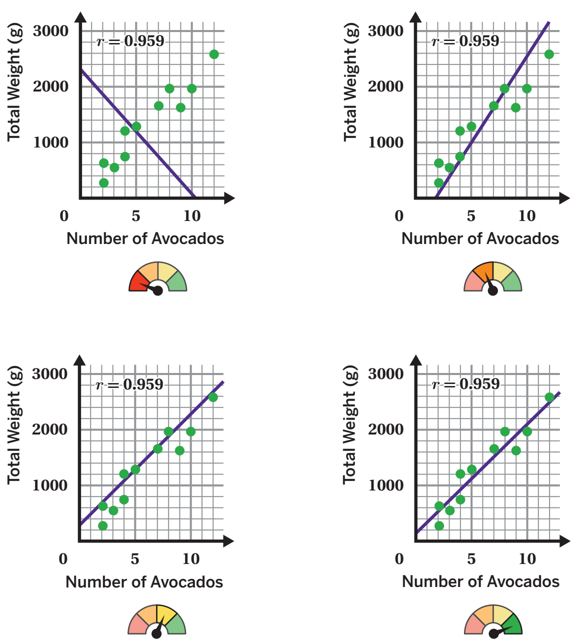
None of these lines fit the data well.

Circle a scatter plot. Explain why the line does not fit the data well.



3 **a** Take a look at these scatter plots. The meters show how well each line fits the data.

b Explain to a classmate how to get a high score on the meter.

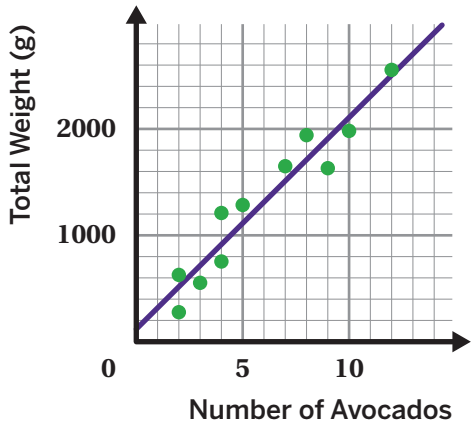


Name: Date: Period:

Predicting With Lines (continued)

4 Brianna has an order to ship 6 avocados.

How could you use this line to predict the weight of the order?



5 The line predicts that 6 avocados will weigh 1,316 grams, but 6 avocados actually weigh 1,740 grams.

What is the difference, in grams, between the actual weight and the predicted weight?

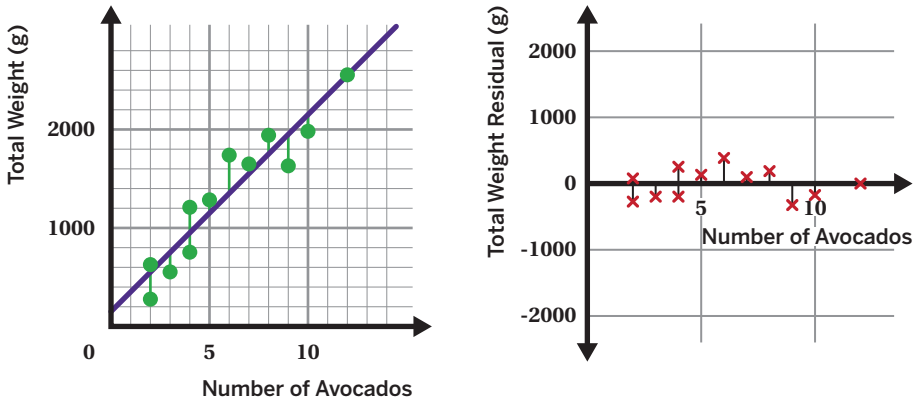
Activity 2

Name: _____ Date: _____ Period: _____

Residual Plots

6 A **residual** is the difference between the predicted and measured weight.

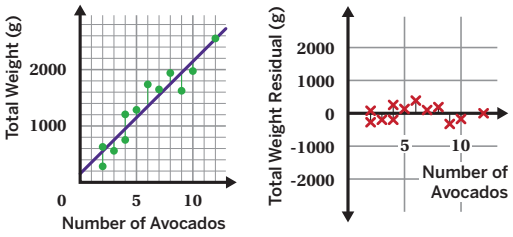
a Let's see how to plot residuals.



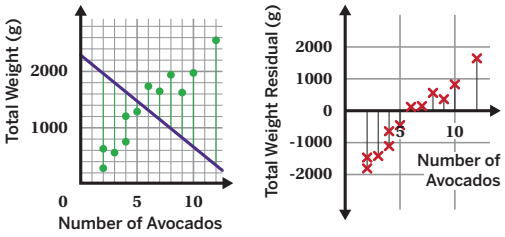
b **Discuss:** How could you make a residual plot?

7 A **residual plot** shows how far each point is from the line of fit.

Fits the Data Well



Doesn't Fit the Data Well



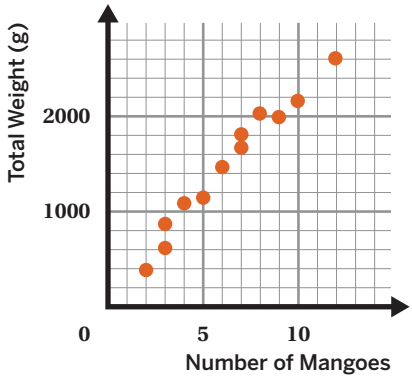
What does the residual plot look like when the line fits the data well? What about when it doesn't fit the data well?

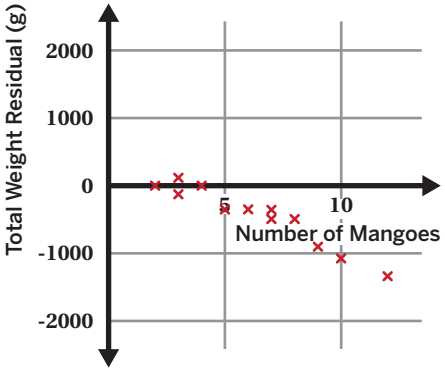
Name: _____ Date: _____ Period: _____

Residual Plots (continued)

8 Here is the residual plot for a line of fit Brianna created.

Sketch the line of fit you think Brianna made.

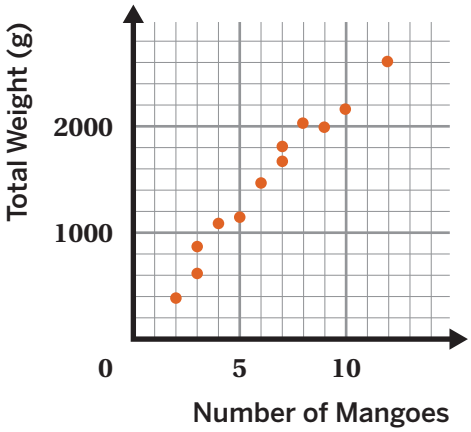




9 Let's look at Brianna's line.

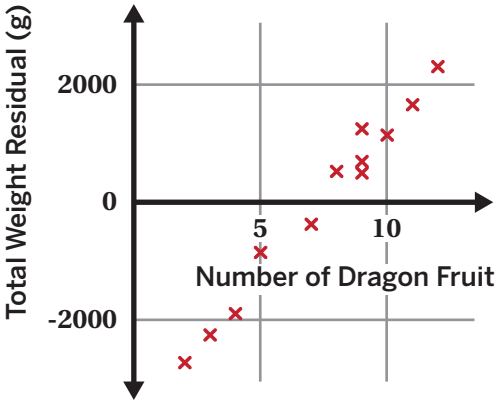
a Draw another line that is a better fit for the data.

b **Discuss:** How will the residual plot change once the line is a better fit for the data?



10 Aditi made a line of fit for this data showing the total weight of different numbers of dragon fruit.

Here is the graph of the residuals from Aditi's line. How well do you think Aditi's line fits the data?

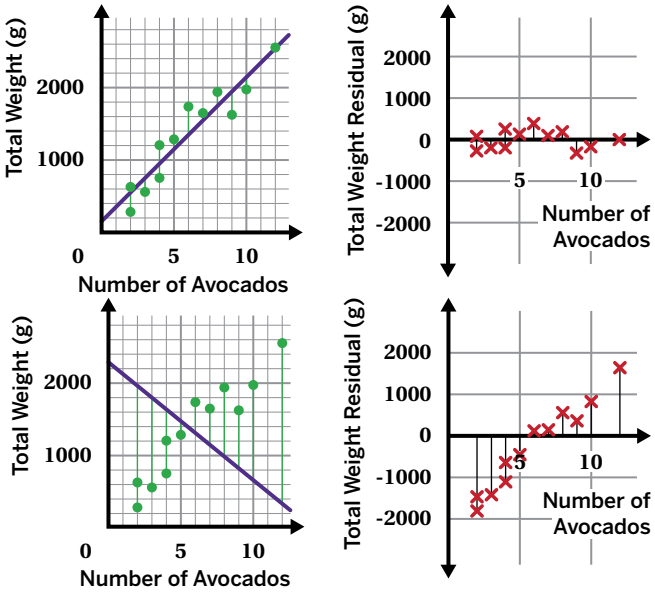


Explore More

11 Use the Explore More Sheet to draw a line of fit.

12 Synthesis

How can you use a residual plot to determine if a line fits the data well?



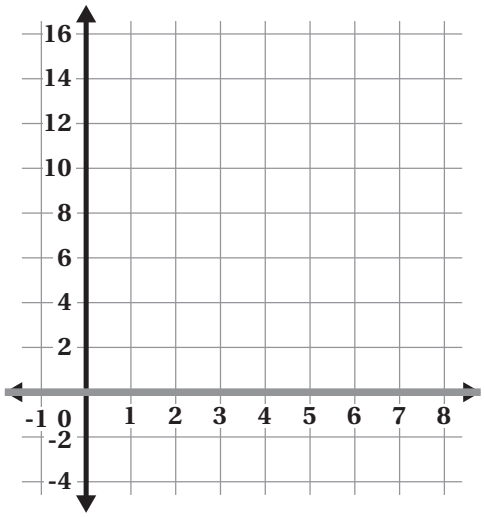
Things to Remember:

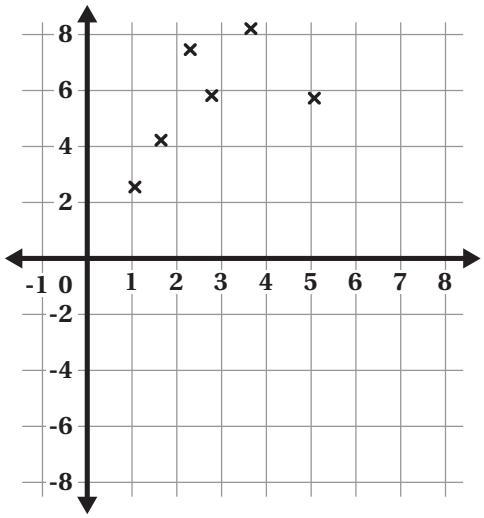
Name: ..... Date: ..... Period: .....

Explore More

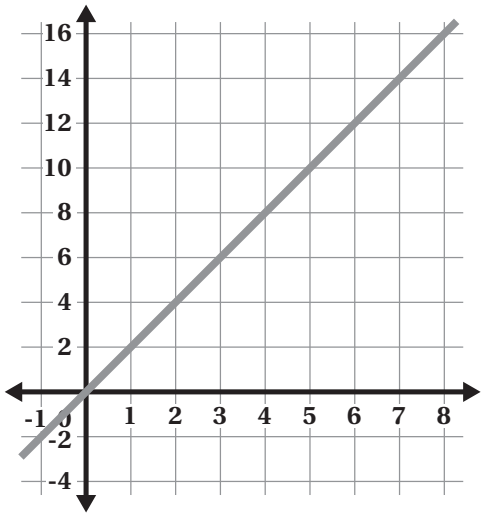
11 The data on each scatter plot is hidden. Use the residual plot to draw a line that fits the data better.

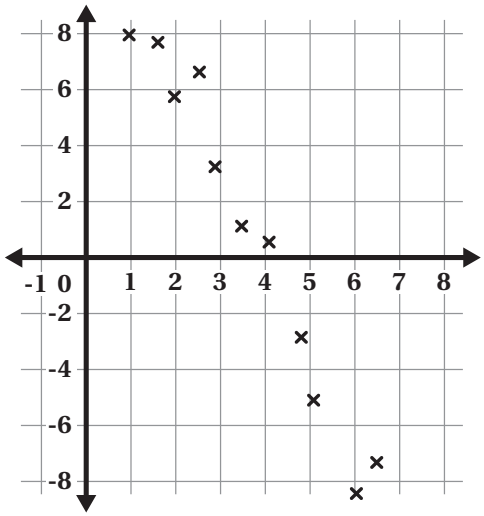
a



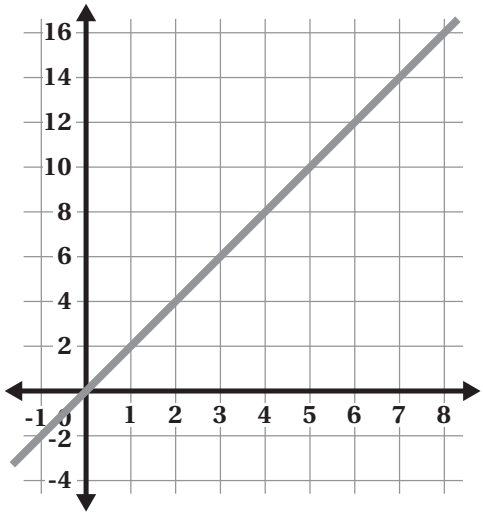


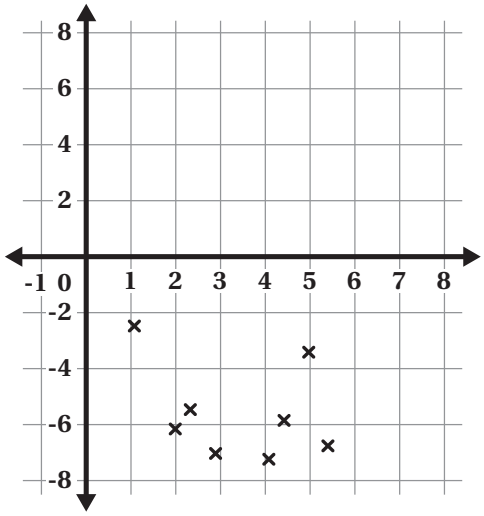
b





c



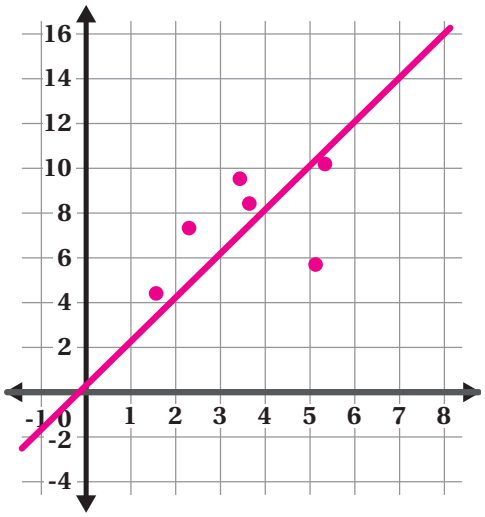


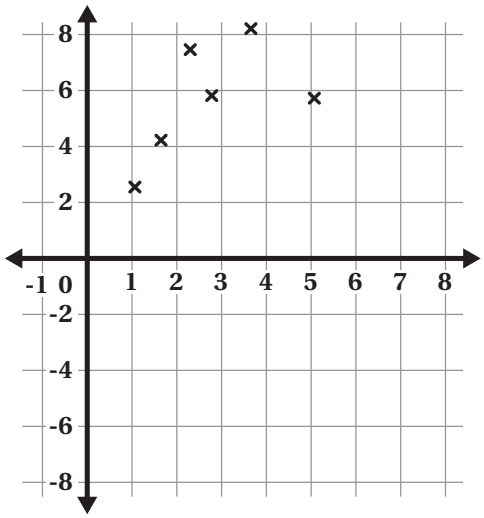
Name: ..... Date: ..... Period: .....

Explore More (answers)

11 The data on each scatter plot is hidden. Use the residual plot to draw a line that fits the data better.

a





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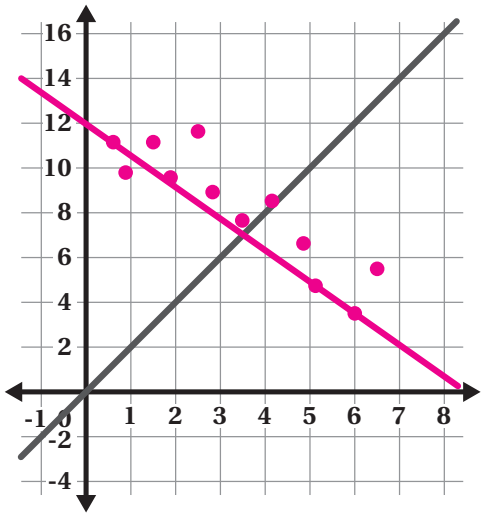

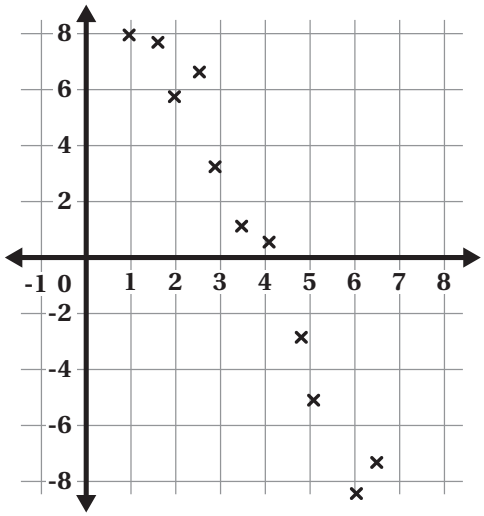

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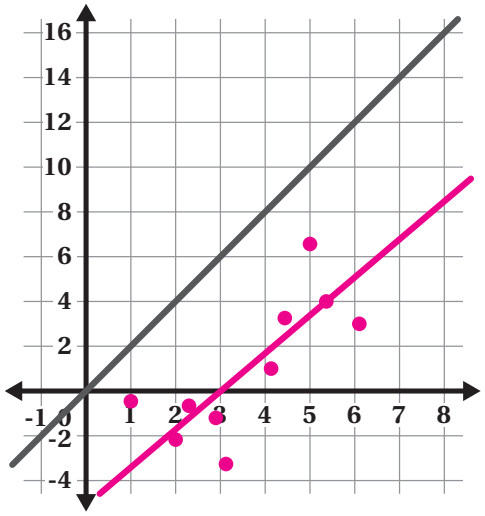
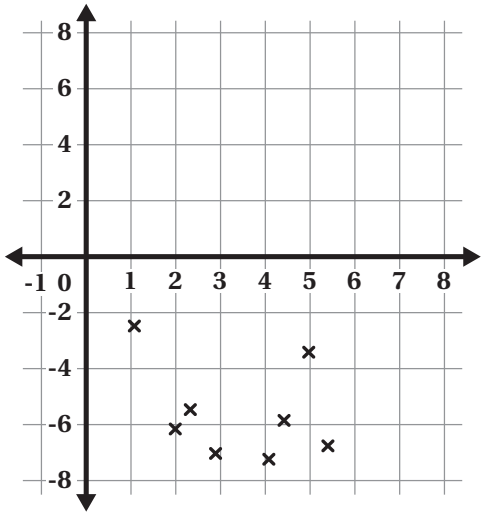




Unit 3
Lesson
17

Name: ..... Date: ..... Period: .....

Penguin Populations

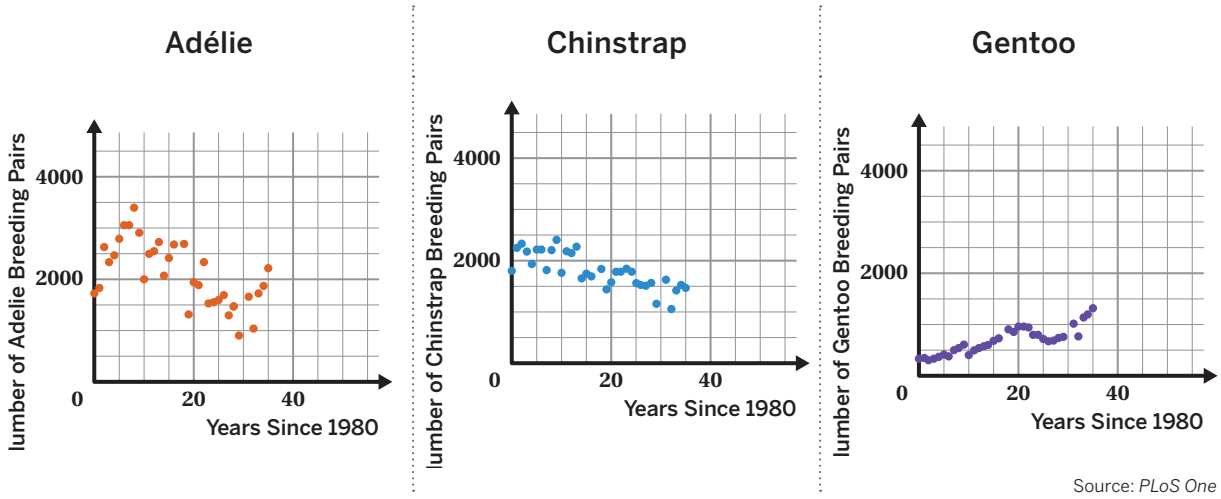
Let's generate and analyze lines of best fit to explore how penguin populations have changed over time.

Warm-Up

1–2 Researchers are conducting a long-term study on the South Orkney Islands in Antarctica.

The goal of this study is to understand how the populations of three species of penguins have changed over the last 40 years.

Here is the study's data for each of the penguin species.



Source: *PLoS One*

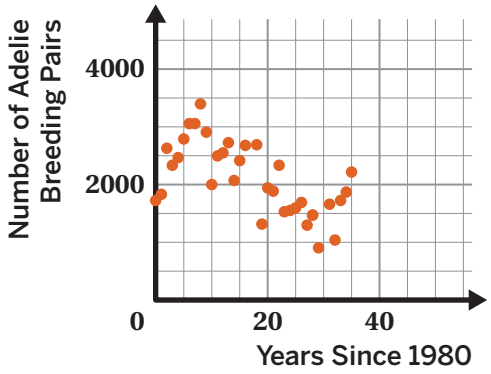
How is the population of each species changing over time? What might be affecting these changes?

Activity 1

Name: _____ Date: _____ Period: _____

Predicting With Lines

3 Let's explore how the Adélie penguin population on the South Orkney Islands has changed over time.



a Draw a line that fits the data.

b What does the line tell you about the penguin population?

4 A calculator can compute the **line of best fit**.

a Let's look at the line of best fit for the Adélie penguin data.

b **Discuss:** What do you notice about the residual plot?

5 Let's look at the equation of the line of best fit for the Adélie penguin data.

a **Discuss:** What x-value represents the year 2030?

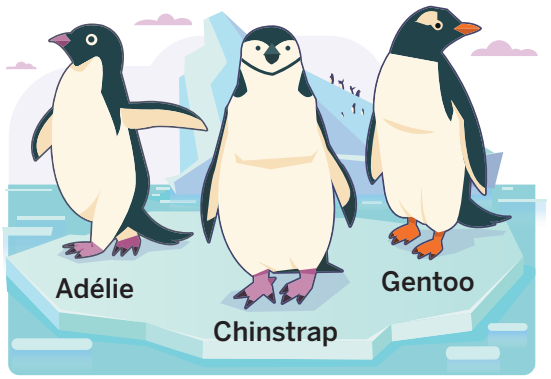
b Use the line of best fit to predict how many breeding pairs of penguins there will be in 2030.

Activity 2

Name: _____ Date: _____ Period: _____

Generating a Line of Best Fit

6–7 In the digital activity, take a look at the table of data showing the Adélie penguin population over time and the equation of the line of best fit.



a. **Discuss:** Why do you think this is called a linear regression?

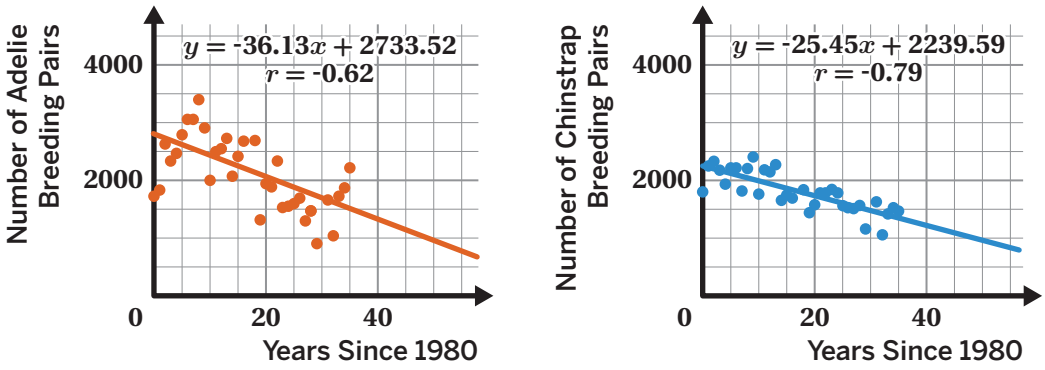
b. **Discuss:** What do the parts of the equation represent?

8 Use the digital activity to generate the line of best fit for the Chinstrap data.

Name: _____ Date: _____ Period: _____

Generating a Line of Best Fit (continued)

9 Compare the data of these two penguin populations.



Discuss: How are they alike? How are they different?

Name: _____ Date: _____ Period: _____

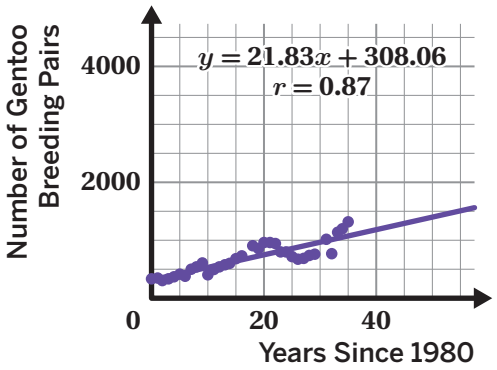
Making Predictions

10 Use the digital activity to analyze the Gentoo data.

Generate the line of best fit and write the equation for the Gentoo data.

11 Here is the line of best fit for the Gentoo data.

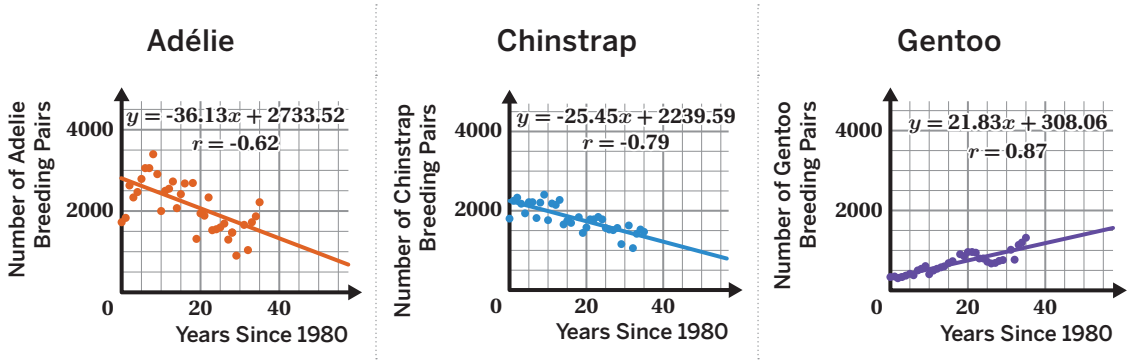
How many breeding pairs of penguins does the line of best fit predict there will be in 2030?



Name: _____ Date: _____ Period: _____

Making Predictions (continued)

12 What questions might researchers have after analyzing the data from the Adélie, Chinstrap, and Gentoo penguin populations?



13 Here's a quote about modeling that you may remember:

All models are wrong, but some are useful.

a Select a model we've explored today.

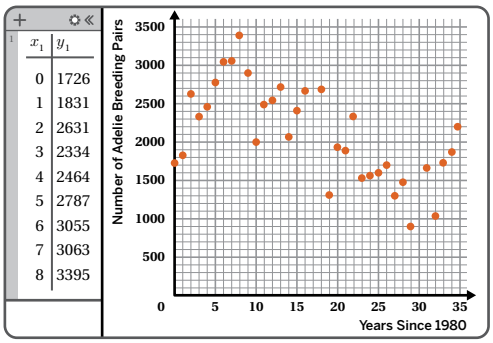
b Explain how that model is wrong and how it is useful.

The model is wrong because . . .

It is useful because . . .

14 Synthesis

Describe how to use the Desmos Graphing Calculator to generate the line of best fit for data.



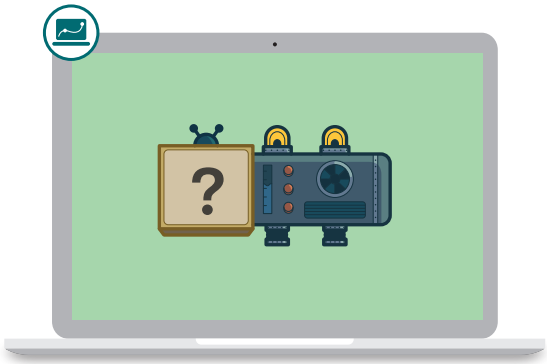
Things to Remember:

Unit 4
Lesson
1

Name: ..... Date: ..... Period: .....

Mystery Rule

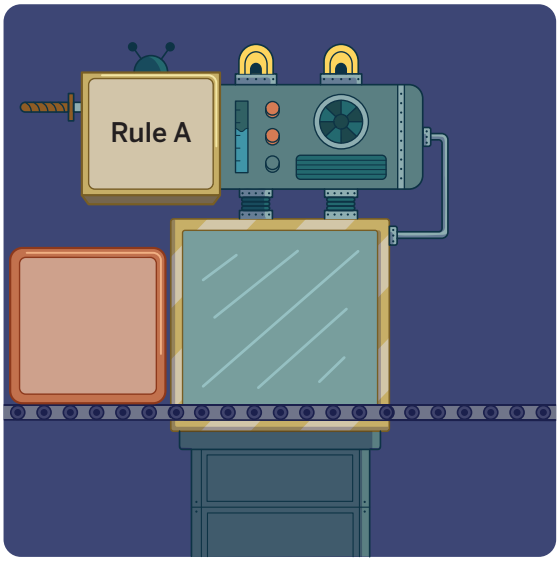
Let's consider whether or not rules are functions.



Warm-Up

1 This machine uses Rule A to turn *inputs* into *outputs*.

Let's test several inputs to see how Rule A works. Record the results in the table.



Input	Output
5	16

2 Predict the output for 101. Explain your reasoning.

Activity 1

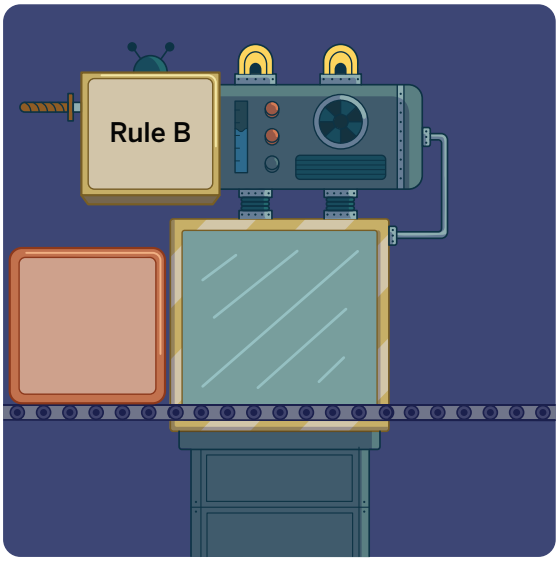
Name: _____ Date: _____ Period: _____

What Is a Function?

3 Rule B's inputs are words.

Let's test several inputs to see how Rule B works. Record the results in the table.

Input	Output
howdy	8



4 Predict the output for "give". Explain your reasoning.

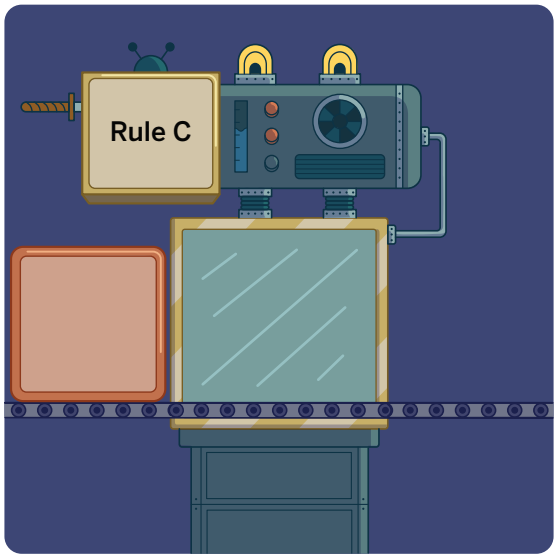
Name: _____ Date: _____ Period: _____

What Is a Function? (continued)

5 Rule C's inputs are whole numbers from 1 to 15.

Let's test several inputs to see how Rule C works. Record the results below.

Input	Output



6 **a** Predict the output for 6.

b Compare your response with a partner's. How are your responses alike and different?

7 Rules A and B are examples of a *function*.

Rule C is *not* a function.

What do you think makes Rule C not a function?

Functions

Rule A

Input	Output
5	16
6	19
0	1
5	16

Rule B

Input	Output
howdy	8
face	6
mountain	13
flower	6

Not a function

Rule C

Input	Output
5	watch
9	vegetable
9	classroom
1	a

Rule H

Input	Output
2	5
1	10
0	15
1	20

Name: _____ Date: _____ Period: _____

Which Rules Are Functions?

8 Here are four rules.

Discuss: Is each rule a function? Why or why not?

Rule D takes temperatures in Fahrenheit and outputs temperatures in Celsius.

Input	Output
50	10
77	25
100	37.8
212	100

Rule E takes any integer and outputs one of its factors.

Input	Output
24	6
13	1
13	13
9	3

Rule F takes any number and rounds it to a whole number.

Input	Output
32.5	33
$\frac{4}{3}$	1
0.1	0
23	23

Rule G takes any word and shifts each letter one place in the alphabet.

Input	Output
bird	cjse
house	ipvtf
hello	ifmmp
world	xpsme

Name: _____ Date: _____ Period: _____

Which Rules Are Functions? (continued)

9 Rule E is *not* a function.

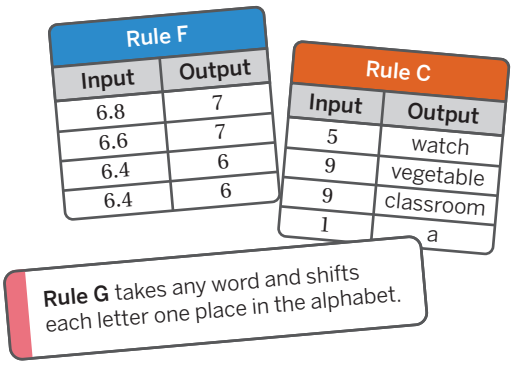
How can you tell by looking at the table?

Rule E takes any integer and outputs one of its factors.

Rule E	
Input	Output
1200	30
1200	10
20	10

10 Synthesis

How can you decide whether a rule is a function?



Rule F

Input	Output
6.8	7
6.6	7
6.4	6
6.4	6

Rule C

Input	Output
5	watch
9	vegetable
9	classroom
1	a

Rule G takes any word and shifts each letter one place in the alphabet.

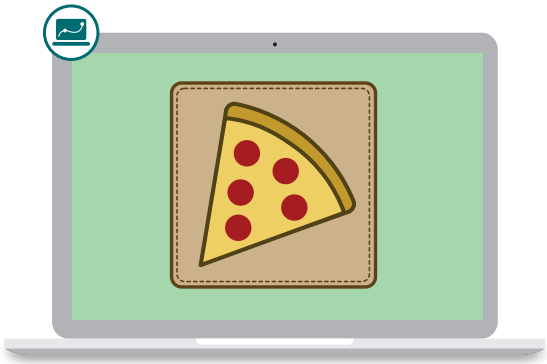
Things to Remember:

Unit 4
Lesson
2

Name: ..... Date: ..... Period: .....

Pricing Pizzas

Let's learn what function notation is and interpret function notation statements in context

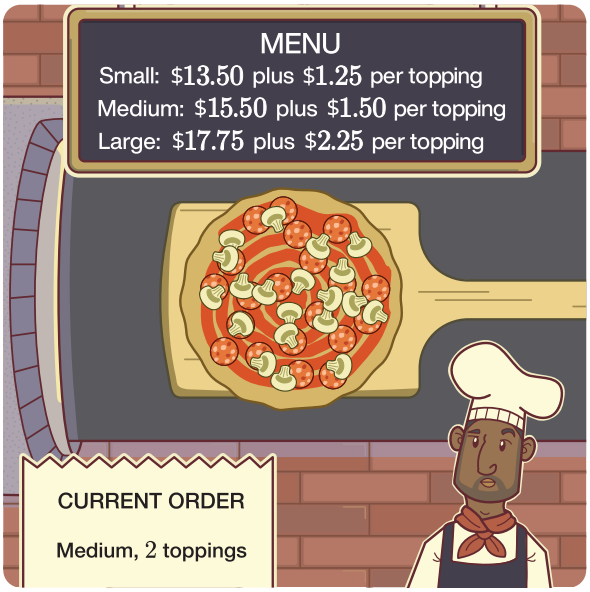


Warm-Up

1 Desmos Pizza offers small, medium, and large pizzas.

Use the menu to determine the price of each pizza order.

Pizza Order	Price ($)
Medium, 2 toppings	
Large, 4 toppings	
Small, 3 toppings	
Large, 6 toppings	



Activity 1

Name: _____ Date: _____ Period: _____

Pricing Pizzas

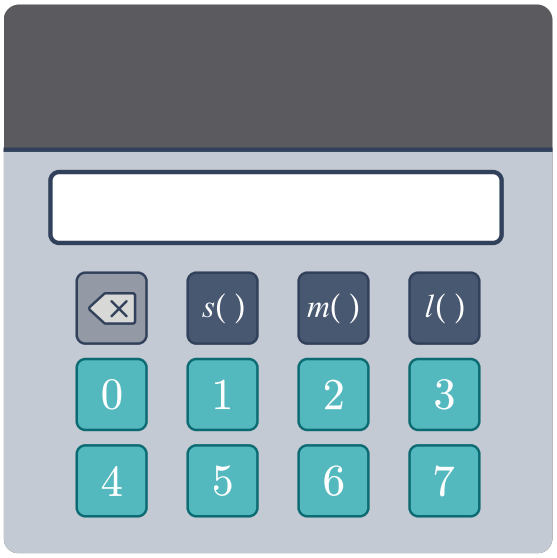
2 A worker at Desmos Pizza made a list of all the large pizza orders one night.

Discuss: How is this list like a function?

3 Desmos Pizza uses a cash register with three functions: one for each size pizza.

In the digital activity, use the functions to determine the price of each pizza order.

Pizza Order	Price ($)
Small, 3 toppings	$s(3) = 17.25$
Large, 2 toppings	
Medium, 1 topping	



Name: Date: Period:

Pricing Pizzas (continued)

4 $s(3)$ is an example of a statement in **function notation**.

We read $s(3)$ as "s of three."

a Say $s(3) = 17.25$ aloud to a classmate.

b Select *all* the ideas that $s(3)$ represents.

- ☐ **A.** The price of a small pizza with 3 toppings.
- ☐ **B.** The price of a small pizza multiplied by 3.
- ☐ **C.** The function s with an input of 3.
- ☐ **D.** The function s with an output of 3.
- ☐ **E.** The price of 3 small pizzas.

5 In the previous problem, Luca said:

s times 3 is 17.25, so a small pizza with 3 toppings will cost $5.75.

What would you say to help him understand his mistake?

Luca

$$\frac{s(3)}{3} = \frac{17.25}{3}$$

$$s = 5.75$$

Activity 2

Name: _____ Date: _____ Period: _____

Interpreting Function Notation

6 Match each function notation statement with its correct interpretation(s). Three cards will have no match.

Card A	Card B	Card C	Card D
The function s with the input 0.	A small pizza that costs $\$0$.	The price of a medium pizza with x toppings.	5 large pizzas

Card E	Card F	Card G
The output of l when the input is 5.	The price of a small pizza with 0 toppings.	The price of a medium pizza multiplied by x.

$l(5)$	$s(0)$	$m(x)$

7 Emma and her friends are texting about their pizza order.

Emma writes: $m(7) < l(5)$.

What do you think this means?

8 Select *all* the true statements.

- ☐ **A.** $s(2) > s(1)$
- ☐ **B.** $m(4) < s(4)$
- ☐ **C.** $l(4) > s(3)$
- ☐ **D.** $l(1) = m(3)$
- ☐ **E.** $l(0) > m(2)$

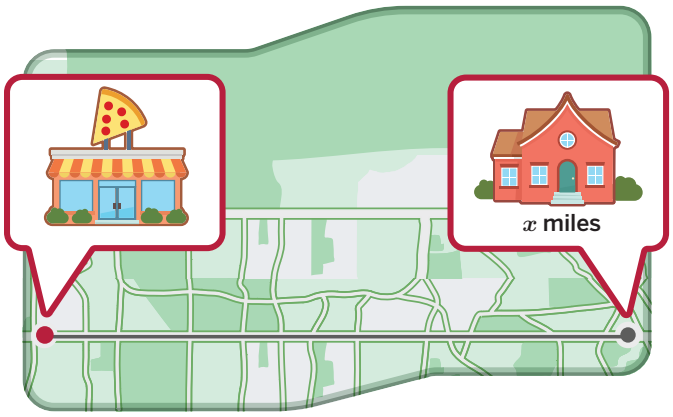
Activity 3

Name: _____ Date: _____ Period: _____

Delivering Pizzas

9 Desmos Pizza uses the function $d(x)$ to estimate the number of minutes it takes to deliver pizza x miles.

What would $d(2) = 30$ mean in this situation?



10 $d(x)$ estimates the number of minutes it takes to deliver pizza x miles. Match each function notation card to a description. One card will have no match.

Card A

It takes longer to deliver 5 miles away than 1 mile away.

Card B

The number of minutes to make a delivery 5 miles away.

Card C

A delivery 1 mile away will take more than 5 minutes.

Card D

A delivery 5 miles away will take more than 1 minute.

Card E

Delivering 5 pizzas takes longer than delivering 1 pizza.

$d(5) > 1$	$d(5)$	$d(5) > d(1)$	$d(1) > 5$

11 Synthesis

This lesson introduced function notation.

a. Say the equation $m(5) = 23$ aloud to a classmate.

b. Describe what each part of the equation means.

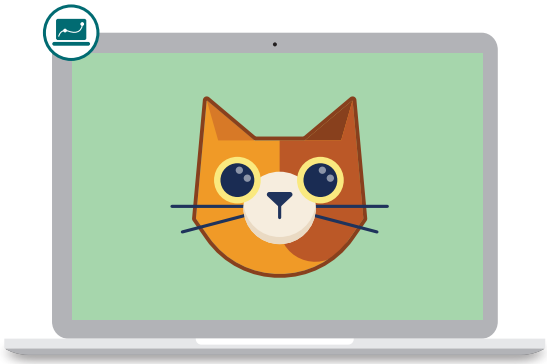
Things to Remember:

Unit 4
Lesson
3

Name: ..... Date: ..... Period: .....

Toy Factory

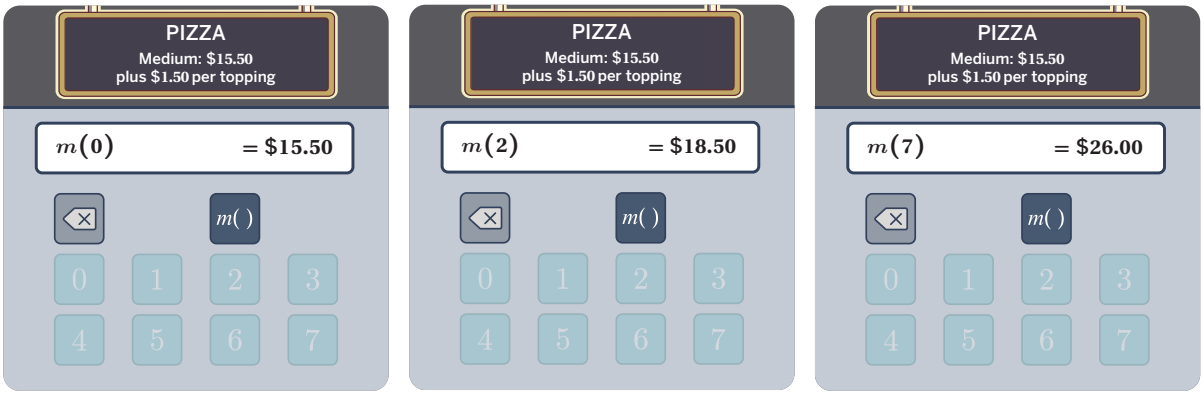
Let's explore functions represented as equations written in function notation.



Warm-Up

1 The cash register uses $m(x)$ to determine the price of a medium pizza with x toppings.

a Here are three pizzas and their prices.



b Describe how the cash register calculates prices.

2 Which equation represents $m(x)$?

A. $m(x) = 15.5 + 1.5$ **B.** $m(x) = 15.5x + 1.5$ **C.** $m(x) = 15.5 + 1.5x$

Explain your thinking.

Activity 1

Name: _____ Date: _____ Period: _____

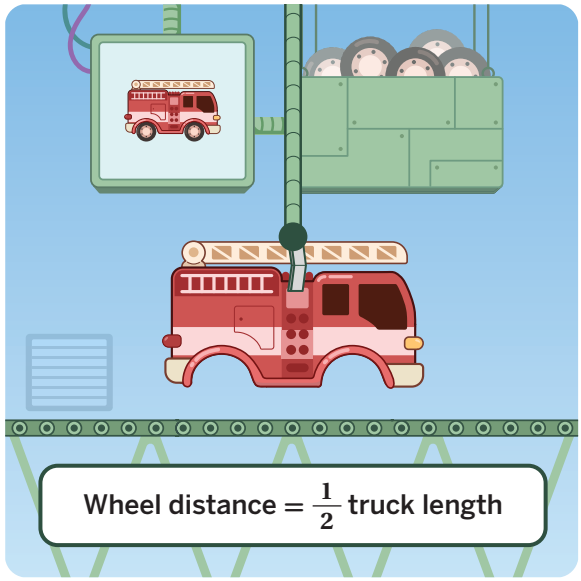
Exploring Equations of Functions

3 A toy factory makes fire trucks in a variety of sizes.

The distance between the truck's wheels is always half the length of the truck.

Complete the table to put wheels on the toy truck.

Truck Length (in.)	Wheel Distance (in.)
10	
6	
3	



4 Kanna wrote the function $d(x) = \frac{1}{2}x$ to determine the wheel distance for a truck length of x.

a **Discuss:** What does $d(x) = \frac{1}{2}x$ mean?

b What is the value of $d(7)$?

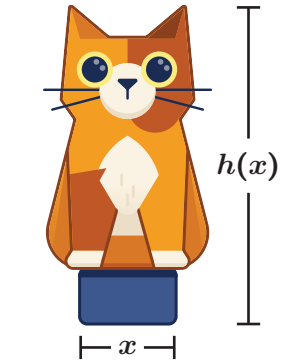
Name: _____ Date: _____ Period: _____

Exploring Equations of Functions (continued)

5 The factory also makes toy cats.

They use this diagram and function to determine where to place the cat's eyes. All units are in inches.

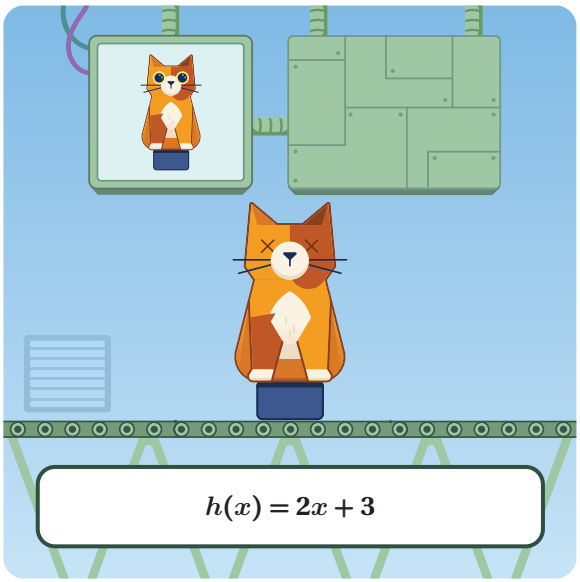
What does $h(x) = 2x + 3$ mean in this situation?



$h(x) = 2x + 3$

6 Calculate the value of each function notation expression.

Expression	Eye Height (in.)
$h(5)$	
$h(3)$	
$h(7.5)$	



Discuss: Why is it useful to write a function as an equation?

Activity 2

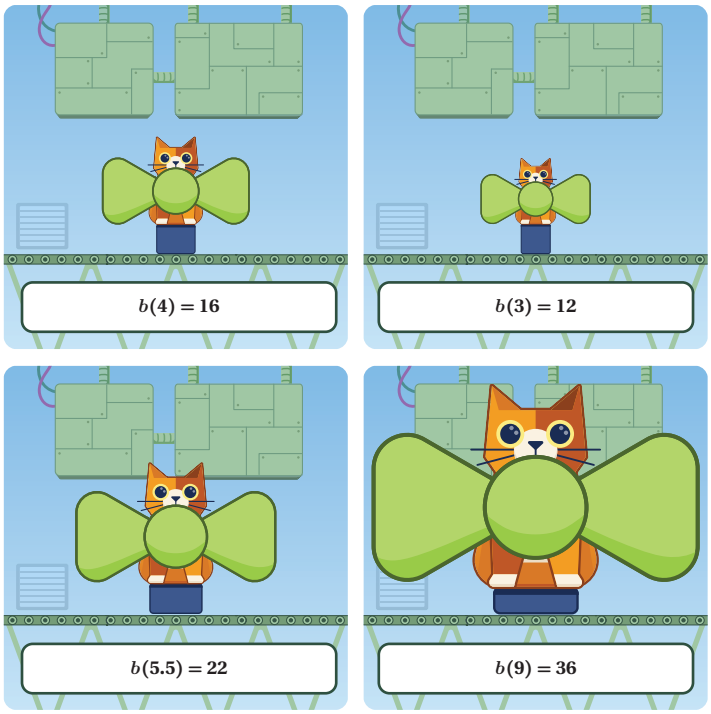
Name: _____ Date: _____ Period: _____

Writing Equations of Functions

7 Each toy cat needs a bow tie. The function $b(x)$ determines the width of the bow tie, where x is the width of the base. All units are in inches.

a Let's see how the function $b(x) = 4x$ works.

b Change the equation $b(x) = 4x$ to make bow ties that fit better.



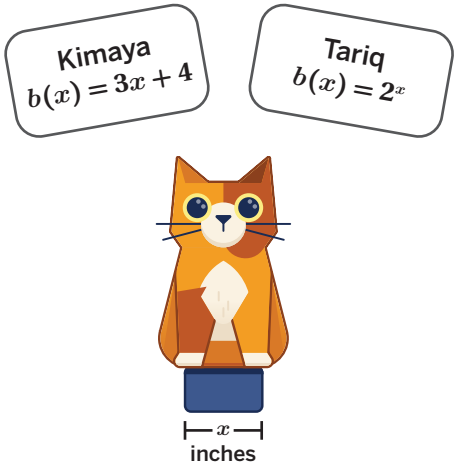
8 $b(x)$ determines the width of the bow tie, where x is the width of the base.

Kimaya says that her function will produce a wider bow tie than Tariq's function for any base width.

Is she correct? Circle one.

Yes No

Explain your thinking.



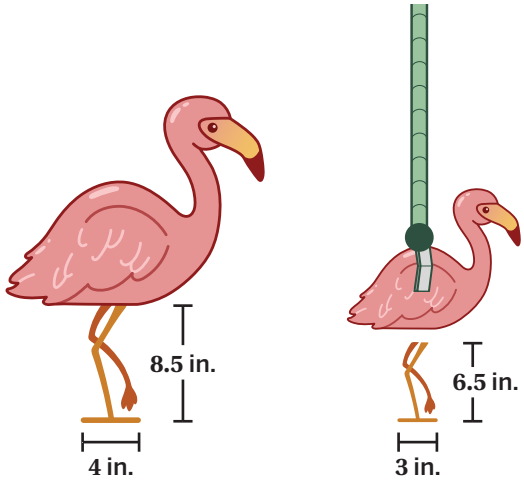
Name: _____ Date: _____ Period: _____

Writing Equations of Functions (continued)

9 Flamingo Frank is made by attaching a flamingo to legs at a specific height.

This height is determined by the width of the base.

Base Width (in.)	Height (in.)
4	8.5
3	6.5
1	2.5
5.5	11.5



How can you determine the height for any base width?

10 This machine assembles Flamingo Frank by attaching it to legs at a specific height. That height, $f(x)$, depends on the width of the base, x. Write an equation for $f(x)$.

Explore More

11 Here are four figures in a visual pattern. The number of tiles is a function of the figure number.

Figure, n	Number of Tiles, $t(n)$
1	3
2	6
4	18
6	38

Write an equation for $t(n)$.

Figure 1

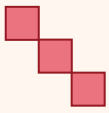


Figure 2

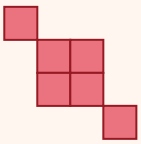


Figure 4

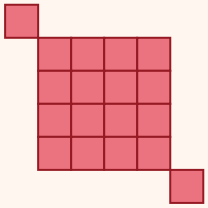
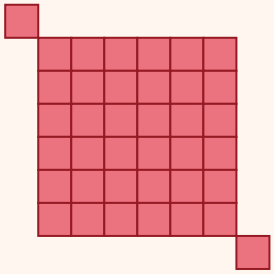


Figure 6

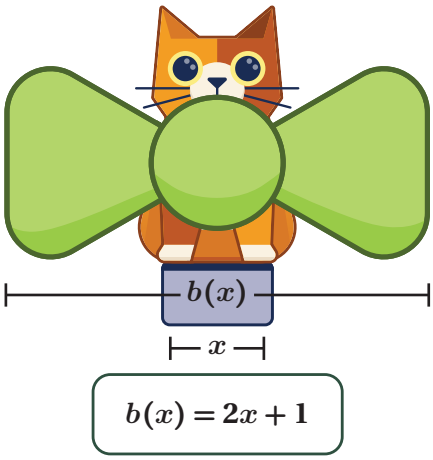


12 Synthesis

A toy factory uses this diagram and function to determine the width of the bow tie. All units are in inches.

What does $b(x) = 2x + 1$ mean in this situation?

What does $b(4) = 9$ mean?



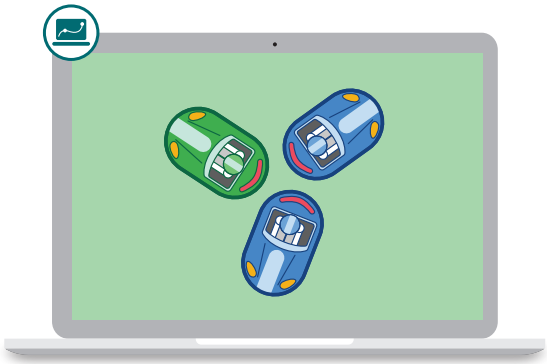
Things to Remember:

Unit 4
Lesson
4

Name: Date: Period:

Function Carnival

Let's create and analyze graphs that represent stories.



Warm-Up

1 Let's watch a video.

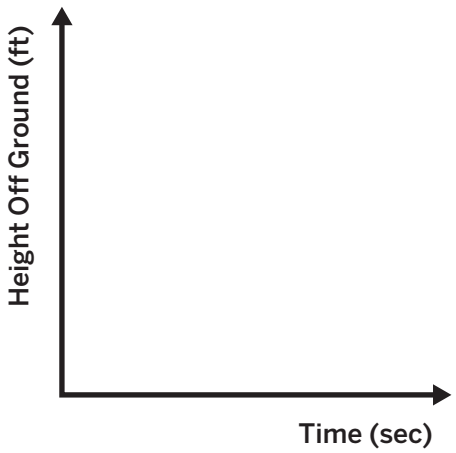
Write a story about what you see.

Name: _____ Date: _____ Period: _____

Cannon Person

2 The function $h(t)$ represents the person's height off the ground at time t.

Sketch a graph of $h(t)$.



3 Let's look at a precise graph of $h(t)$.

Select a true statement.

A. $h(3) > h(5)$

B. $h(3) = h(5)$

C. $h(0) < h(10)$

D. $h(0) = h(10)$

Explain what the statement says about the Cannon Person.

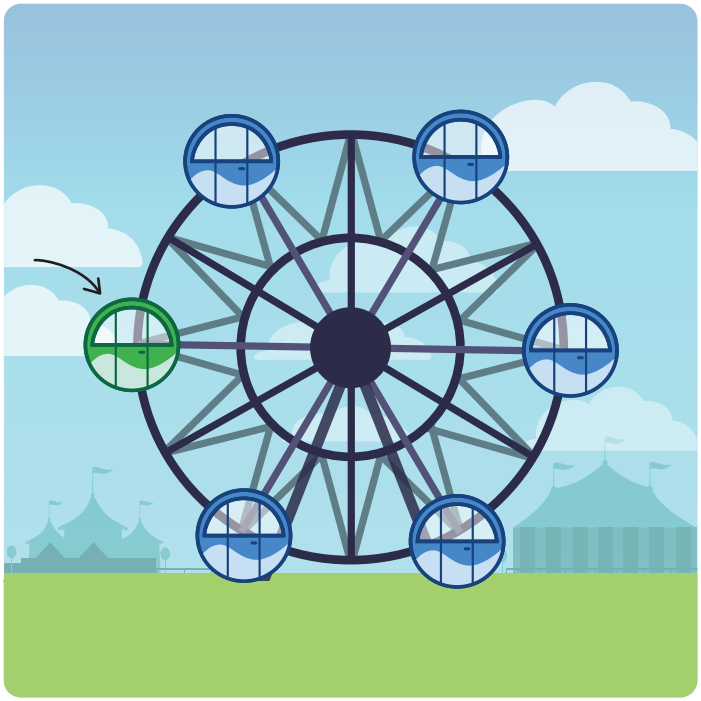
Activity 2

Name: Date: Period:

Ferris Wheel

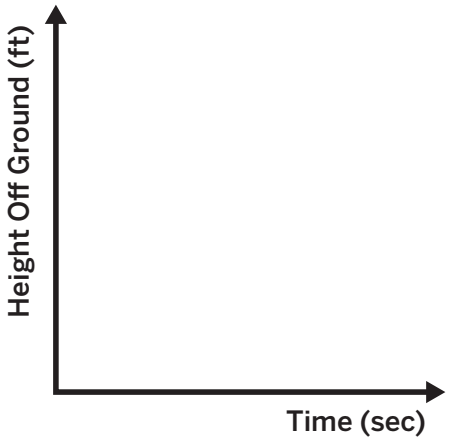
4 Let's watch a video of a Ferris wheel.

Describe what happens to the height of the green cart.



5 The function $h(t)$ represents the height of the Ferris wheel at time t.

Sketch a graph of $h(t)$.



6 Let's look at the graph that Liam drew.

What do you think happened to the green cart when Liam pressed play?

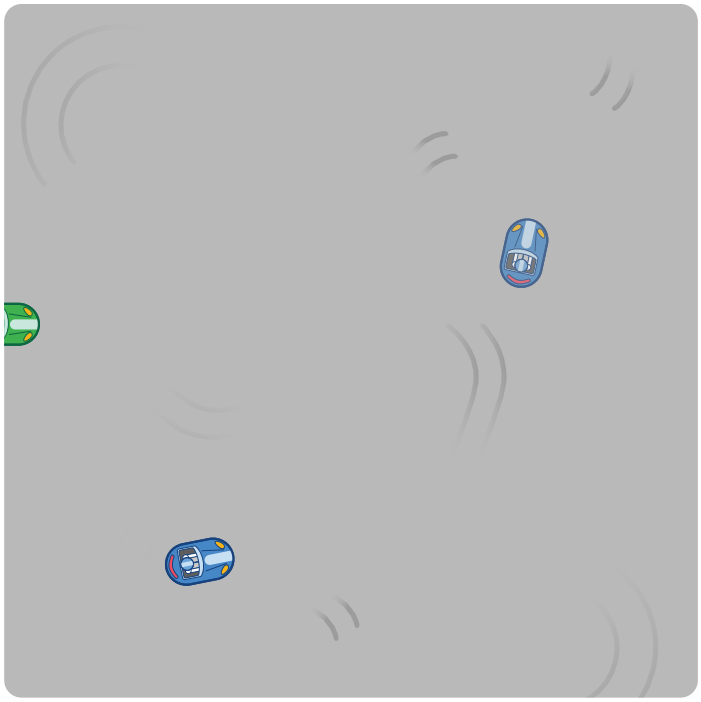
Activity 3

Name: _____ Date: _____ Period: _____

Bumper Cars

7 Let's watch a video of some bumper cars.

What are some different things you could measure and display in a graph?



8 The function $d(t)$ represents the distance traveled by the green car at time t.

Sketch a graph of $d(t)$.

Name: _____ Date: _____ Period: _____

Bumper Cars (continued)

9 Match each description to its graph. Two graphs will have no match.

Note: Distance measures distance from the start.

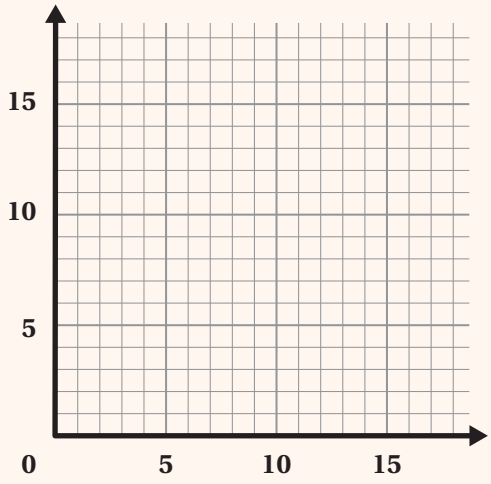
a The roller coaster moves slowly, stops for a moment, then goes very fast.

b The roller coaster moves forward, stops, and then backs up.

c The roller coaster starts in the middle, backs up, and then splits into multiple roller coasters.

Explore More

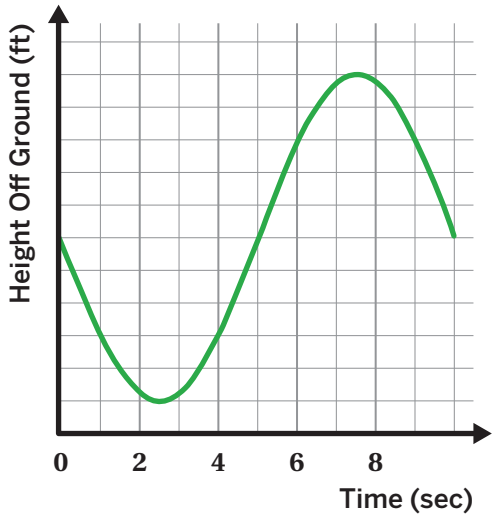
10 Draw your own graph and label the x-axis and y-axis. Then write a story about what it describes.



12 Synthesis

How can a graph help tell a story about a situation?

Use this example of a Ferris wheel graph if it supports your explanation.



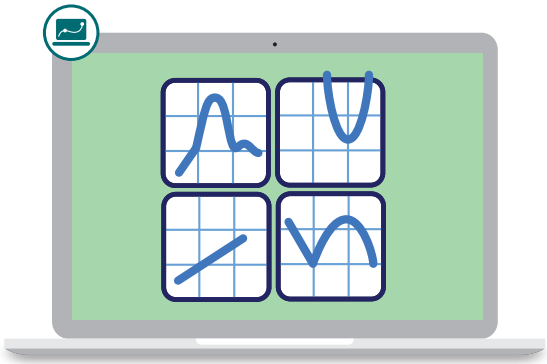
Things to Remember:

Unit 4
Lesson
5

Name: ..... Date: ..... Period: .....

Craft-a-Graph

Let's describe and create graphs of functions using key features.



Warm-Up

1 Play a few rounds of Polygraph with your classmates!

You will use a Warm-Up Sheet with functions. In each round:

- You and your partner will take turns being the Picker and the Guesser.
- Picker: Select a function from the Warm-Up Sheet. Keep it a secret!
- Guesser: Ask the Picker yes-or-no questions, eliminating functions until you're ready to guess which function the Picker chose.

Record helpful questions from each round in the space below.

Activity 1

Name: ..... Date: ..... Period: .....

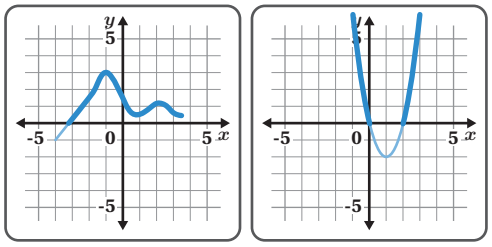
Describe It

2 Here are some functions from Polygraph, along with some terms that describe parts of their graphs.

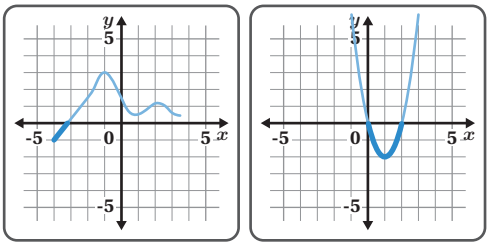
a Take a look at each term and where it appears on the graph.

b **Discuss:** What does each term mean?

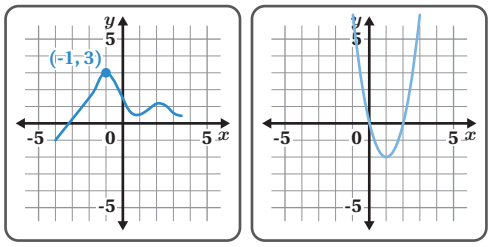
Positive



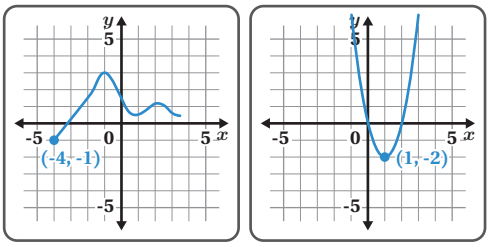
Negative



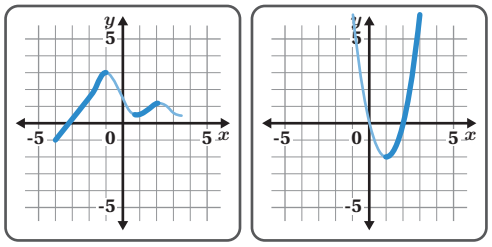
Maximum



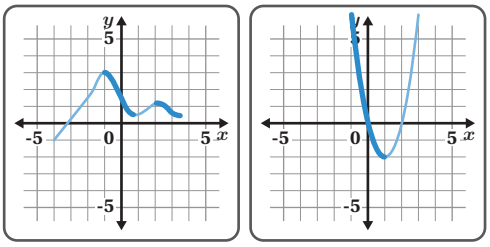
Minimum



Increasing



Decreasing



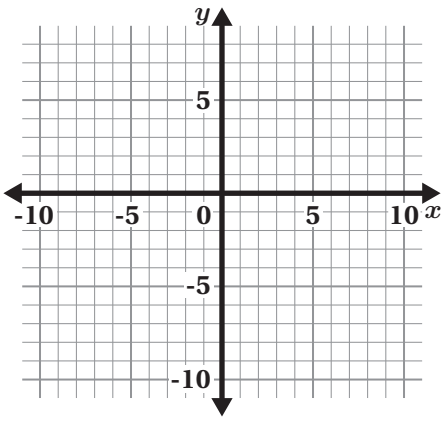
Activity 2

Name: Date: Period:

Build It

3 Now it's your turn to make a function!

Describe your function using some of the terms from the previous problem.

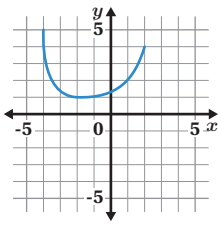


4 Latifah described her function this way:

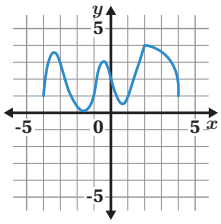
- My function is always positive.
- The maximum is at $(2, 4)$.

Select *all* the functions that could be Latifah's.

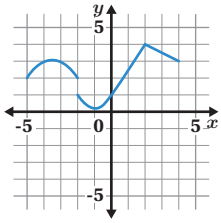
☐ A.



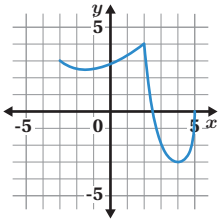
☐ B.



☐ C.



☐ D.



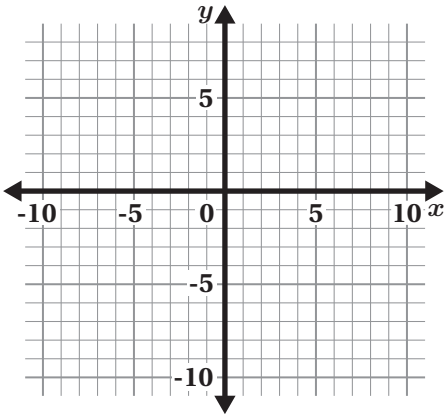
Name: Date: Period:

Build It (continued)

5 Haruto described his function this way:

- My function is increasing when $x > 0$.
- It has a minimum at $(-3, -4)$.

Sketch a function that could be Haruto's.



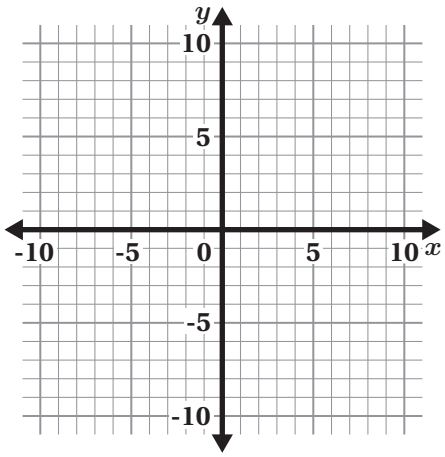
6 Andrea described her function this way:

- Positive when $x > -2$.
- Decreasing when $x > 1$.

Is it possible for a function to have both features? Use the graph if it helps with your thinking. Circle one.

Possible Impossible I'm not sure

Explain your thinking.



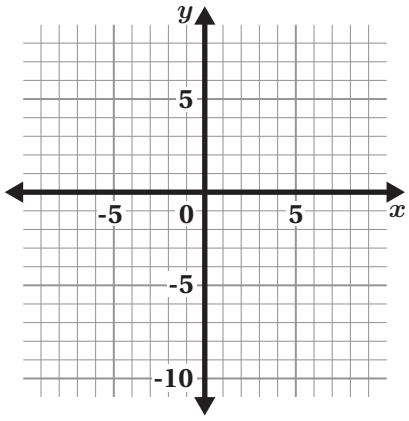
Activity 3

Name: Date: Period:

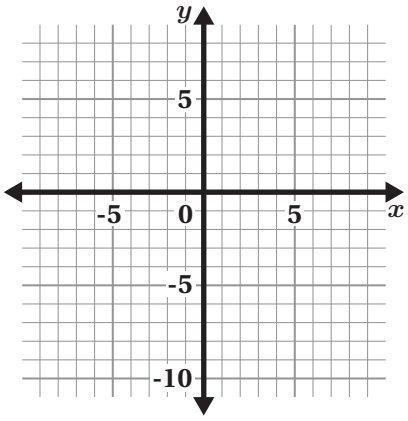
Repeated Challenges

7 Sketch a function that meets these criteria.

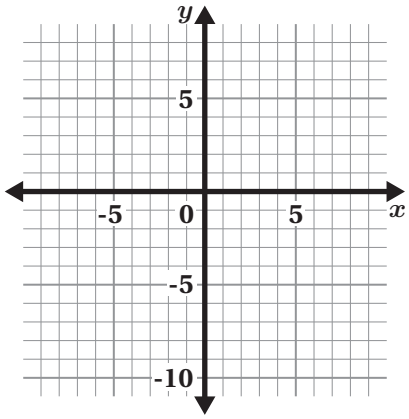
Positive when $x < 4$
Decreasing when $x > -1$



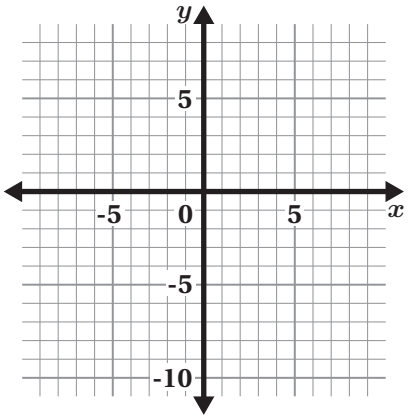
Increasing when $x > 1$
Maximum at (-3, 5)



Negative when $x < 2$
Decreasing when $x > -5$



Positive when $x > -2$
Minimum at (-3, -1)

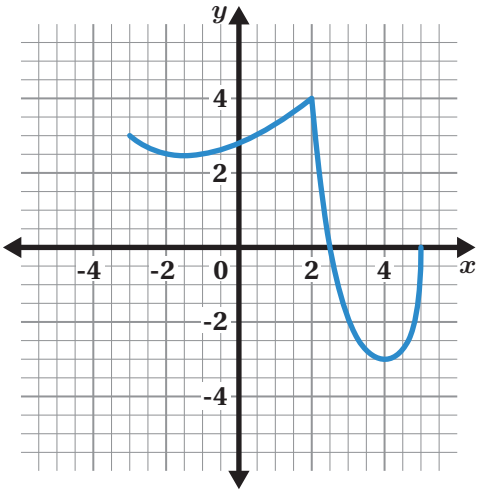


8 Synthesis

Here are some of the terms to describe functions that we learned about today.

Positive	Negative
Maximum	Minimum
Increasing	Decreasing

Select three terms. Write the meaning of each term you selected.

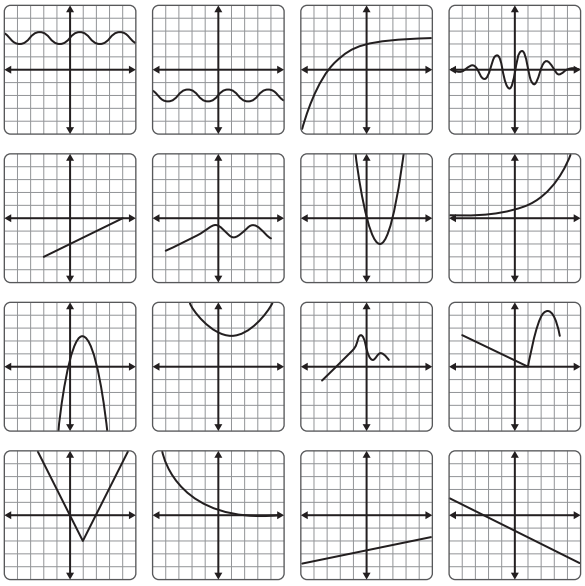


Things to Remember:

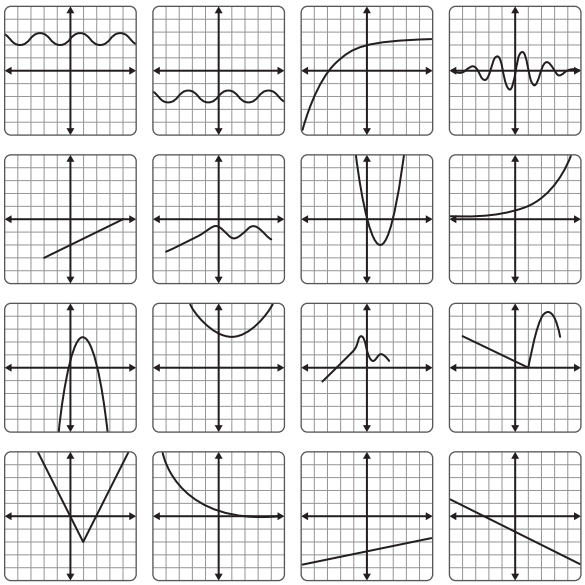
Name: ..... Date: ..... Period: .....

Polygraph Set A

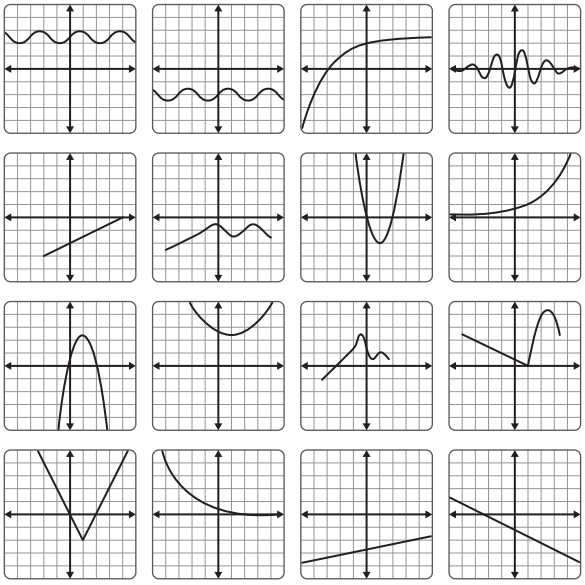
Round 1



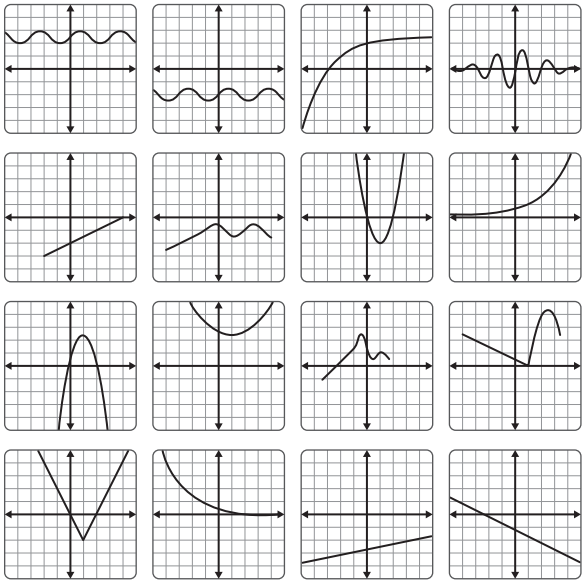
Round 2



Round 3

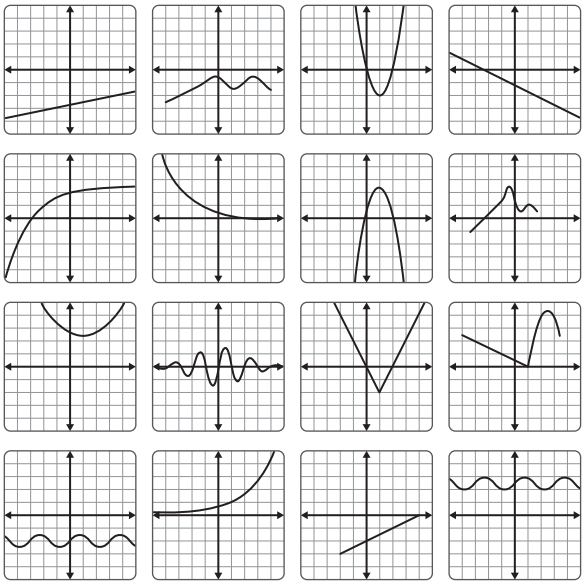


Round 4

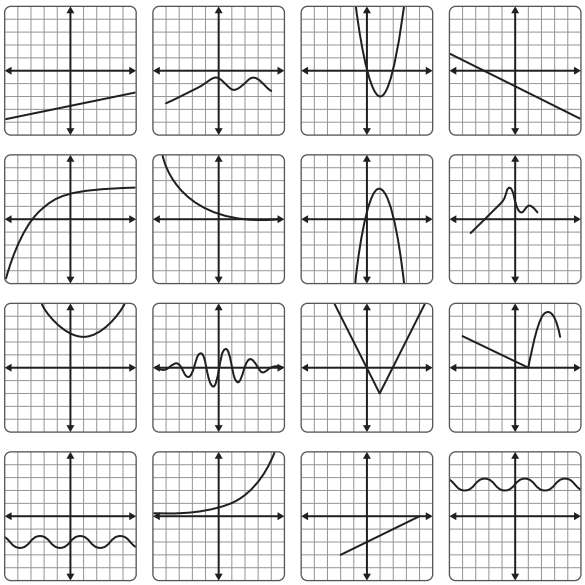


Polygraph Set B

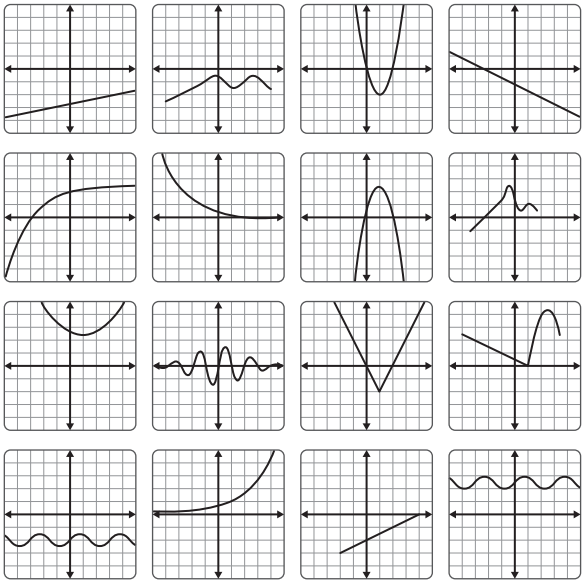
Round 1



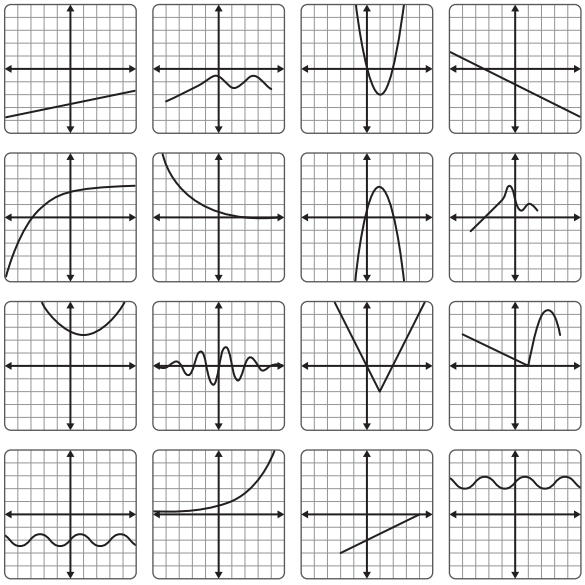
Round 2



Round 3



Round 4



Unit 4
Lesson
6

Name: ..... Date: ..... Period: .....

Plane, Train, and Automobile

Let's calculate the average rate of change over a specified interval.

Warm-Up

1 A wedding is happening in New York City! Many relatives are coming from out of town.

a Take a look at three wedding guests and how they traveled to the wedding.

Arjun

Troy

Mayra

b What do you notice? What do you wonder?

I notice:

I wonder

Activity 1

Name: _____ Date: _____ Period: _____

Arjun's Automobile Trip

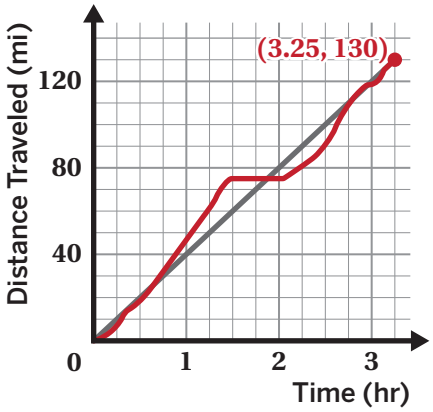
2 Arjun's car trip was 130 miles and took 3.25 hours.

Imagine Arjun traveled at a constant speed the whole trip.

What would be Arjun's speed in miles per hour?

3 **a** Take a look at the map and the graph of Arjun's actual trip from Hartford, Connecticut to New York City.

b Tell a story about Arjun's trip.

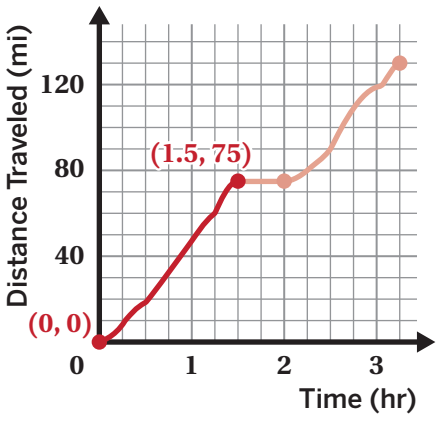


4 The **average rate of change** is the *slope* of the line that connects two points.

For Arjun's trip, the average rate of change was 40 miles per hour.

We can also look at average rate of change for an **interval**, such as 0 to 1.5 hours (highlighted in the graph).

How would you calculate the average rate of change for that interval?

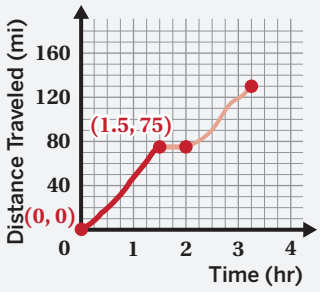
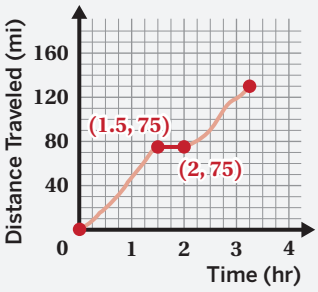
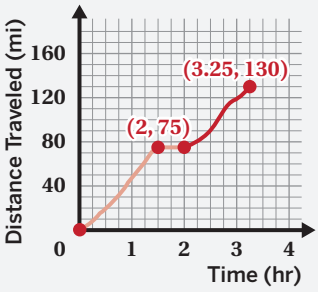


Name: _____ Date: _____ Period: _____

Arjun's Automobile Trip (continued)

5 Let's examine some intervals of Arjun's trip.

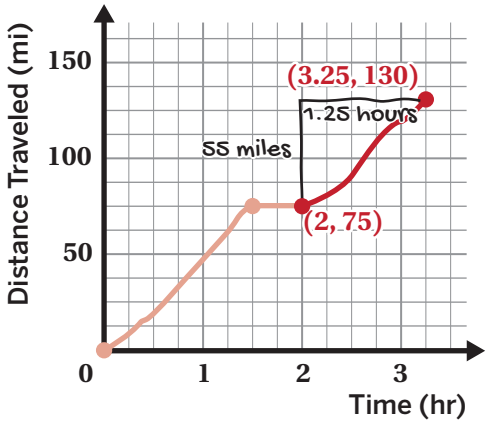
Determine the average rate of change of each interval.

Interval	0 to 1.5 hours	1.5 to 2 hours	2 to 3.25 hours
Graph			
Average Rate of Change (mph)			

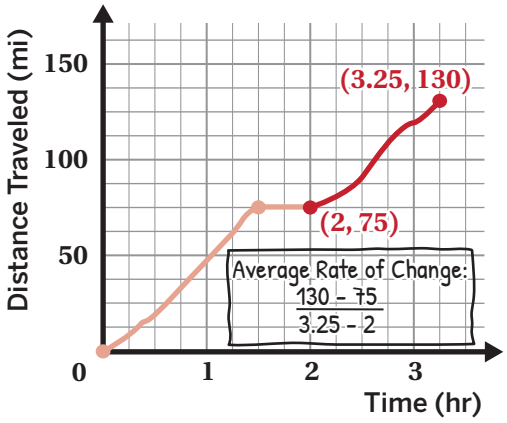
6 Two students calculated the average rate of change between 2 and 3.25 hours.

a Take a look at Naoki's and LaShawn's work.

Naoki



LaShawn



b How do their methods compare?

Activity 2

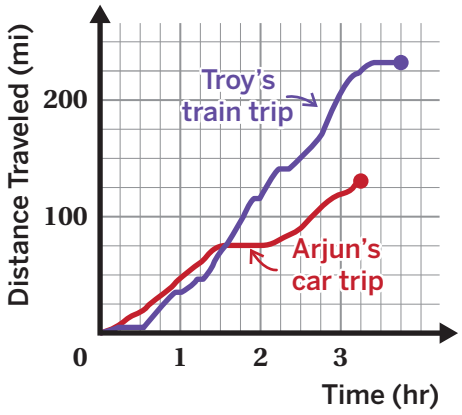
Name: Date: Period:

Troy's Train Trip

7 Let's watch an animation of Troy's trip.

Here is the graph for Arjun's and Troy's trips.

What are some ways Troy's trip is different from Arjun's?

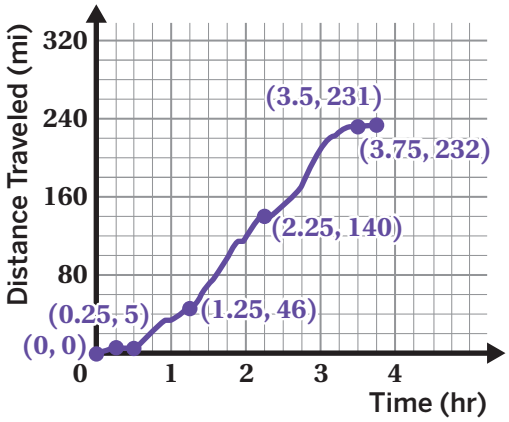


8 Let's examine some intervals of Troy's trip.

Choose two points on the graph. Then calculate the average rate of change for the interval you selected.

Can you find:

- Troy's average rate of change for the full trip?
- An interval where Troy moved fast? Slow?



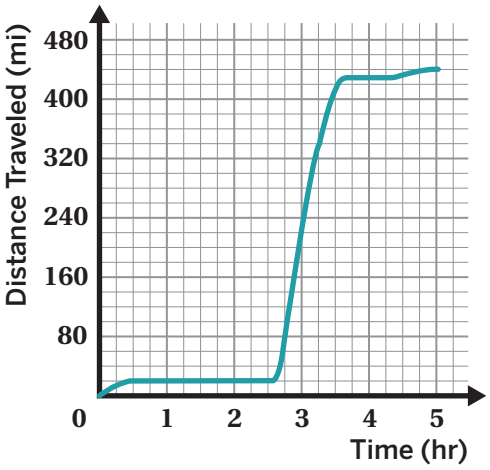
Interval	Average Rate of Change (mph)

Activity 3

Name: _____ Date: _____ Period: _____

Mayra's Flight

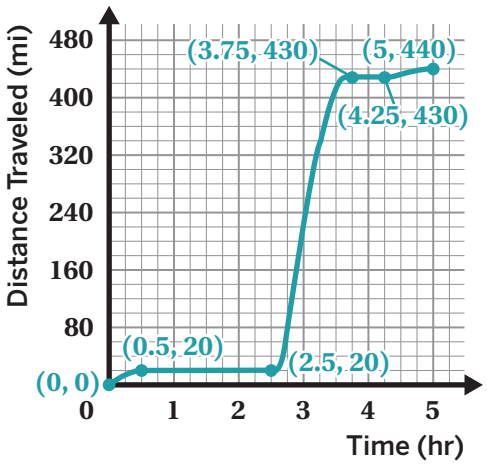
9 Mayra's plane trip from Pittsburgh was 440 miles and took 5 hours.



a **Discuss:** What questions do you have about Mayra's trip?

b Label three or more intervals of the graph with what you think was happening at that time.

10 Mayra said: *The flight felt fast, but the trip felt slow.*



a Choose two points on the graph. Calculate the average rate of change for the interval you selected.

b Can you find:

- Mayra's average rate of change for the full trip?
- Mayra's average rate of change *during the flight*?

Interval	Average Rate of Change (mph)

Name: _____ Date: _____ Period: _____

Mayra's Flight (continued)

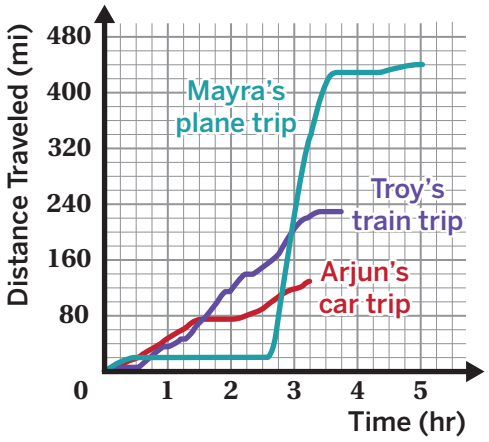
11 There are many reasons why someone may choose to travel by car, train, or plane.

Here are the graphs for all three people's trips.

Circle the trip you think is best.

Arjun (car) Troy (train) Mayra (plane)

Explain why you chose that trip.

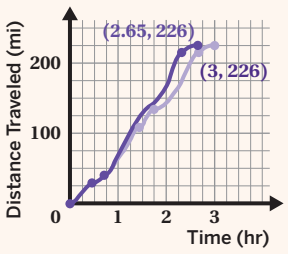


Explore More

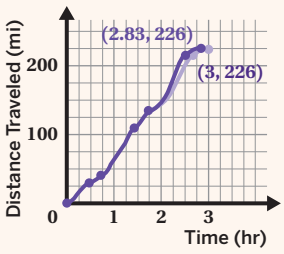
12 Public transit agencies often look at average speed (rate of change) when they look for ways to improve service.

a Explore these three proposals to improve the train from Washington to New York:

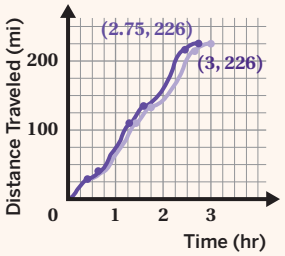
Proposal A: Skip stops along the way.



Proposal B: Upgrade one section of the tracks.



Proposal C: Buy trains with a higher top speed.



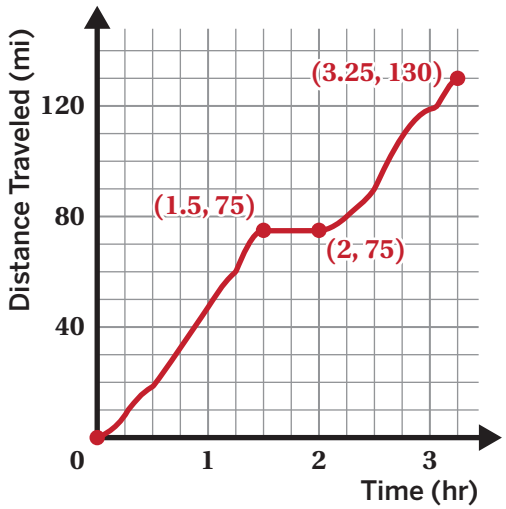
b **Discuss:**

- What is the train's average speed?
- Who might benefit and who might be harmed?

13 Synthesis

How can you calculate the average rate of change for an interval of a function?

Use this graph if it helps to show your thinking.



Things to Remember:

Unit 4
Lesson
7

Name: ..... Date: ..... Period: .....

Space Race

Let's make connections between function notation and key features of graphs.

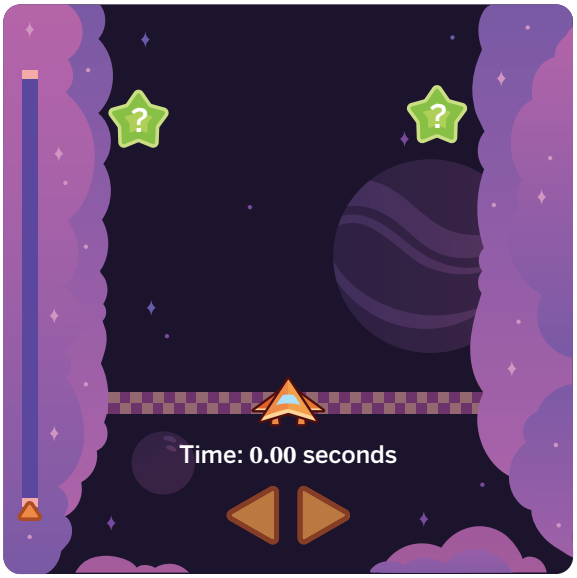
Warm-Up

1 It's time to play a game called Space Race!

Your mission: Get your spaceship to the finish line as quickly as possible.

a Use the digital activity to play Round 1.

b **Discuss:** How does this game work?



2 Use the digital activity to race your spaceship several more times. Then compare your graphs with a partner's graphs.

Activity 1

Name: Date: Period:

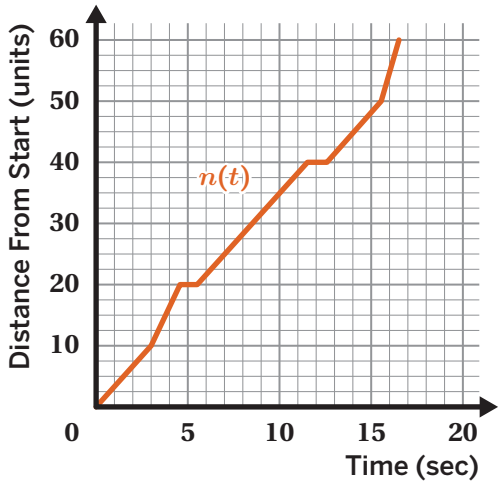
Comparing Graphs By Key Features

3 Nekeisha also played a round of Space Race.

$n(t)$ represents the distance of Nekeisha's spaceship after t seconds.

What is a value of t for which $n(t) = 10$?

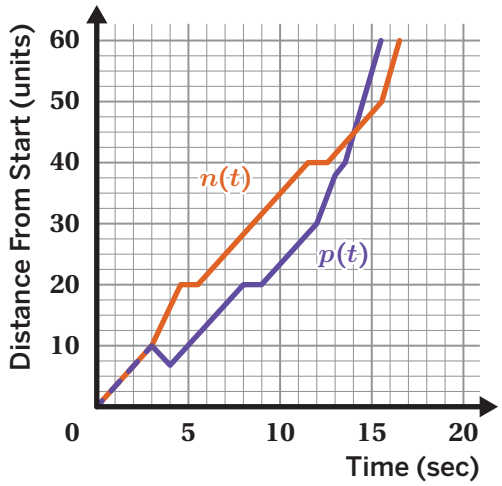
What does that tell you about the situation?



4 Let's watch Nekeisha and Polina race their spaceships.

Discuss:

- What were some interesting moments during this race?
- Where do you see those moments on the graph?



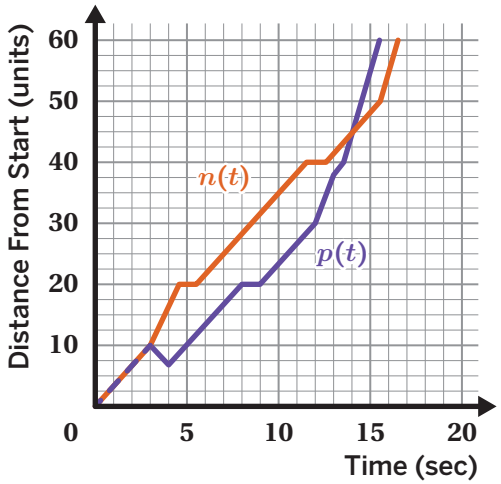
Name: _____ Date: _____ Period: _____

Comparing Graphs By Key Features (continued)

5 $n(t)$ and $p(t)$ represent the distances of Nekeisha's and Polina's spaceships after t seconds.

One student described part of the graph like this: *Polina's function was decreasing, and Nekeisha's function was increasing.*

Write a value of t that is in this interval of time.



Describe what was happening in the race at that moment.

6 Who had a greater average rate of change from 12 to 14 seconds?

A. Polina **B.** Nekeisha **C.** Their average rates of change were the same

Explain your thinking.

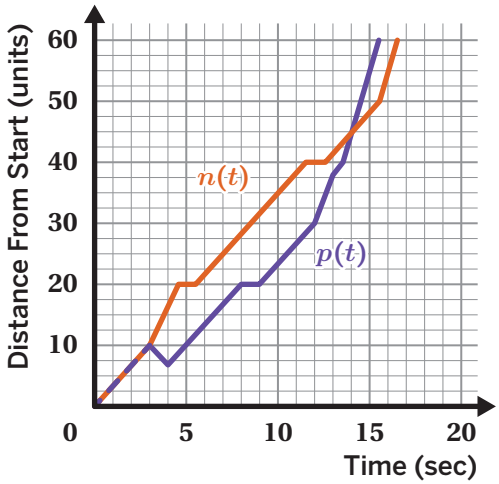
Activity 2

Name: _____ Date: _____ Period: _____

Comparing Using Function Notation

7 Nekeisha traveled farther than Polina after 11 seconds.

You can express that by writing $n(11) > p(11)$.



a **Discuss:** How can you use the graph to tell that $n(11) > p(11)$?

b Which statement could you use to show who traveled farther after 15 seconds?

A. $n(15) > p(15)$

B. $n(15) = p(15)$

C. $p(15) \geq n(15)$

D. $p(15) > n(15)$

8 We know that $n(11) > p(11)$.

What is a different value of t for which $n(t) > p(t)$?

Name: _____ Date: _____ Period: _____

Comparing Using Function Notation (continued)

9 What is a value of t for which $n(t) = p(t)$?

Describe what was happening in the race at that moment.

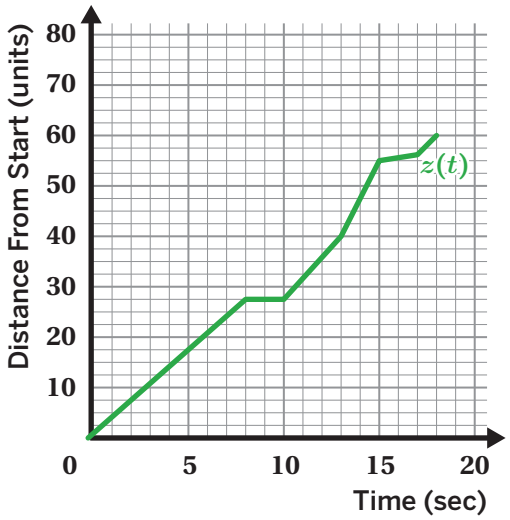
10 Valeria and Zion also raced their spaceships.

$v(t)$ and $z(t)$ represents the distances of Valeria's and Zion's spaceships after t seconds.

Here is some information about their race:

- $v(0) = z(0)$
- $v(4) < z(4)$
- $v(10) = 20$
- $v(13) = z(13)$
- $v(15) > z(15)$

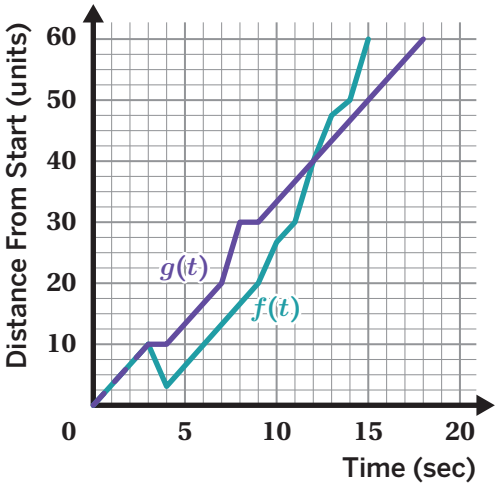
Make a graph that could represent Valeria's distance traveled after t seconds.



8 Synthesis

How can statements in function notation and terms like *maximum*, *increasing*, and *average rate of change* help us compare graphs of functions?

Use the example if it helps to show your thinking.



Things to Remember:

Unit 4
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Name: ..... Date: ..... Period: .....

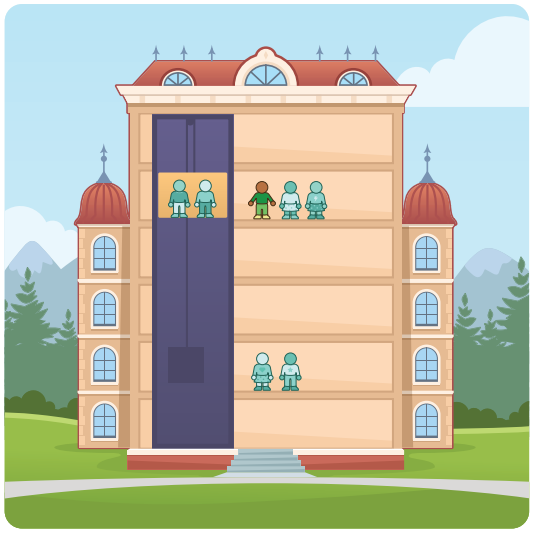
Elevator Stories

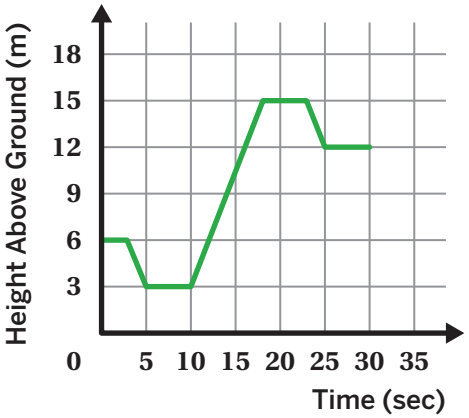
Let's use compound inequalities to describe the domain and range of functions from their graphs.

Warm-Up

1 Let's watch an animation of Amari's elevator ride.

Tell a story about what you see.





Activity 1

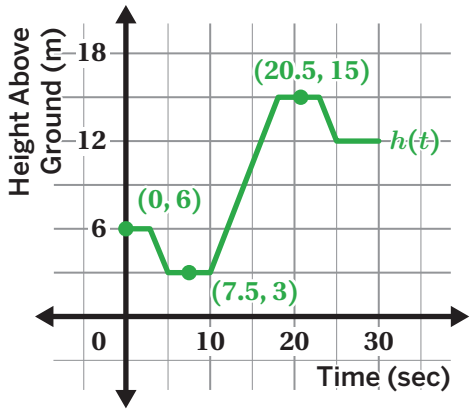
Name: _____ Date: _____ Period: _____

Describing the Domain

2 Here is a graph of $h(t)$, which represents the height of the elevator at a certain time, t, during Amari's ride.

Complete the input and output table for $h(t)$.

t	$h(t)$
5	
20	
30	



3 **a** Select *all* the numbers that are in the domain of $h(t)$.

- ☐ **A.** -5
- ☐ **B.** $\frac{1}{2}$
- ☐ **C.** 2
- ☐ **D.** 12
- ☐ **E.** 18
- ☐ **F.** 23.5
- ☐ **G.** 32
- ☐ **H.** 60

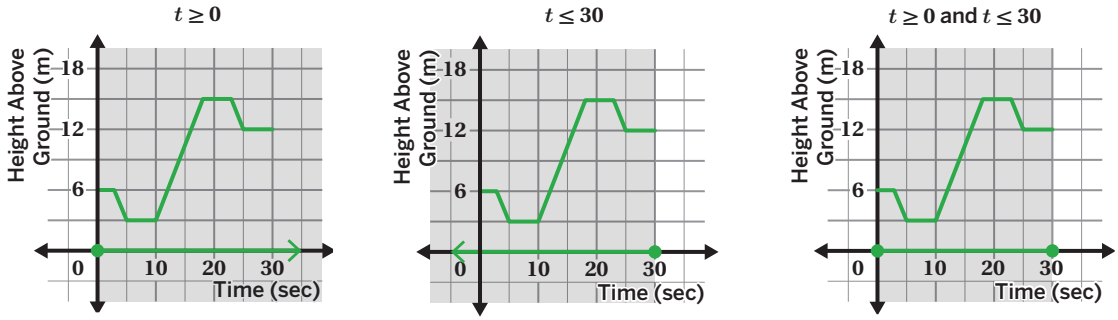
b Describe the domain of $h(t)$.

Name: _____ Date: _____ Period: _____

Describing the Domain (continued)

4 The domain of $h(t)$ is all the numbers from 0 to 30.

a Think about these inequalities and their relationship with the graph of $h(t)$.



b **Discuss:**

- Which inequality describes the domain of $h(t)$?
- Why don't the other inequalities describe the domain?

5 Each of these **compound inequalities** accurately describes the domain of $h(t)$.

a **Discuss:** How are they alike? How are they different?

$t \geq 0$ and $t \leq 30$

$0 \leq t$ and $t \leq 30$

$0 \leq t \leq 30$

b Explain how a compound inequality can help you describe the domain of $h(t)$.

Activity 2

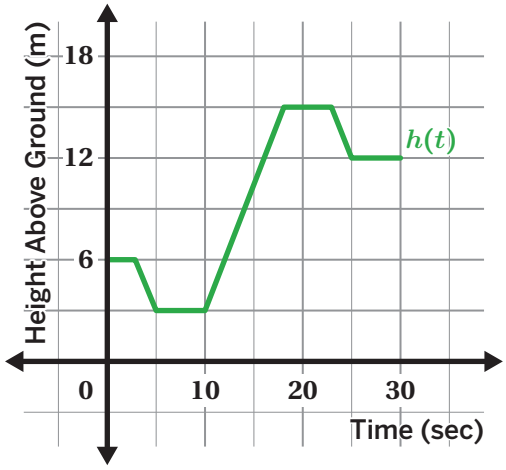
Name: _____ Date: _____ Period: _____

Distinguishing Domain and Range

6 **a** Write a compound inequality that describes the range of $h(t)$.

_____ $\leq h(t) \leq$ _____

b Describe in words what the compound inequality says about the range.



7 Here is the graph of another elevator ride.

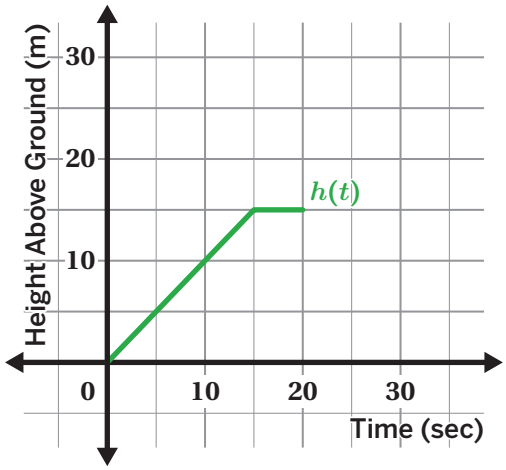
Two students described the range of this function.

- Ali says the range is $0 \leq t \leq 15$.
- Shanice says the range is $0 \leq h(t) \leq 15$.

Whose thinking is correct? Circle one.

Ali's Shanice's Both Neither

Explain your thinking.

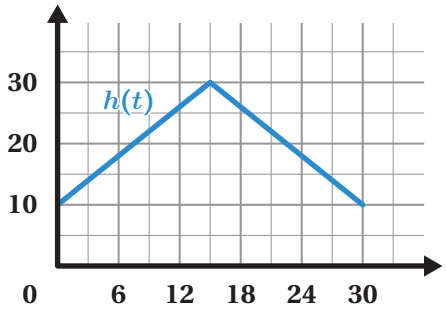


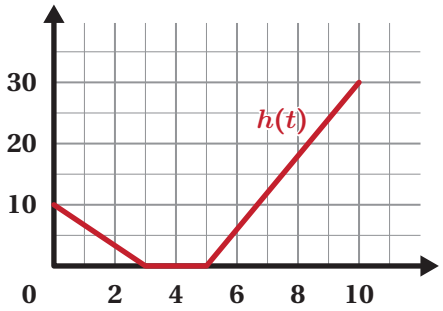
Name: _____ Date: _____ Period: _____

Distinguishing Domain and Range (continued)

8 Match each graph of a function with its domain and range. Two inequalities will have no match.

$0 \le t \le 10$	$0 \le h(t) \le 30$	$0 \le t \le 30$
$10 \le h(t) \le 30$	$0 \le h(t) \le 10$	$10 \le t \le 30$





9 The function $h(t)$ represents the height of the elevator at a certain time, t.

The range of $h(t)$ is $-30 \le h(t) \le 0$.

What could you say about this elevator ride?

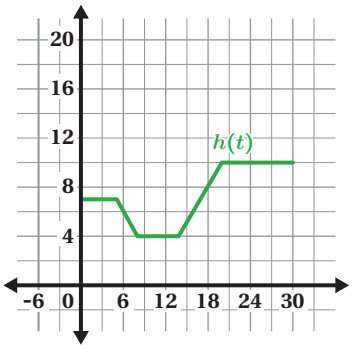
Activity 3

Name: _____ Date: _____ Period: _____

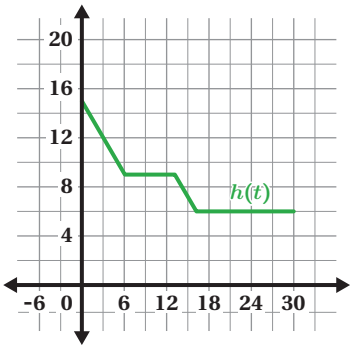
Writing Compound Inequalities

10 Complete the compound inequality to describe the range of $h(t)$.

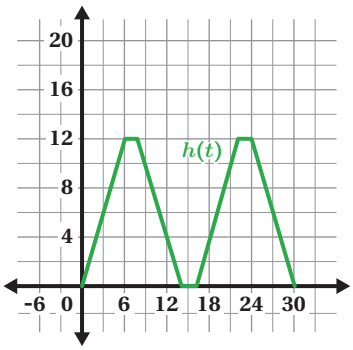
_____ $\leq h(t) \leq$ _____



_____ $\leq h(t) \leq$ _____



_____ $\leq h(t) \leq$ _____



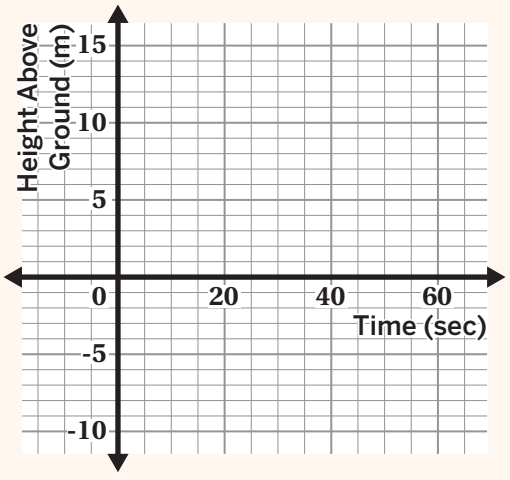
Explore More

11 $h(t)$ represents the height of an elevator at a certain time, t, during an elevator ride.

- The domain is $0 \leq t \leq 60$.
- The range is $-6 \leq h(t) \leq 12$.

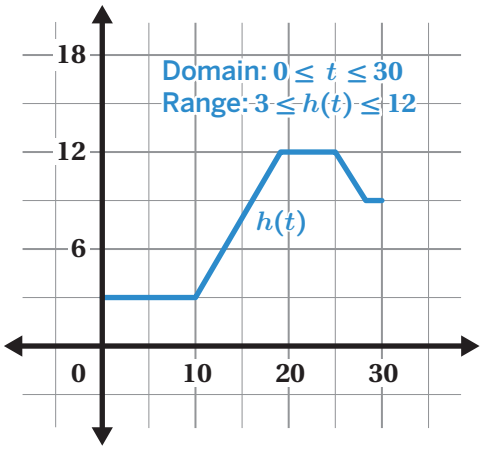
a Create a graph that matches the domain and range.

b Tell a story about your graph.



12 Synthesis

How can you determine the domain and range of a function from its graph? Draw on the graph if it helps with your thinking.



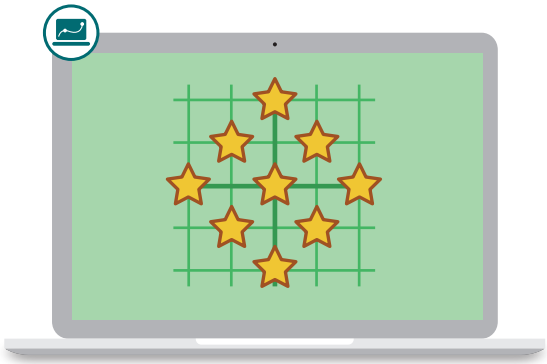
Things to Remember:

Unit 4
Lesson
10

Name: Date: Period:

Marbleslides

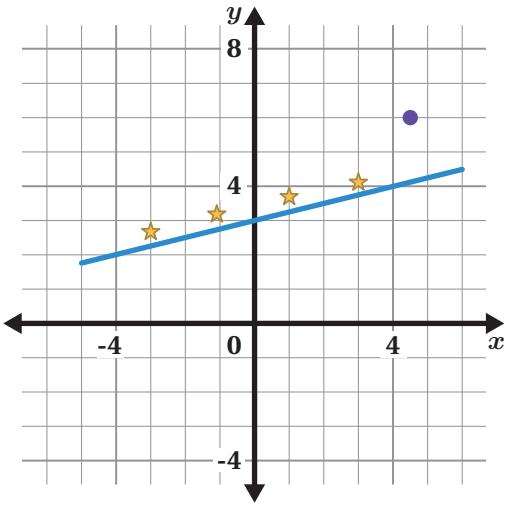
Let's practice restricting the domain and range of a graph.



Warm-Up

1 Your goal is to capture all the stars.

Use the digital screen to see what happens when we press "Launch."



Activity 1

Name: _____ Date: _____ Period: _____

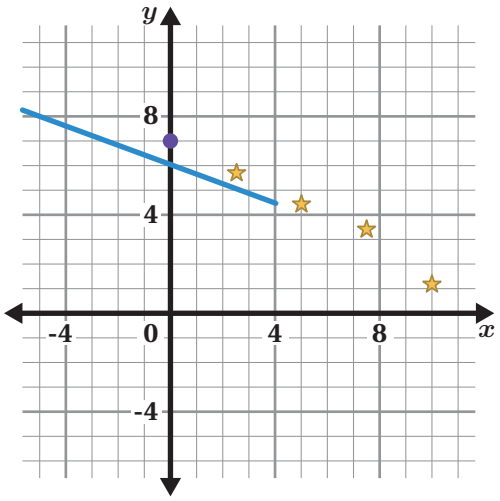
Restrict the Domain and Range

Use the digital screens for the next two problems.

2 Change one number to capture all the stars.

Original equation:
$y = -0.4x + 6\ \{x < 4\}$

Your equation:

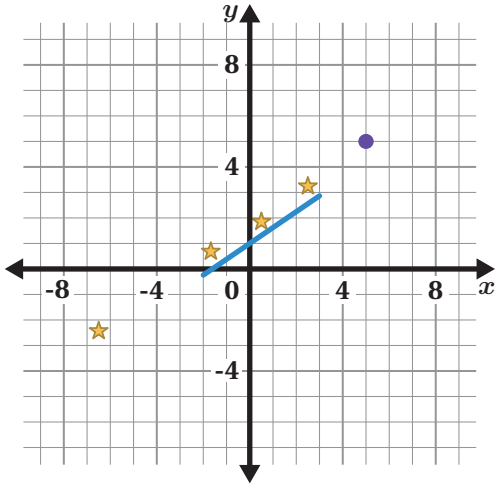


3 Change the domain to fix the Marbleslide.

Original equation:
$y = 0.6x + 1\ \{-2 < x < 3\}$

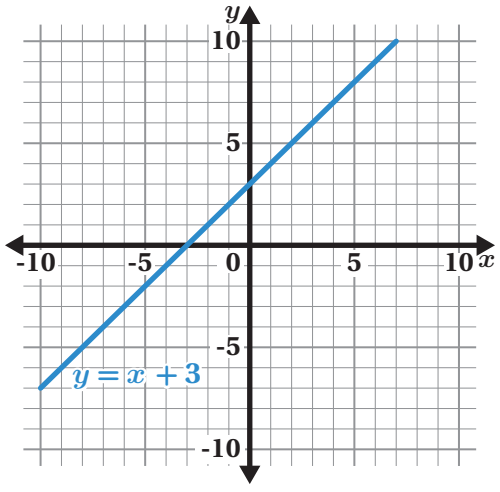
Your equation:

$y = 0.6x + 1\ \{\qquad\qquad\}$



4 If we included the range restriction $\{-2 < y < 4\}$, what would happen to the graph?

Draw on the graph if it helps with your thinking.



Name: ..... Date: ..... Period: .....

Restrict the Domain and Range (continued)

Use the digital screens for the next two problems.

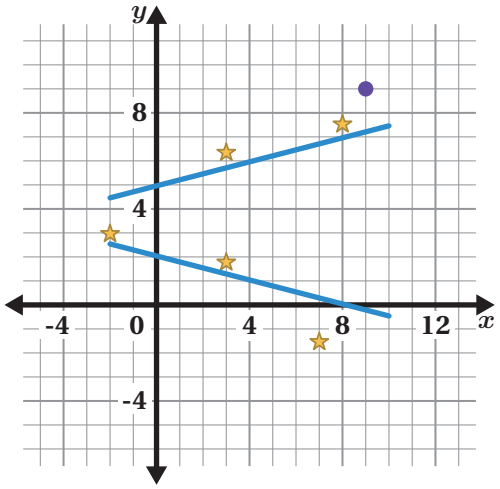
5 Change the domain, range, or equations to collect all the stars.

Original equations:

$y = \frac{1}{4}x + 5 \{-2 < x < 10\}$

$y = -\frac{1}{4}x + 2 \{-0.5 < y < 2.5\}$

Your equations:



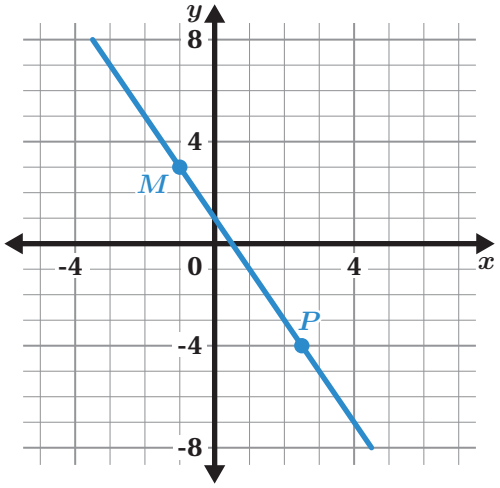
6 **a** Change the domain or range so that the line only appears from point M to point P.

Original equation:

$y = -2x + 1$

Your equation:

$y = 2x + 1 \{$.....$\}$



b **Discuss:** What was your strategy?

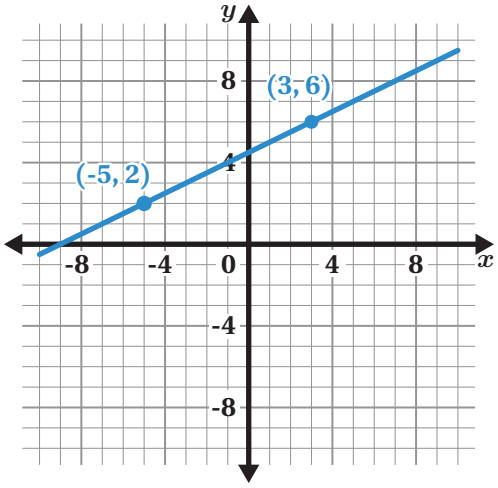
7 Two students are discussing how to restrict the graph of $y = \frac{1}{2}x + 4.5$ between (-5, 2) and (3, 6).

Hailey says to include $\{-5 < x < 3\}$.
Ricardo says to include $\{2 < y < 6\}$.

Whose thinking is correct? Circle one.

Hailey's Ricardo's Both Neither

Explain your thinking.



Activity 2

Name: Date: Period:

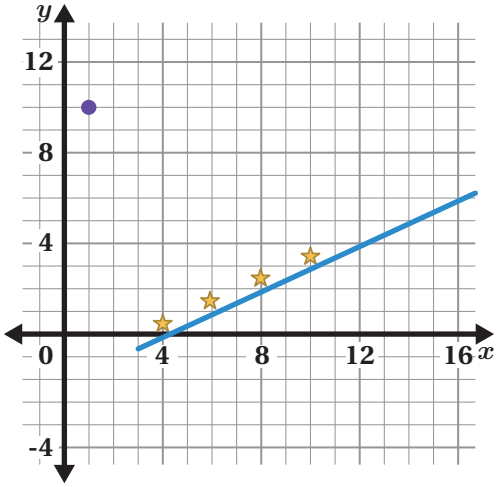
More Challenges!

Use the digital screens for this activity.

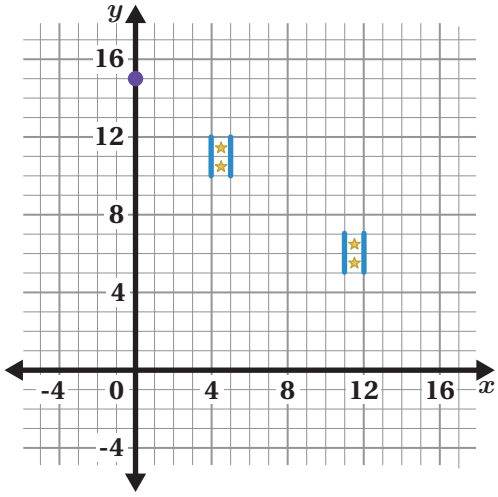
8 Create as many equations as you need to collect all the stars.

We have included the equation of a line that might help you start.

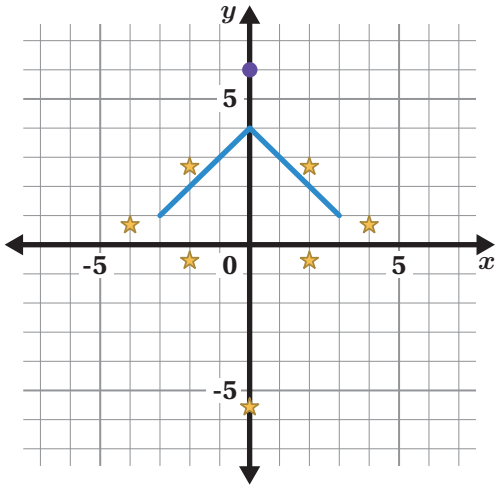
$$y = \frac{1}{2}x - 2.1\{x > 3\}$$



9 Create as many equations as you need to collect all the stars.



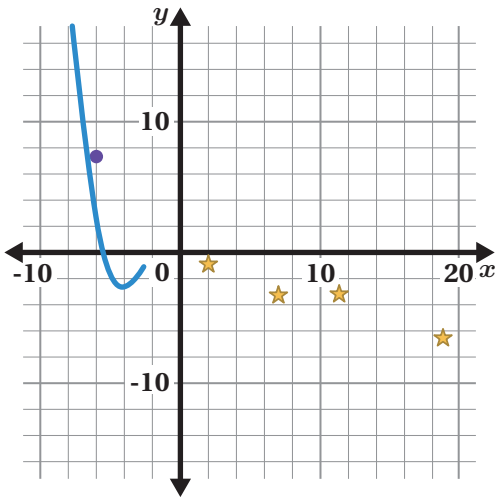
10 Create as many equations as you need to collect all the stars.



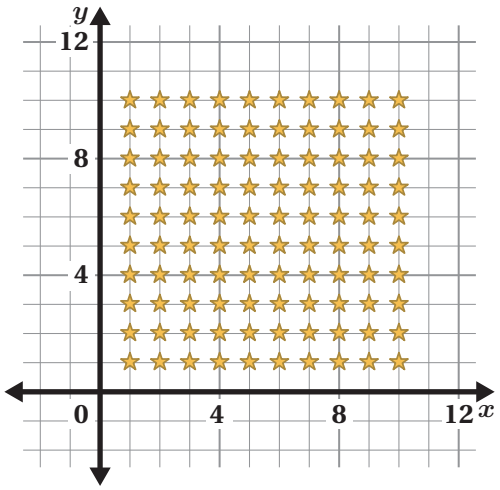
Name: _____ Date: _____ Period: _____

More Challenges! (continued)

11 Create as many equations as you need to collect all the stars.

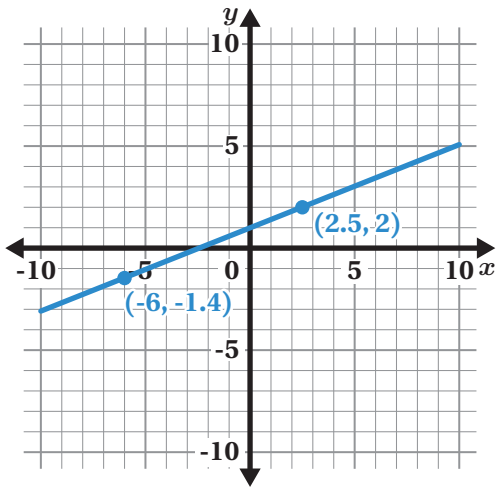


12 Challenge yourself to collect as many stars as you can!



13 Synthesis

a How can you use the domain to restrict the graph from $(-6, -1.4)$ to $(2.5, 2)$?



b How can you use the range?

Things to Remember:

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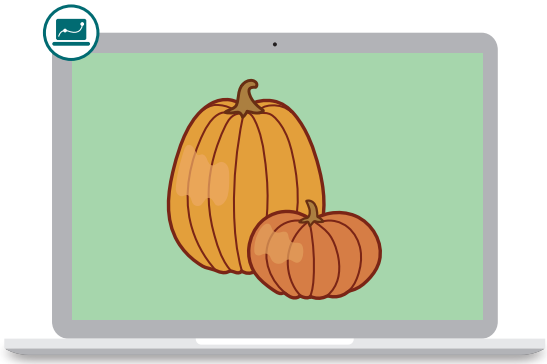


Pumpkin Prices

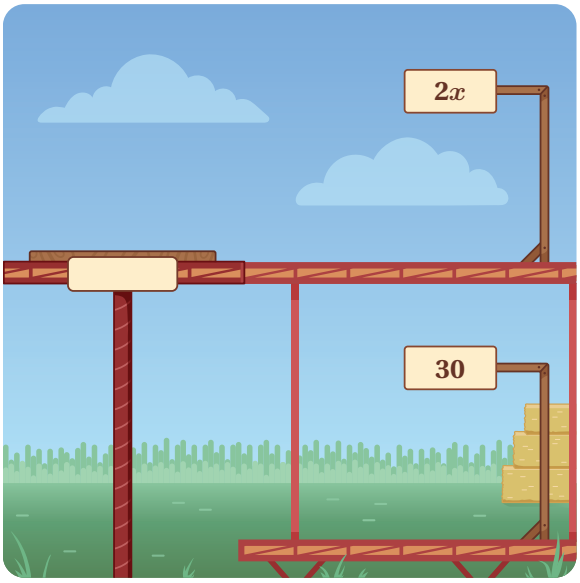
Let's make sense of piecewise-defined functions.

Warm-Up

1 Fran's Farm sells different sizes of pumpkins.

a Let's look at the prices of different pumpkins. Record the pumpkin weight and prices.

Pumpkin Weight (lb)	Price ($)



b **Discuss:** What do you notice and wonder?

Name: _____ Date: _____ Period: _____

Piecewise-Defined Functions

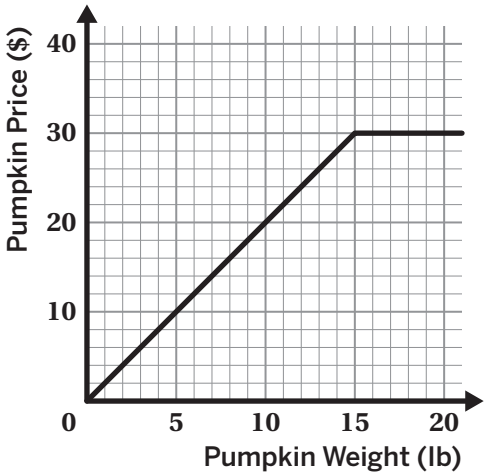
2 This table shows the prices of different pumpkins.

Pumpkin Weight (lb)	3	7	10	14	15	18	20	97
Price ($)	6	14	20	28	30	30	30	30

How can you determine the price of *any* size pumpkin?

3 The sign, the **piecewise-defined function** equation, and the graph each show pumpkin prices at Fran's Farm.

$$f(x) = \begin{cases} 2x & 0 \le x \le 15 \\ 30 & x > 15 \end{cases}$$



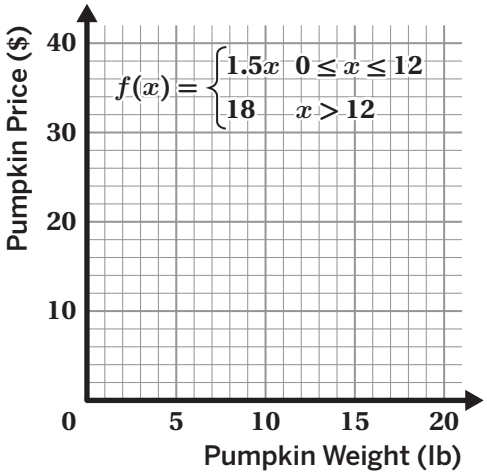
Discuss: Where do you see the same information in each representation?

Activity 2

Name: _____ Date: _____ Period: _____

Pricing Plans

4 Fran's Farm is having a sale on pumpkins. Draw a graph of the sale prices.

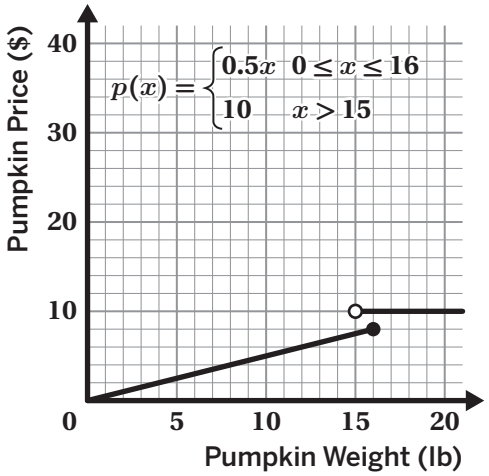


5 Pumpkins-a-Plenty also sells pumpkins.

The piecewise-defined function $p(x)$ represents the price of a pumpkin that weighs x pounds.

Customers are complaining that these prices are confusing.

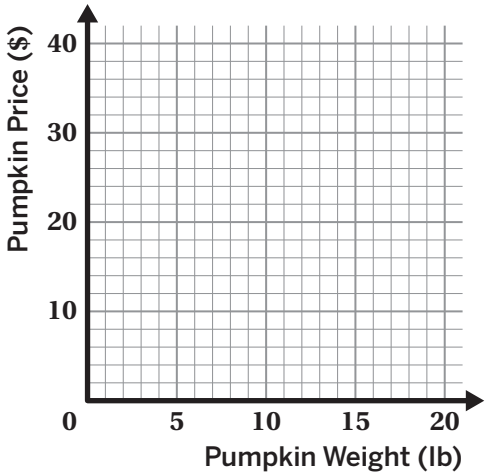
Discuss: Why might someone think these prices are confusing?



6 Pumpkins-a-Plenty wants you to fix the pumpkin prices.

a Draw a new graph so that each pumpkin only has one price.

b Explain what your graph says about the price of pumpkins.



Activity 3

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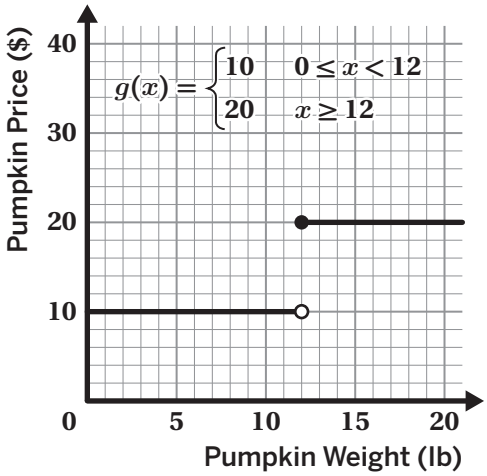
Evaluating Piecewise-Defined Functions

7 $g(x)$ represents the price of a pumpkin that weighs x pounds at Gourds, Gourds, Gourds.

Discuss:

- Where do you see $g(12)$ in the graph?
- Where do you see $g(12)$ in the equation?



8 $g(x)$ represents the price of a pumpkin that weighs x pounds at Gourds, Gourds, Gourds.

$$g(x) = \begin{cases} 10 & 0 \le x < 12 \\ 20 & x \ge 12 \end{cases}$$

What is $g(3)$?

9 Let's watch what Abdel did to determine $g(3)$.

a **Discuss:** How would you describe Abdel's strategy?

b Describe how to use Abdel's strategy to determine $g(35)$.

Abdel

$$g(x) = \begin{cases} 10 & 0 \le x < 12 \\ 20 & x \ge 12 \end{cases}$$

3

$g(3) = 10$

Name: Date: Period:

Evaluating Piecewise-Defined Functions (continued)

10 Here are some piecewise-defined functions.

$f(x) = \begin{cases} 3 & 0 < x \leq 5 \\ 8 & x > 5 \end{cases}$ What is $f(2)$?

$g(x) = \begin{cases} 4 & 0 \leq x \leq 8 \\ 6 & 8 < x < 16 \end{cases}$ What is $g(8)$?

$h(x) = \begin{cases} 5 & 0 < x < 9 \\ 12 & x \geq 9 \end{cases}$ What is $h(4)$?

$a(x) = \begin{cases} 1.5x & 0 < x < 9 \\ 12 & x \geq 9 \end{cases}$ What is $a(9)$?

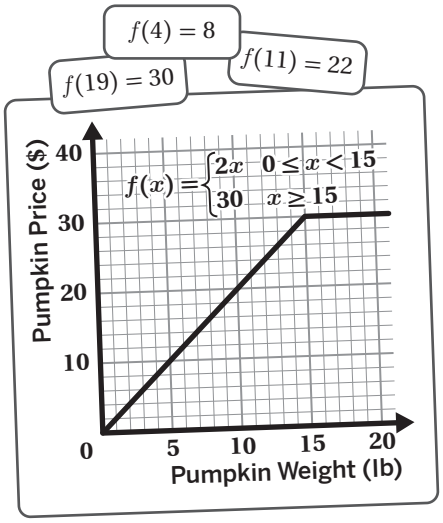
$b(x) = \begin{cases} x & 0 \leq x < 3 \\ x + 2 & x \geq 4 \end{cases}$ What is $b(5)$?

$c(x) = \begin{cases} x + 3 & 0 \leq x < 4 \\ 3x & 4 \leq x < 7 \end{cases}$ What is $c(4)$?

11 Synthesis

a How can you use a graph to evaluate a piecewise-defined function?

b How can you use an equation to evaluate a piecewise-defined function?



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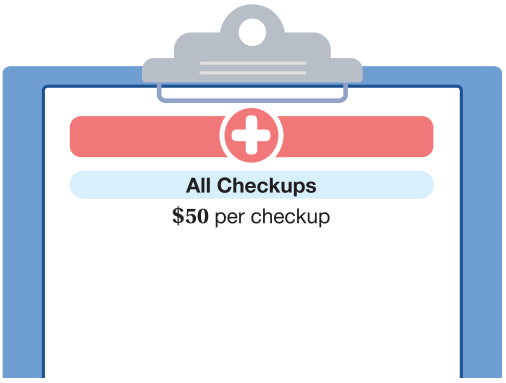
Doctor Doctor

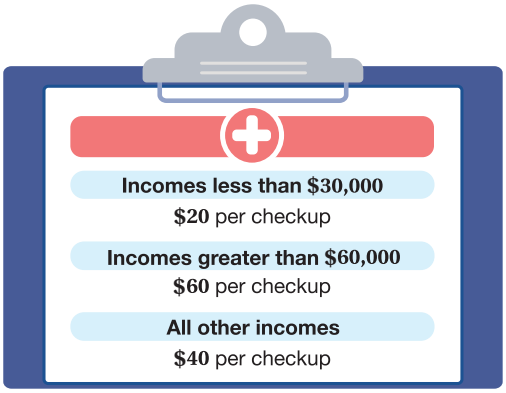
Let's explore ways to represent pricing plans.

Warm-Up

Doctor Remy's Office



Doctor Dyani's Office



1. What do you notice? What do you wonder?

I notice:

I wonder:

Activity 1

Name: _____ Date: _____ Period: _____

Dr. Dyani

Dr. Remy is interested in learning more about how Dr. Dyani's office charges for checkups.

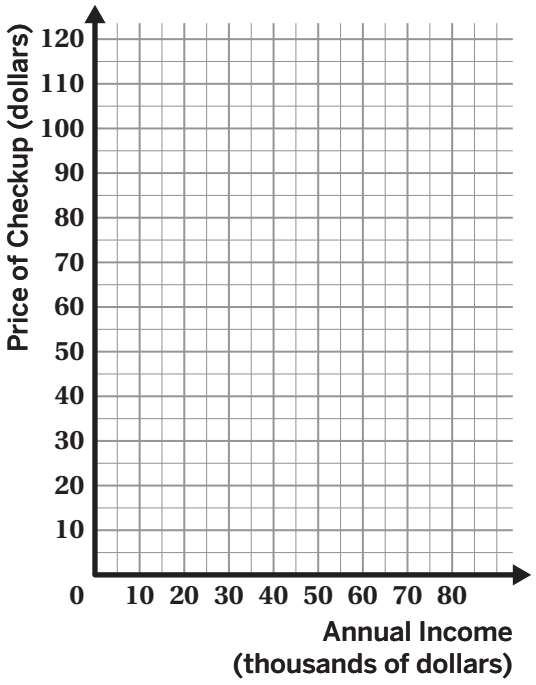
This is a summary of the checkup prices at Dr. Dyani's office.

Annual Incomes Less Than $30,000	Annual Incomes Greater Than $60,000	All Other Income
$20 per checkup	$60 per checkup	$40 per checkup

2. Complete the table.

Annual Income ($)	Price of Checkup ($)
20,000	
40,000	
60,000	
75,000	

3. Create a graph that shows the price of a checkup for any annual income.



4. The graph that represents the pricing plan for Dr. Dyani's office is an example of a special type of piecewise-defined function, called a **step function**. Why do you think it is called a step function?

Name: _____ Date: _____ Period: _____

Dr. Dyani (continued)

Hoang wrote a function to describe the price of a checkup at Dr. Dyani's office, $f(x)$, for an annual income of x dollars.

$$f(x) = \begin{cases} 30000 & 0 \le x \le 20 \\ 40 & 30000 < x < 60000 \\ 60 & x > 60000 \end{cases}$$

5. Some of Hoang's work is correct and some of Hoang's work is incorrect.

Discuss: What might be the mistake in Hoang's work?

6. Write a function that correctly describes the price of a checkup at Dr. Dyani's office.

$$f(x) = \begin{cases} \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ \end{cases}$$

7. Neena says that the pricing plan at Dr. Remy's office is more fair than the plan at Dr. Dyani's office because everyone pays the same amount for a checkup.

Ichiro disagrees.

Discuss: Who do you agree with? Why?

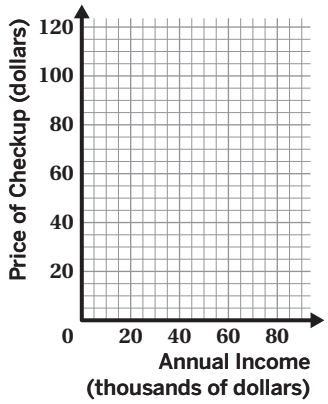
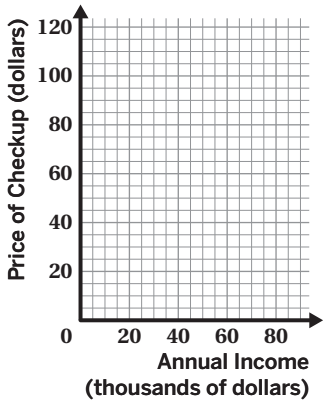
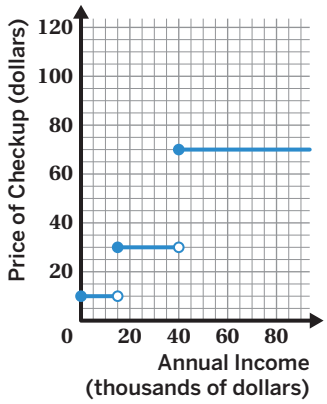
Activity 2

Name: _____ Date: _____ Period: _____

Dr. Remy

Dr. Remy's office currently charges a flat fee of $50 per checkup. They are considering changing to one of these three pricing plans for checkups.

8. Fill in the missing graphs and piecewise-defined functions.

Plan A	Plan B	Plan C
Incomes less than $30,000 $20 per checkup Incomes greater than $70,000 $60 per checkup All other incomes $40 per checkup	Incomes less than $60,000 $25 per checkup Incomes greater than or equal to $80,000 $100 per checkup All other incomes $50 per checkup	Incomes less than $15,000 $10 per checkup Incomes greater than or equal to $40,000 $70 per checkup All other incomes $30 per checkup
		
$a(x) = \begin{cases} \\ \\ \end{cases}$	$b(x) = \begin{cases} 25 & 0 \le x < 60000 \\ 50 & 60000 \le x < 80000 \\ 100 & x \ge 80000 \end{cases}$	$c(x) = \begin{cases} \\ \\ \end{cases}$

Name: Date: Period:

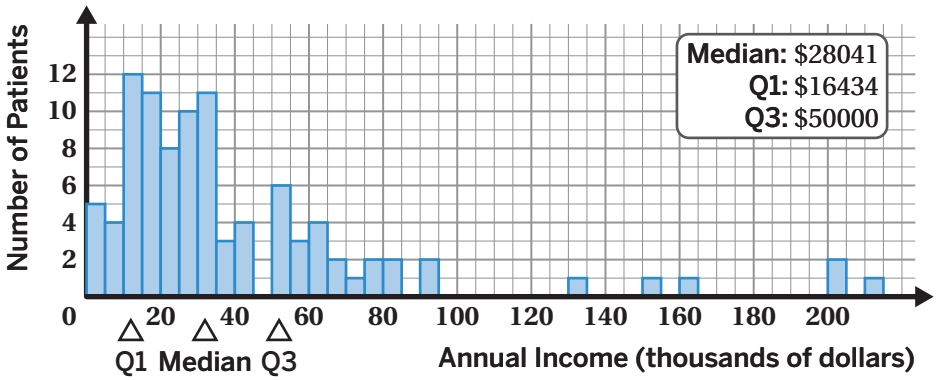
Dr. Remy (continued)

9. Choose one plan to analyze further. Circle one.

Plan A Plan B Plan C

According to the plan you selected, what is the price of a checkup for someone who has a median annual income of $28,041?

10. This histogram shows the annual incomes of Dr. Remy's patients.

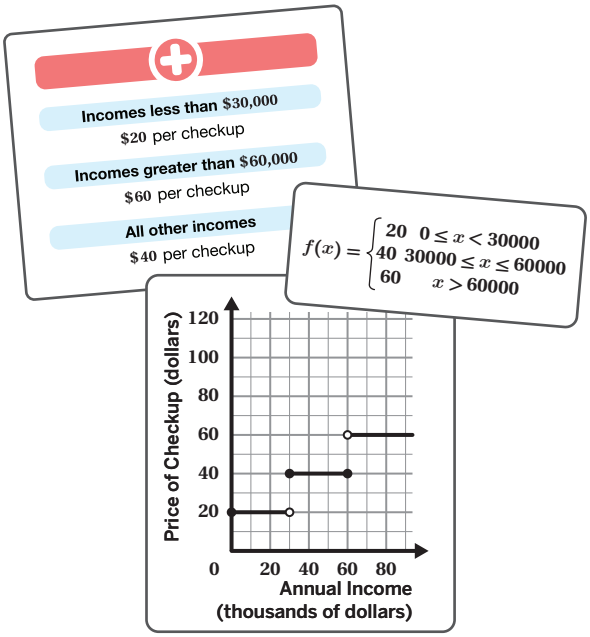


Use the histogram to help you describe how switching from a $50 flat fee to the plan you chose could impact patients of different annual incomes and Dr. Remy's office.

Synthesis

11. What is important to remember when graphing and writing equations of piecewise-defined functions?

Use the example if it helps with your thinking.



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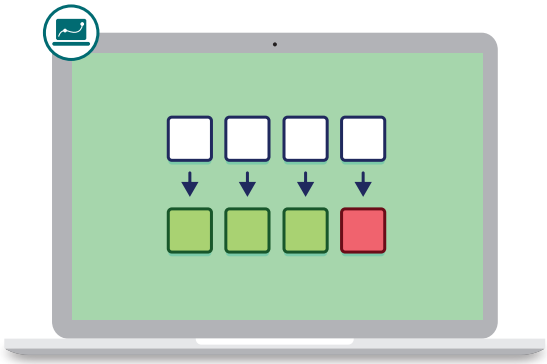
Recursion Excursion

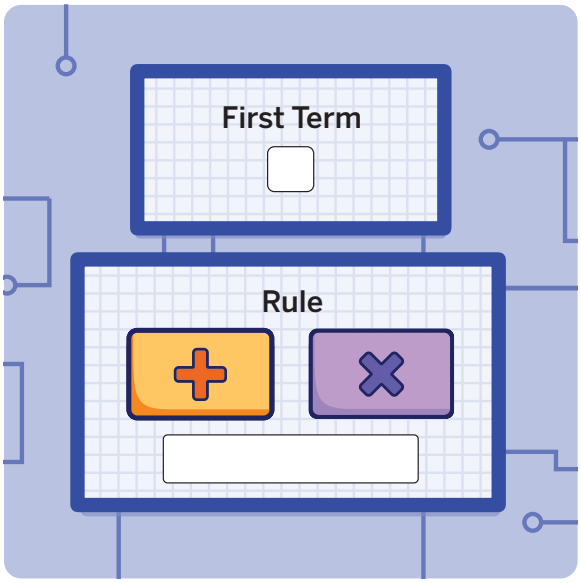
Let's write a recursive definition for a sequence using function notation.



Warm-Up

1 Let's play with the first term and rule, and notice what happens to the *sequence*, graph, and table.

Discuss: What do you remember about sequences?



Activity 1

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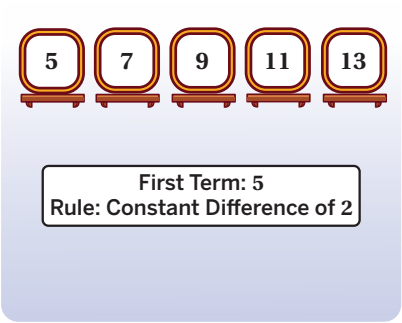
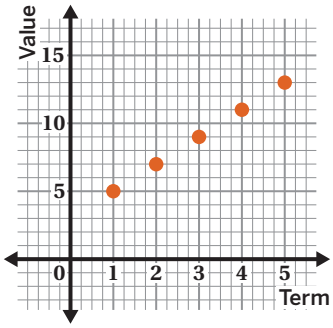
Sequences as Functions

2 Ariel made this sequence during the Warm-Up.





Term	Value
1	5
2	7
3	9
4	11
5	13

Ariel said: *If the domain is the term number, then all sequences are functions.*

Explain what Ariel might have been thinking.

3 Let $f(n)$ represent the value of term n in this sequence.

a **Discuss:** What does $f(4) = 11$ mean?

b What is the value of $f(7 - 1)$?

4 A sequence's *recursive definition* is made up of its first term and rule.

a Here are some recursive definitions.

$5, 7, 9, 11, 13$	$10, 6, 2, -2, -6$	$80, 40, 20, 10, 5$
First Term: 5 Constant Difference: 2	First Term: 10 Constant Difference: -4	First Term: 80 Constant Ratio: 0.5
$f(n) = \begin{cases} 5 & n = 1 \\ f(n-1) + 2 & n \geq 2 \end{cases}$	$f(n) = \begin{cases} 10 & n = 1 \\ f(n-1) + (-4) & n \geq 2 \end{cases}$	$f(n) = \begin{cases} 80 & n = 1 \\ f(n-1) \cdot 0.5 & n \geq 2 \end{cases}$

b **Discuss:** What does $f(n - 1)$ mean?

Name: ..... Date: ..... Period: .....

Sequences as Functions (continued)

5 These two recursive definitions will make the same sequence.

$$f(n) = \begin{cases} 7 & n = 1 \\ f(n-1) + 2 & n \geq 2 \end{cases}$$

$$f(1) = 7$$
$$f(n) = f(\boldsymbol{n} - 1) + 2$$

How are the definitions alike? How are they different?

Alike:

Different:

6 Match each recursive definition to the sequence that it makes. One sequence will have two matches.

Recursive Definition	Sequence
a. $d(n) = \begin{cases} 3 & n = 1 \\ d(n-1) + n & n \geq 2 \end{cases}$	..... $3, 5, 8, 12, \ldots$
b. $h(1) = 5$ $h(n) = h(n-1) \cdot 3$	..... $3, 8, 13, 18, \ldots$
c. $k(n) = \begin{cases} 3 & n = 1 \\ k(n-1) + 5 & n \geq 2 \end{cases}$	..... $5, 8, 11, 14, \ldots$
d. $m(1) = 5$ $m(n) = m(n-1) + 3$	..... $5, 15, 45, 135, \ldots$
e. $v(1) = 3$ $v(n) = v(n-1) + 5$	

Name: _____ Date: _____ Period: _____

Sequence Challenges

7 Here is the recursive definition for a sequence:

$h(1) = 5$

$h(n) = h(n-1) \cdot 2$

Write the first five terms of the sequence.

Term, n	Value
1	
2	
3	
4	
5	

8 Complete the recursive definition so that it makes the sequence 7, 12, 17, 22, 27.

$g(1) =$ _____

$g(n) = g(n-1)$ _____

Term, n	Value
1	7
2	12
3	17
4	22
5	27

Name: _____ Date: _____ Period: _____

Sequence Challenges (continued)

9 Here's a recursive definition Nyanna wrote for the sequence 7, 12, 17, 22, 27.

a What did Nyanna do well?

Nyanna's Recursive Definition

$g(1) = \boxed{7}$

$g(n) = \boxed{5(n-1)}$

b What would you recommend Nyanna change to get a recursive definition that creates the sequence?

10 Write a recursive definition that will make the sequence 448, 224, 112, 56, 28.

$f(1) =$ _____

$f(n) =$ _____

Term, n	Value
1	448
2	224
3	112
4	56
5	28

Activity 3

Name: _____ Date: _____ Period: _____

Choose Your Own Sequence

11 The Fibonacci sequence is a sequence in which each term is the sum of the two terms that come before it.

A common Fibonacci sequence is $1, 1, 2, 3, 5, 8, 13$.

Write a recursive definition for it.

$f(1) =$ _____

$f(2) =$ _____

$f(n) =$ _____

12 You will be designing a challenge for your classmates to solve.

a **Make It! Create a sequence.**

- Write the first five terms of your sequence.
- Write a recursive definition that will make your sequence. Your definition can include $f(n-1)$, $f(1)$, and/or n.

My Challenge	Recursive Definition
_____, _____, _____, _____, _____	$f(1) =$ $f(n) =$

b **Solve It!**

- Share your sequence with a classmate. Keep your recursive definition a secret!
- Write a recursive definition that will make their sequence.

Challenges	Recursive Definition
_____'s Sequence _____, _____, _____, _____, _____	$f(1) =$ $f(n) =$
_____'s Sequence _____, _____, _____, _____, _____	$f(1) =$ $f(n) =$
_____'s Sequence _____, _____, _____, _____, _____	$f(1) =$ $f(n) =$

13 Synthesis

Explain the different parts of a recursive definition and what they tell you about a function.

Use one or both examples if they help with your thinking.

$$f(n) = \begin{cases} 25 & n = 1 \\ f(n-1) + 5 & n \geq 2 \end{cases}$$

$$f(1) = 25$$

$$f(n) = f(n-1) + 5$$

Things to Remember:

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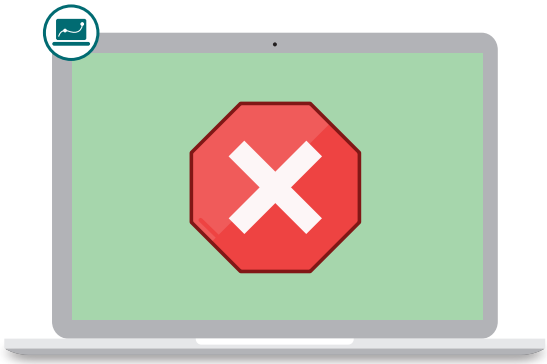
What's Your Score?

Let's make sense of absolute value functions.



Warm-Up

1 Which one doesn't belong?

A. $x = |\text{-}3|$

B. $|x| = 3$

C. $x = |9| - |12|$

D. $|9 - 12| = x$

Explain your thinking.

Activity 1

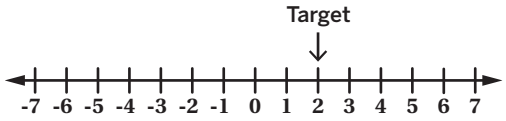
Name: _____ Date: _____ Period: _____

Target Numbers

2 Let's play a game. On the digital screen, press "Stop" to stop the arrow and get a score.

a Play up to five times.

b **Discuss:** How are the scores determined?



3 Here are Adriana's scores.

Adriana got a score of 4 on her next try.

What number do you think she stopped on? Why?

Number	Score
5	3
1	1
2	0
-4	6

4 Now there is a mystery target in this game!

a Play several rounds of the game on the digital activity. Record your number and score in the table.

b **Discuss:** What do you think the target is? Why?

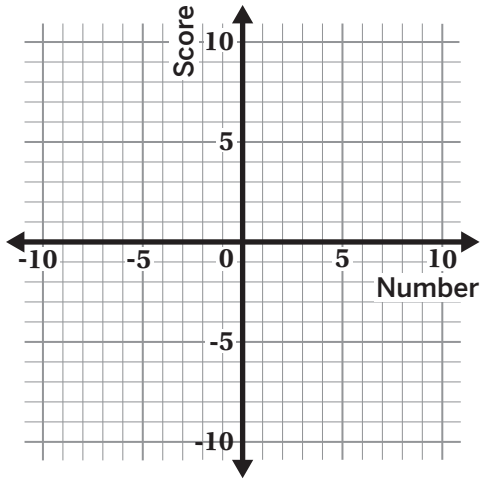
Number	Score

Name: _____ Date: _____ Period: _____

Target Numbers (continued)

5 **a** Plot the scores on the graph.

Number	Score
5	1
-1	5
3	1
0	4
-3	7



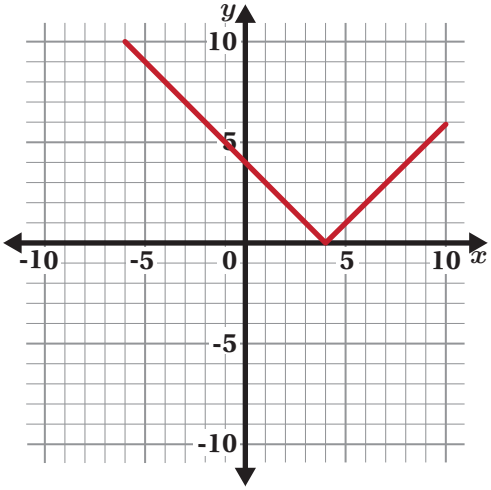
b **Discuss:**

- What do you think the graph of all possible scores looks like?
- Where can you see the mystery target?

6 The function $f(x) = |x - 4|$ is an example of an **absolute value function**.

This particular function tells you how far away you are from a target value of 4.

What is the value of $f(-2)$?



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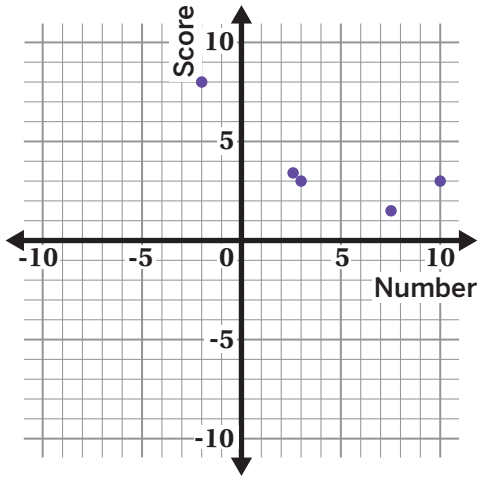
Absolute Value Functions

7 Use the digital screen to play another game. In this game, your score is how far away your guess is from a mystery number.

a Use the digital screen to enter up to five guesses.

b Tell a partner what the mystery number is and why.

8 Here are some guesses and scores.
Which function gives the score for each guess in this game?



A. $a(x) = |x| + 6$

B. $b(x) = |x + 6|$

C. $c(x) = |x| - 6$

D. $d(x) = |x - 6|$

Explain your thinking.

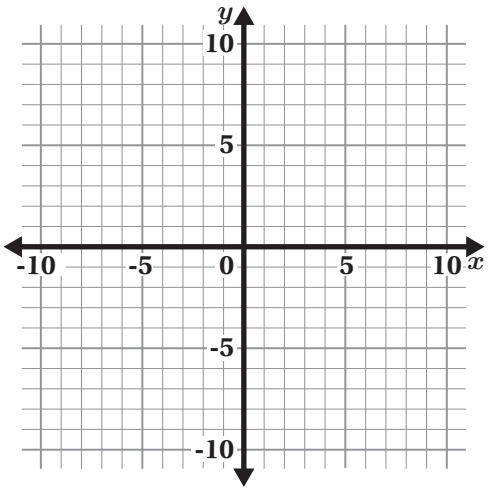
Name: _____ Date: _____ Period: _____

Absolute Value Functions (continued)

9 There is a new mystery number. The function $f(x) = |x + 3|$ gives the score for each guess, x.

Complete the table and plot the ordered pairs.

x	$f(x)$
5	
-1	
-5	
-3	
3	



Explore More

10 Here are some guesses and scores for a new mystery number. Can these be scores for the same mystery number? Circle one.

Yes No

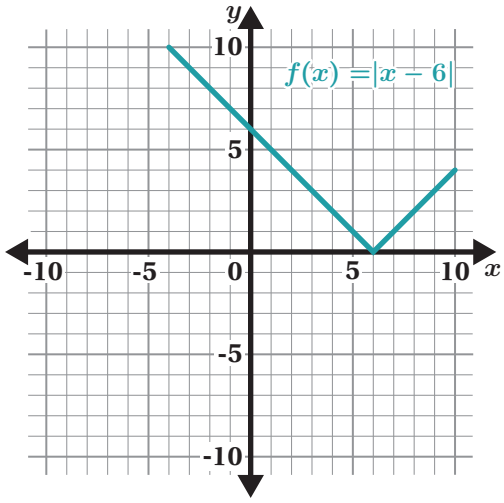
Explain your thinking.

Guess	Score
1	4
6	2

11 Synthesis

How is an absolute value function related to the distance from a number?

Use the graph and equation if they help with your thinking.



Things to Remember:

Unit 4
Lesson
16

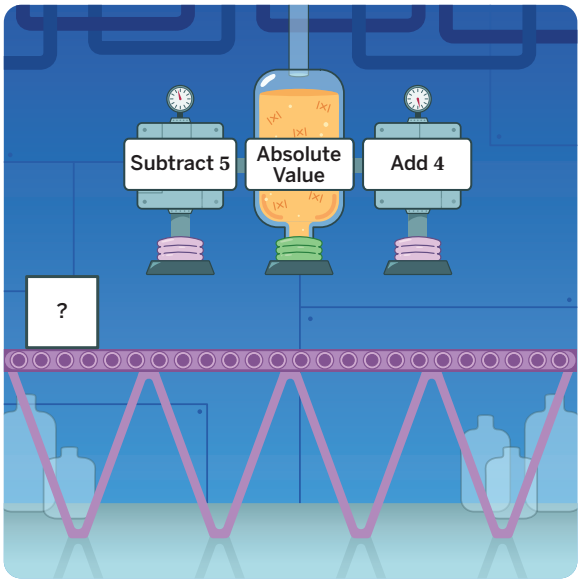
Name: Date: Period:

Absolute Value Machines

Let's graph absolute value functions.

Warm-Up

1 Here is a machine for $f(x) = |x - 5| + 4$.



a Let's watch how the machine works. Write down what happens to each input value at each solving step.

x	$x - 5$	$\lvert x - 5 \rvert$	$\lvert x - 5 \rvert + 4$

b **Discuss:** What do you notice and wonder?

Activity 1

Name: _____ Date: _____ Period: _____

Features of Absolute Value Functions

2 Kiri tried the numbers in the table.

She says: *The minimum value the machine can make is 4.*

Do you agree? Circle one.

Yes No Not enough information

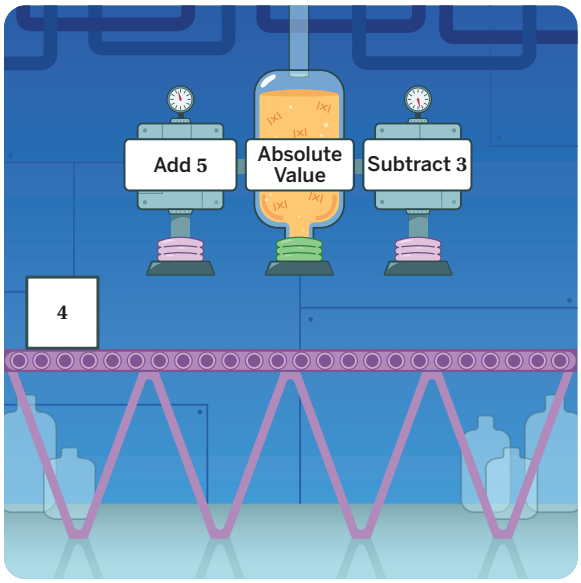
Explain your thinking.

x	$x-5$	$\lvert x-5\rvert$	$\lvert x+5\rvert+4$
-1	-6	6	10
2	-3	3	7
5	0	0	4
6	1	1	5

3 Here is a machine for $g(x) = |x + 5| - 3$.

What number will come out of the machine if we enter 4, -1, and -6?

Use the table if it helps with your thinking.



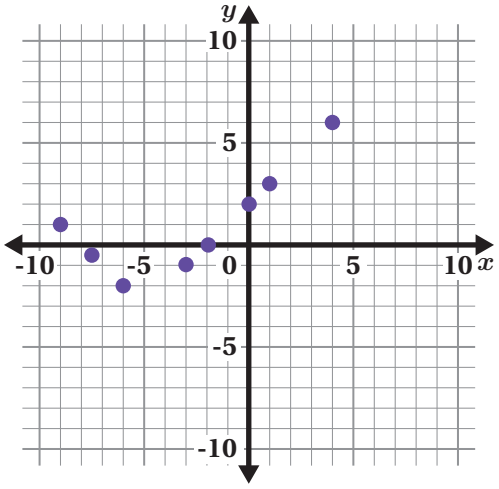
	x	$x+5$	$\lvert x+5\rvert$	$\lvert x+5\rvert-3$
a	4			
b	-1			
c	-6			

Name: ..... Date: ..... Period: .....

Features of Absolute Value Functions (continued)

4 Here are some points on the graph of $g(x) = |x + 5| - 3$.

a Draw a sketch that shows what all the points look like.



b Describe your sketch using some of these terms:

positive	maximum	increasing	domain
negative	minimum	decreasing	range
symmetry		piecewise-defined function	

5 Here are descriptions of $g(x) = |x + 5| - 3$ from other students.

Select *all* the descriptions that are true.

- ☐ **A.** The domain is all numbers.
- ☐ **B.** The minimum is at (-6, -2).
- ☐ **C.** The range is $g(x) \geq -3$.
- ☐ **D.** $g(x)$ is increasing when $x > -6$.
- ☐ **E.** The minimum is at (-5, -3).

Activity 2

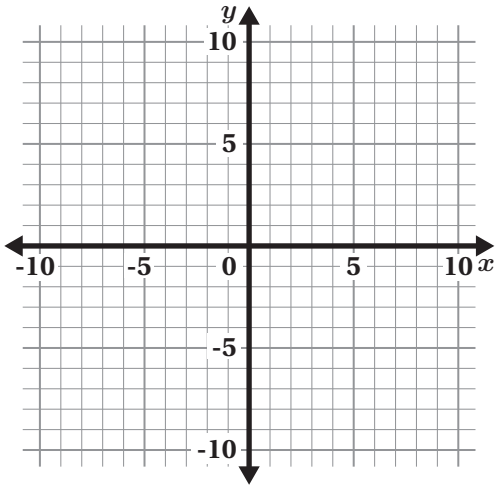
Name: Date: Period:

Graphing Absolute Value Functions

6 Here is a new function: $f(x) = |x - 6| + 2$.

Complete the table and then plot the graph of $f(x)$.

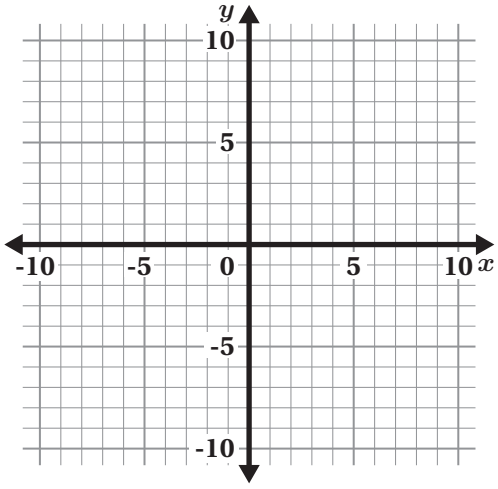
x	$f(x)$
9	
2	
0	
6	
-1	



7 Draw a graph of $j(x) = |x + 3| + 1$.

Use the table if it helps with your thinking.

x	$j(x)$



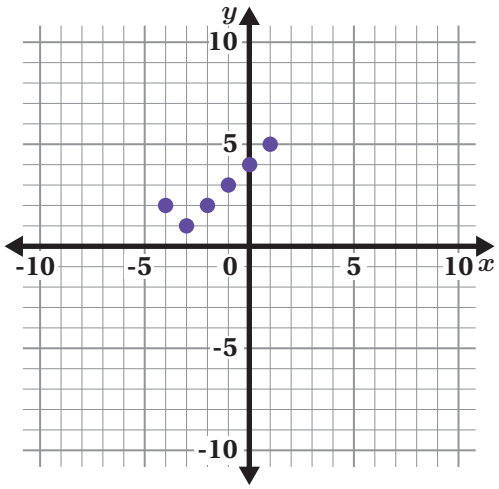
Name: Date: Period:

Graphing Absolute Value Functions (continued)

8 Here are the points that Tiana plotted for $j(x) = |x + 3| + 1$.

Tiana says: *I can use symmetry to plot more points on the graph.*

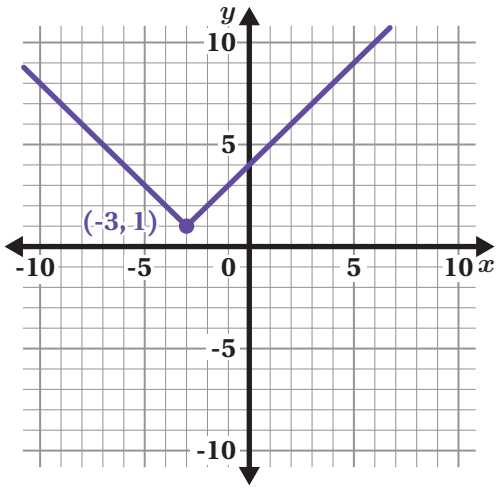
Show or describe what you think this means.



9 Here is the graph of $j(x) = |x + 3| + 1$.

The minimum value is shown.

How can you see the minimum value in the equation?



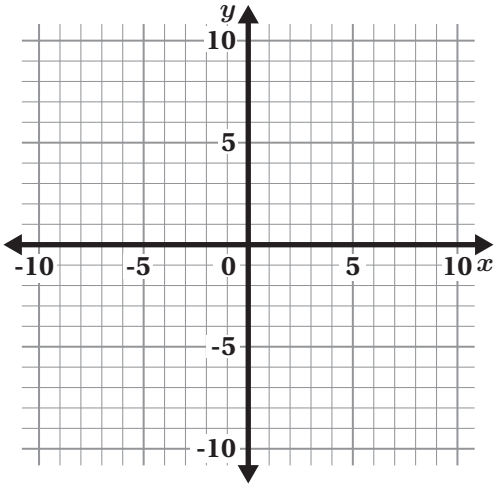
Activity 3

Name: _____ Date: _____ Period: _____

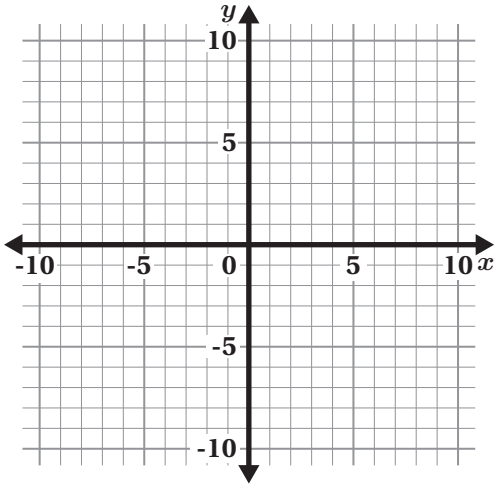
Repeated Challenges

10 Draw the graph of each function.

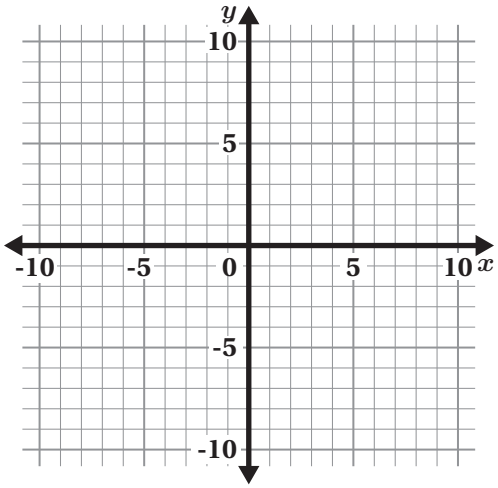
a $f(x) = |x - 4| + 3$



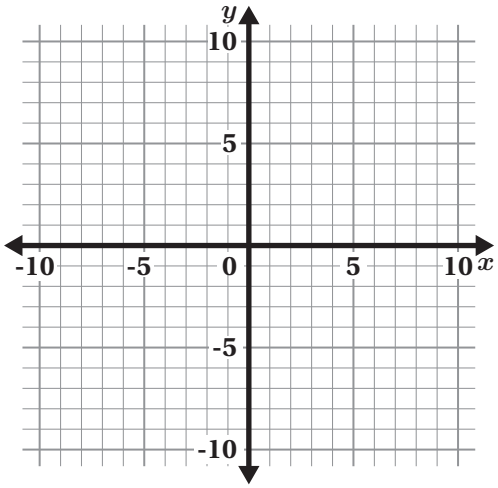
b $f(x) = |x + 2| + 1$



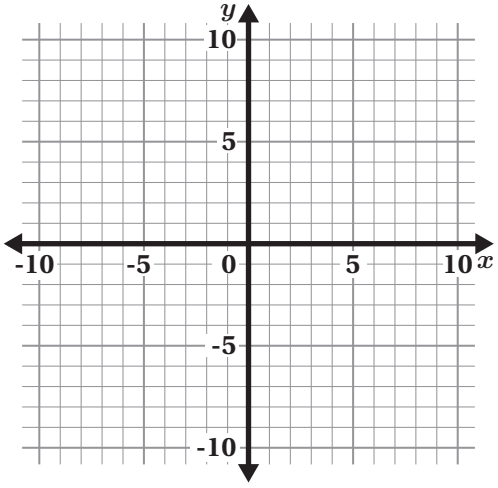
c $f(x) = |x - 1| + 7$



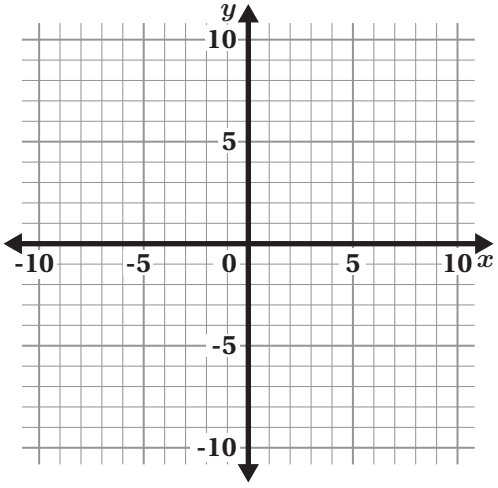
d $f(x) = |x - 5| - 4$



e $f(x) = |x + 4| + 3$



f $f(x) = |x + 6| - 1$



11 Synthesis

What can you know about the graph of an absolute value function by looking at its table or equation?

Use the example if it helps with your thinking.

$f(x) = |x - 4| + 3$

x	$f(x)$
-2	9
0	7
2	5
4	3
6	5

Things to Remember:

Unit 4
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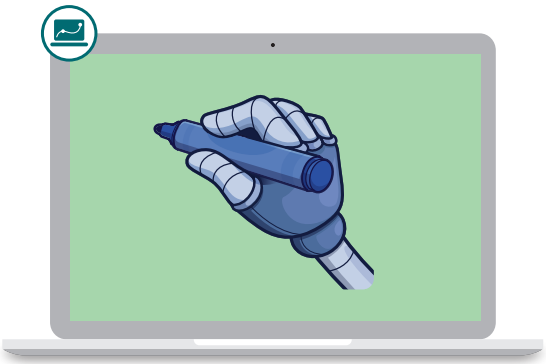
Name: ..... Date: ..... Period: .....

Chip the Robot

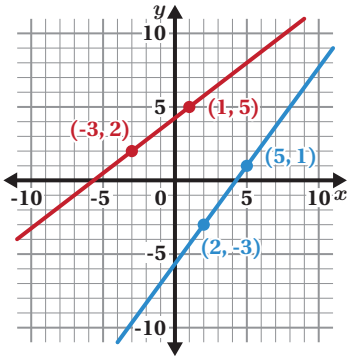
Let's make connections between a function and its inverse using graphs.

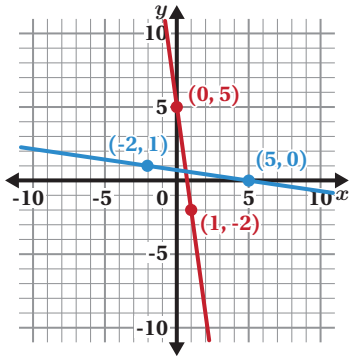


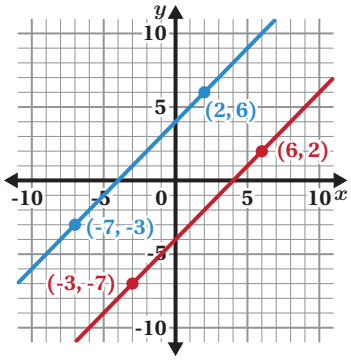
Warm-Up

1 Let's play a game with Chip the Robot!

When you create a red line, Chip draws a blue line using a rule.







Explain how Chip's rule works.

Activity 1

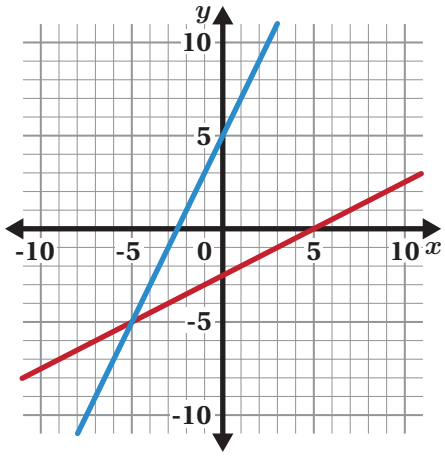
Name: _____ Date: _____ Period: _____

Chip's Rule

2 Here is a line Vihaan plotted, along with Chip's copy.

Vihaan says: *All of the points on Chip's line are reversed from my points.*

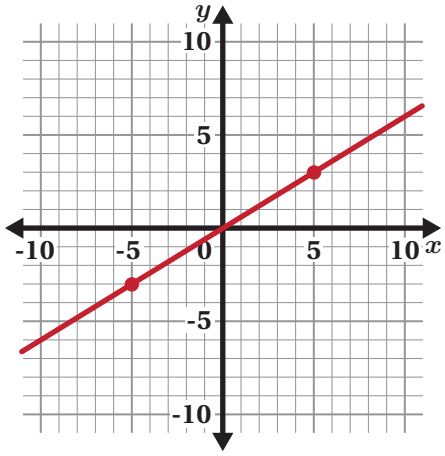
Show or describe what you think this means.



3 Here is a line that Aki plotted.

a Sketch the line you think Chip would draw.

b **Discuss:** What is your strategy?

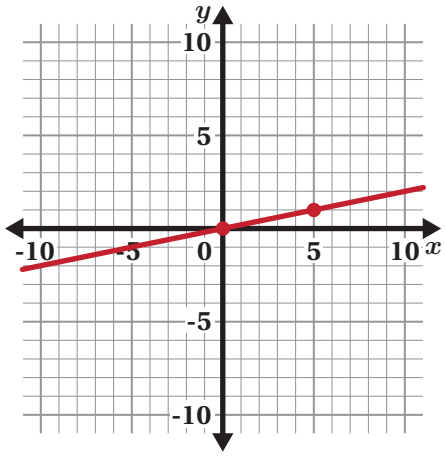


4 Chip can also use function notation to draw the line.

Mariam plotted the function $f(x) = \frac{1}{5}x$.

What is the function you think Chip will plot?

$g(x) =$



Name: Date: Period:

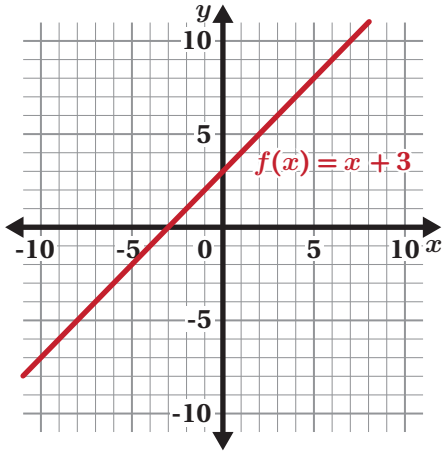
Chip's Rule (continued)

5 Chip the Robot plots **inverse functions**.

In general, if a function has a point at (h, k), then the inverse function has a point at (k, h).

Watch the screen to see the inverses of each of the different functions plotted on the graph.

Which function is the inverse of $f(x) = x + 3$?



A. $a(x) = x$

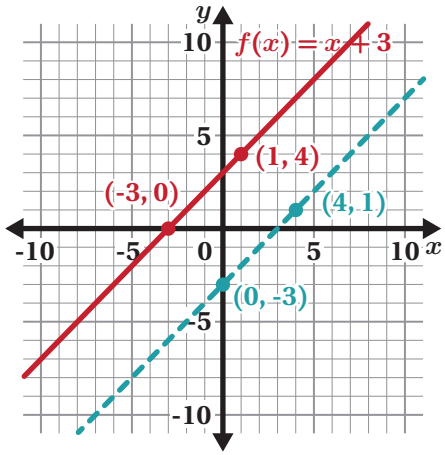
B. $b(x) = 3x$

C. $c(x) = -x + 3$

D. $d(x) = x - 3$

6 Fatima says that the inverse of $f(x) = x + 3$ goes through the points $(0, -3)$ and $(4, 1)$.

Show or describe how you can write the equation of the inverse function using these two points.



Activity 2

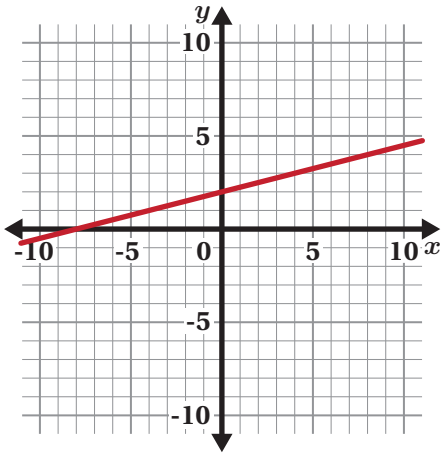
Name: Date: Period:

Inverse Challenges, Challenging Inverses

7 Here is the function $f(x) = \frac{1}{4}x + 2$.

Use the graph of the function to write an equation for the inverse function.

$g(x) =$



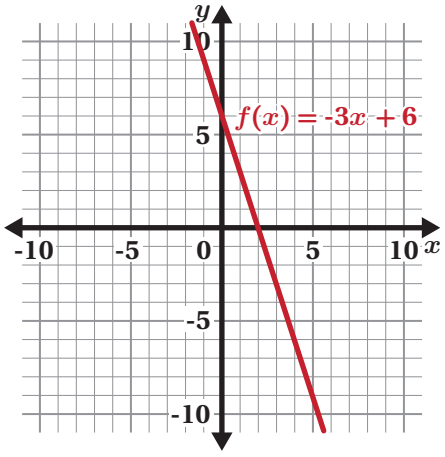
8 Here is the function $f(x) = -3x + 6$.

Jin says the inverse of $f(x)$ will pass through $(0, -6)$.

Is Jin correct? Circle one.

Yes No I'm not sure

Explain your thinking.



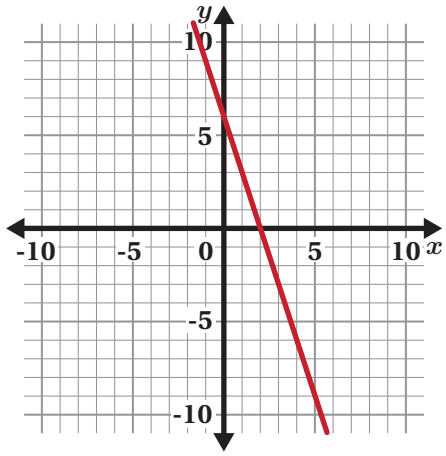
Name: _____ Date: _____ Period: _____

Inverse Challenges, Challenging Inverse (continued)

9 Here is the function $f(x) = -3x + 6$.

Use the graph of the function to write an equation for the inverse function.

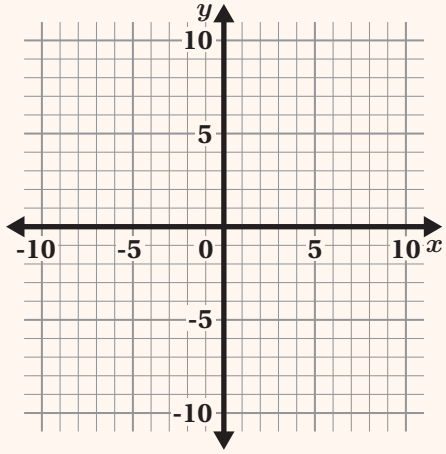
$g(x) =$



Explore More

10 Find as many pairs of linear inverse functions as you can. Try to find a pair that you think no one else in your class will find!

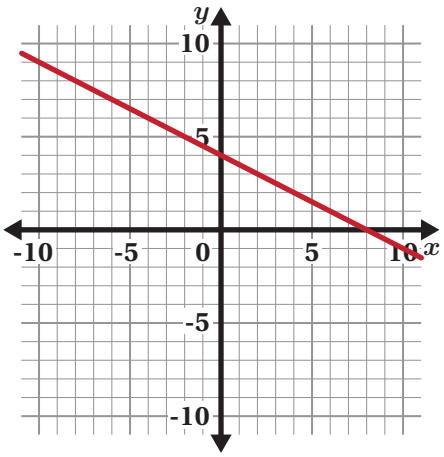
Function	Inverse Function
$f(x) =$	$g(x) =$
$f(x) =$	$g(x) =$
$f(x) =$	$g(x) =$
$f(x) =$	$g(x) =$



11 Synthesis

Describe a strategy for determining the inverse of a linear function.

Use the example if it helps with your thinking.



Things to Remember: