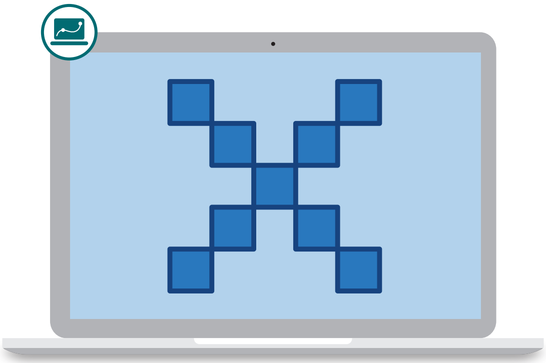
Unit 1
Lesson
1

Name: ________ Date: ________ Period: ________

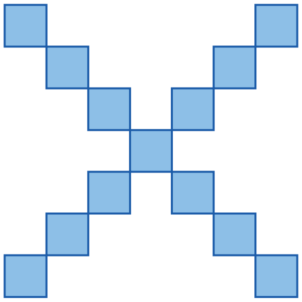


Visual Patterns

Let's explore visual patterns.

Warm-Up

1 Without counting one by one, determine how many tiles are in this figure.



Show or explain your thinking.

13 tiles. *Explanations vary.*

- **The shape has 4 arms and each arm has 3 tiles. There is also 1 tile in the middle.**
- **I noticed that the figure is symmetrical. I counted 6 tiles in the bottom half and assumed the top half would also have 6 tiles. There's also a tile in the middle. $2(6) + 1 = 13$**

Activity 1

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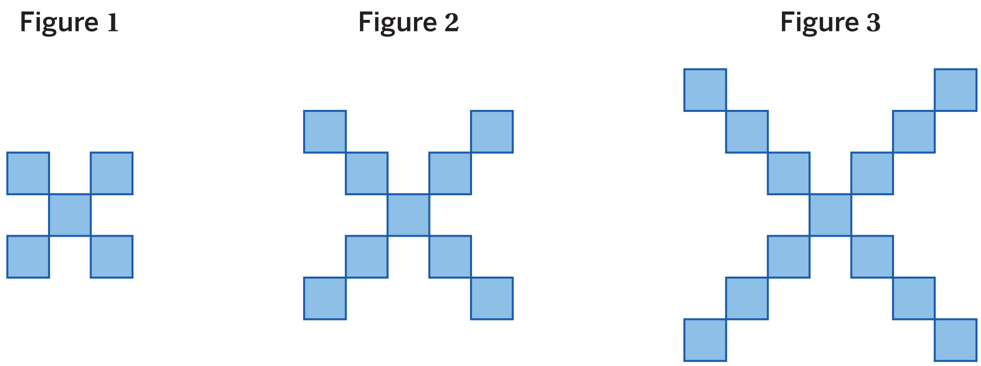
Pattern A

2 The figure in the Warm-Up is part of a visual pattern.



a **Discuss:** What do you think Figure 4 will look like?

***Responses vary*. I think Figure 4 will have 4 arms with 4 tiles each, arranged diagonally, plus 1 tile in the middle.**

b How many tiles will there be in Figure 4?

17 tiles

3 Here are the number of tiles in Figures 1–3.

How many tiles will there be in Figure 10?

41 tiles

Figure	Number of Tiles
1	5
2	9
3	13

Activity 2

Name: Date: Period:

Pattern B

4 Here is a new visual pattern.

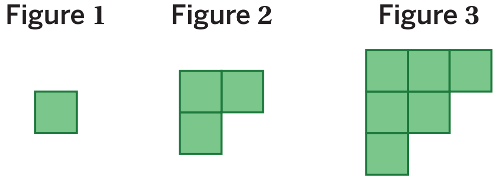


Figure	Number of Tiles
1	1
2	3
3	6

Matias says Figure 4 will have $6 + 4$ tiles.

Do you agree? Circle one.

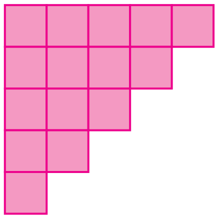
Yes No I'm not sure

Explain your thinking.

Explanations vary.

- **Matias is correct because each time the figure number increases by 1, the number of tiles added is equal to the figure number. When going from Figure 2 to 3, 3 tiles are added. When going from Figure 3 to 4, 4 tiles will be added.**
- **I agree. I noticed that the right side of the table has a pattern. The difference between Figure 2 and Figure 1 is 2 tiles. The next difference is 3 tiles. It makes sense that the next figure would grow by 4 tiles.**

5 Draw Figure 5 of the pattern.



6 How many tiles will there be in Figure 10?

55 tiles

Activity 3

Name: .... Date: .... Period: ....

Pattern C

7 Here are the two visual patterns we've seen.

Pattern A

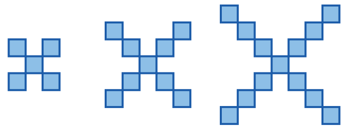


Figure 1 Figure 2 Figure 3

Figure	Number of Tiles
1	5
2	9
3	13
4	17

Pattern B

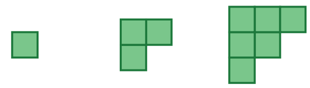


Figure 1 Figure 2 Figure 3

Figure	Number of Tiles
1	1
2	3
3	6
4	10

How are these patterns alike? How are they different?

Alike	Different
Responses vary. • In both patterns, each figure has more tiles than the previous one. • For both patterns, I can use a table to predict how many tiles are in the larger figures.	*Responses vary.* • The patterns grow in different ways: Pattern A grows by 4 every time, but Pattern B doesn't grow by the same number every time. • Pattern B starts with fewer tiles, but it has more tiles in Figure 10 than Pattern A. • The patterns have different shapes and colors.

8 Here is a new visual pattern.

Figure 1

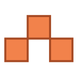


Figure 2

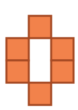


Figure 3

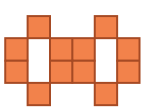


Figure 4

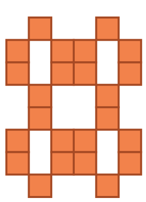


Figure	Number of Tiles
1	3
2	6
3	12
4	24

How many tiles will there be in Figure 7?

192 tiles

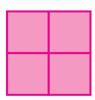
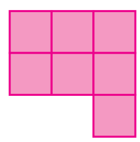
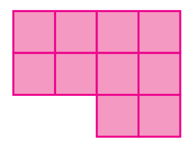
Name: .... Date: .... Period: ....

Pattern C (continued)

9 Here are the number of tiles in Figures 1–3 of another new visual pattern.

Figure	Number of Tiles
1	4
2	7
3	10

a Draw three figures to match the pattern in the table. ***Patterns vary.***

Figure 1	Figure 2	Figure 3
		

b How many tiles will there be in Figure 8?

25 tiles

Explore More

10 There are many possibilities for Figure 3 in this visual pattern.

Make two different versions that each continue the pattern in some way. ***Responses vary.***

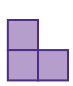
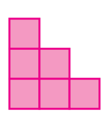
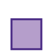
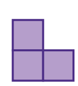
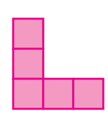
Figure 1	Figure 2	Figure 3
		

Figure 1	Figure 2	Figure 3
		

11 Synthesis

Describe *at least one* strategy for determining the number of tiles in Figure 7 of a visual pattern. Use these examples if they help with your thinking.

Pattern B

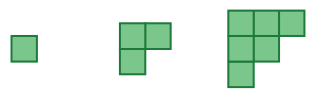


Figure 1 Figure 2 Figure 3

Figure	Number of Tiles
1	1
2	3
3	6
4	10

Pattern C

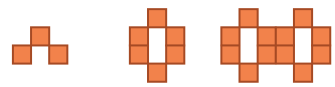


Figure 1 Figure 2 Figure 3

Figure	Number of Tiles
1	3
2	6
3	12
4	24

Responses vary.

- **I would draw what I think Figure 7 would look like based on the pattern and then count the number of tiles. For example, in Pattern B, Figure 7 would have 7 rows. The top row would have 7 tiles, and each row would have one fewer tile than the row above it.**
- **I would look for patterns in the table and then extend the table until I got to Figure 7. For example, the number of tiles doubles in every row of the table for Pattern C. I can keep doubling until I get to Figure 7.**

Things to Remember:

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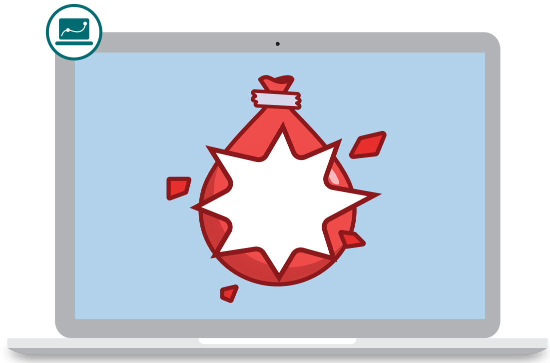
Sequence Carnival

Let's explore sequences.



Warm-Up

1 Here is a **sequence**: a list of numbers in a particular order.

Let's watch an animation to see how the sequence is made.

What do you notice? What do you wonder?

Responses vary.

I notice:

- **I notice that the machine makes each card 4 higher than the last card.**
- **I notice the numbers grow by 4 each time.**

I wonder:

- **I wonder if the machine can make different patterns.**
- **I wonder how far the machine keeps going.**

Activity 1

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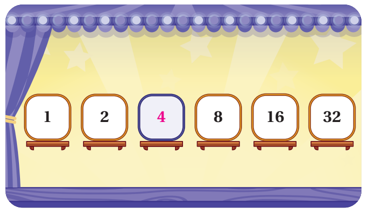
Seeing Sequences

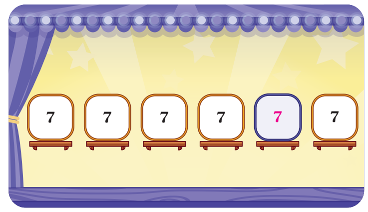
2 Each sequence follows a pattern and has a missing term. Write the missing term for each sequence. ***Responses shown on illustrations.***

Sequence 1

Sequence 2



Sequence 3



Sequence 4

Sequence 5

Sequence 6

3 Here are two new sequences.

A $5, 8, 11, 14, 17, \ldots$

B $5, 15, 45, 135, 405, \ldots$

How are they alike? How are they different? ***Responses vary.***

Alike	Different
• Both sequences start with 5. • Both sequences are increasing. • Both sequences change by 3.	• Sequence A grows by adding 3 each time. Sequence B grows by multiplying by 3 each time. • Sequence A shows even and odd numbers. Sequence B only shows odd numbers.

4 Sequence A changes by a **constant difference**. Sequence B changes by a **constant ratio**.

What type of change does Sequence C show? Circle one.

C $40, 20, 10, 5, \ldots$

Constant difference   Constant ratio   Neither

Explain your thinking.

Explanations vary.

- Each term is half of the one before it, which is the same as multiplying by 0.5.
- The differences are not constant (-20, -10, -5), but the ratio between terms is always $\frac{1}{2}$.

Activity 2

Name: ______ Date: ______ Period: ______

Sequence Challenges

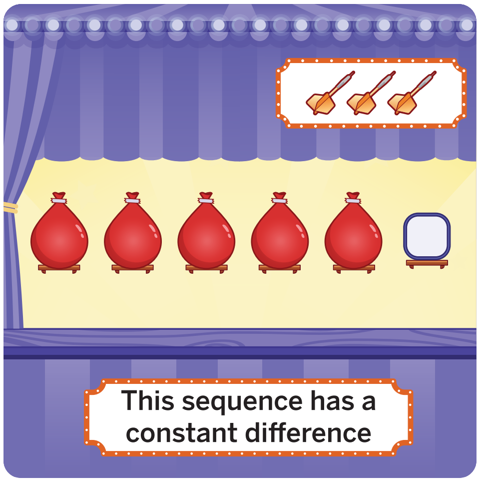
5 Let's look at a new sequence.

This sequence has a constant difference. The terms are hidden by balloons.

As a class, decide which balloons to pop. You can pop up to *three* balloons.

What is the missing term?

50



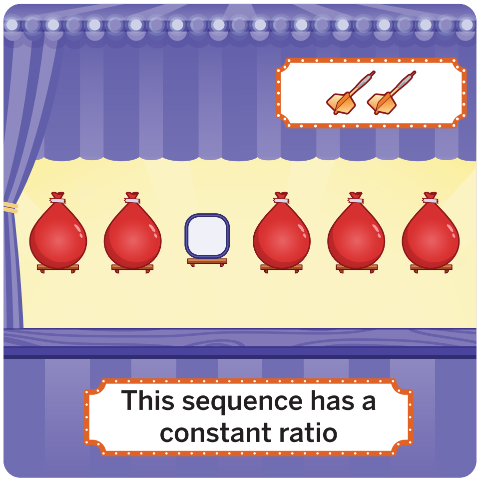
6 Let's look at a new sequence.

This sequence has a constant ratio.

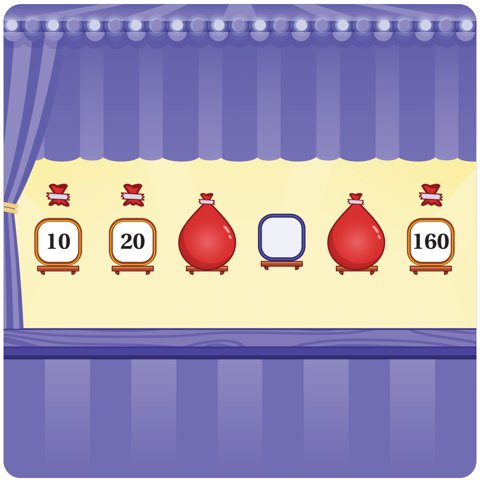
As a class, decide which balloons to pop. You can pop up to *two* balloons.

What is the missing term?

2



7 Moon and Gabriel looked at this sequence.



Moon said: *This sequence has a constant difference of* 10.

Gabriel said: *This sequence has a constant ratio of* 2.

Who is correct? Circle one.

Moon   Gabriel   Both   Neither

Explain your thinking.

Explanations vary.

- It can't be a constant difference because then the final term would be 60 instead of 160, and it can't be a constant ratio because then the last term would be 320 instead.
- There is no constant difference or constant ratio that will take you from 10, to 20, to 160 on the 6th term.

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Sequence Challenges (continued)

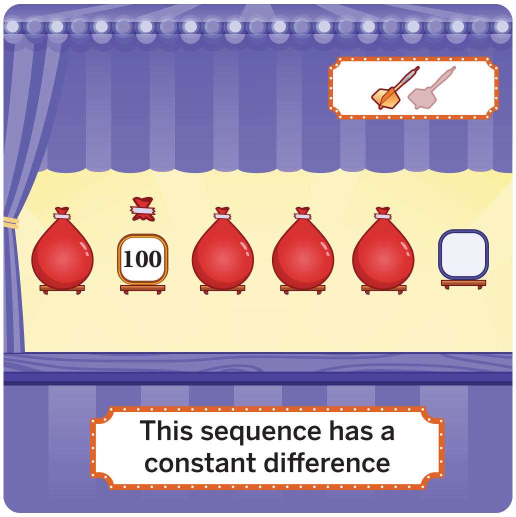
8 Laila is working on this sequence, which has a constant difference.

She can pop *one more* balloon.

Which balloon do you think she should pop? Explain your thinking.

Responses vary.

- **Laila should pop a balloon that is next to 100 so she can see the constant difference.**
- **Laila should pop the fourth balloon in the sequence. That balloon will be the middle value between 100 and the unknown term.**



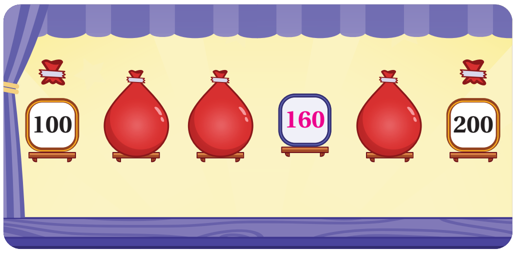
9 Let's see which balloon Laila chose to pop.

What is the missing term?

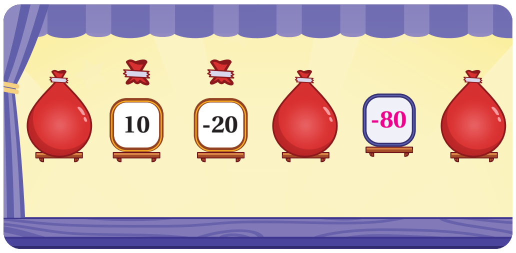
64

10 What is the missing term in each sequence? ***Responses shown on illustrations.***

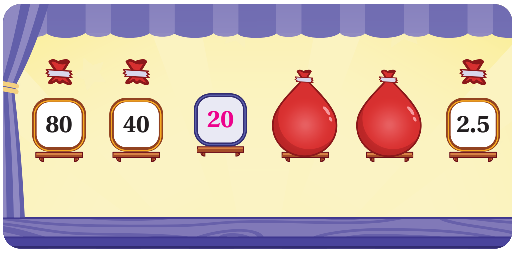
a This sequence has a constant difference.



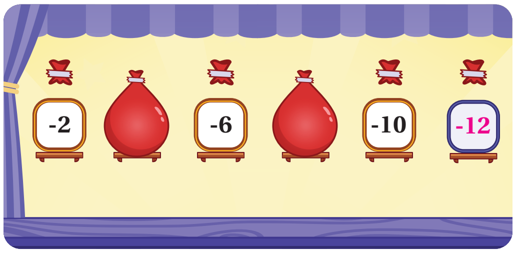
b This sequence has a constant ratio.



c This sequence has either a constant ratio *or* a constant difference.



d This sequence has either a constant ratio *or* a constant difference.

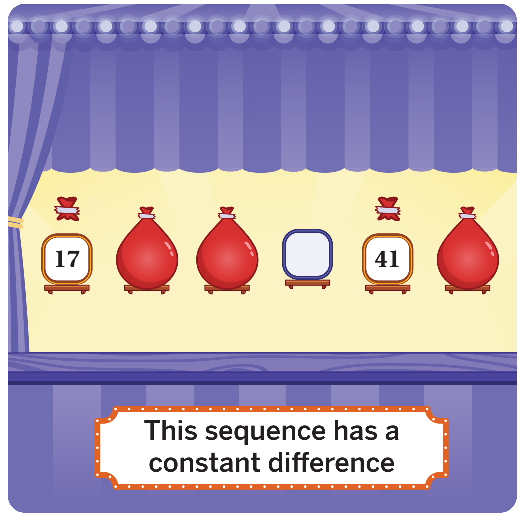


11 Synthesis

Describe a strategy for finding a missing term in a sequence.

Use the example if it helps with your thinking.

Responses vary. First, determine if the sequence has a constant difference or constant ratio. If you know it has a constant difference, use subtraction and division to figure out what the constant difference is. Then apply the constant difference to a known term to find the unknown term. In this sequence, the constant difference is $\frac{(41-17)}{4} = 6$. The unknown term is $41 - 6 = 35$.



Things to Remember:

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Recursion Machine

Let's write recursive definitions of sequences to meet certain requirements.

Warm-Up

1 **a** Let's watch an animation to see how a machine created this sequence.

b How would you describe what the machine is doing?

Responses vary.

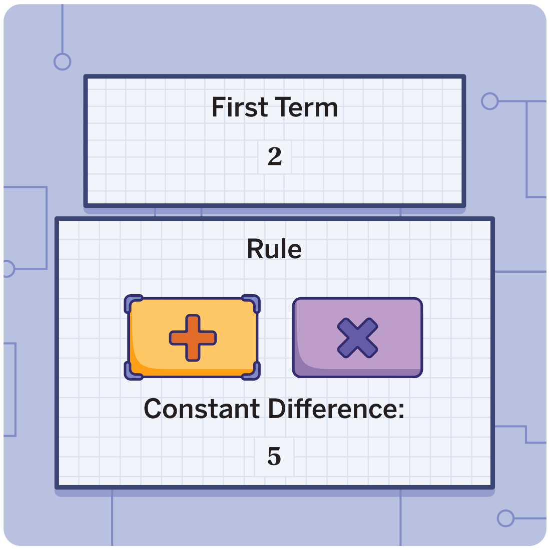
- The machine is multiplying each term by -10 to get the next term.
- The machine is applying a constant ratio of -10 to each term.

Activity
1

Name: .... Date: .... Period: ....

Recursive Definitions

2 The machine creates a sequence using a first term and a rule. Together, a first term and a rule make a **recursive definition**.



a Take a look at these sequences that were made with different *recursive definitions*.

Sequence	First Term	Rule
2, 7, 12, 17, 22	2	Constant difference: 5
3, 6, 12, 24, 48	3	Constant ratio: 2
48, 24, 12, 6, 3	48	Constant ratio: $\frac{1}{2}$

b Create as many sequences as you can that include the number 12. ***Responses vary.***

Sequence	First Term	Rule
12, 12, 12, 12, 12	12	Constant difference: 0
3, -6, 12, -24, 48	3	Constant ratio: -2
8, 9, 10, 11, 12	8	Constant difference: 1

Activity
2

Name: ________ Date: ________ Period: ________

Recursive Challenges

3 In this sequence, the first term is $1{,}600$. Create a rule that will produce this sequence.

Sequence	First Term	Rule
$1600, 400, 100, 25, 6.25$	$1{,}600$	Constant ratio: $\frac{1}{4}$ (or equivalent)

4 In this sequence, the first term is 3. Create a rule that will make the fourth term 24.

Sequence	First Term	Rule
3, ___, ___, 24, ___ **3, 10, 17, 24, 31** or **3, 6, 12, 24, 48**	3	Constant difference: 7 or Constant ratio: 2

5 Create a recursive definition for a sequence that makes the second term 25 and the fourth term 1.

Sequence	First Term	Rule
___, 25, ___, 1, ___ **37, 25, 13, 1, -11** or **125, 25, 5, 1,** $\frac{1}{5}$	37 or 125	Constant difference: -12 or Constant ratio: 0.2 (or equivalent)

Activity 2

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Recursive Challenges (continued)

6 Troy made a mistake when he created a recursive definition on the previous problem.

Troy

____, 25, ____, 1, ____

First term: 49

Rule: Constant difference of -24

a **Discuss:** How do you think Troy created this recursive definition?

Responses vary. **I think Troy subtracted 1 from 25 to find the constant difference. He got -24, and then worked backwards from 25 to get a first term of 49.**

b What is something Troy can improve on?

Responses vary. **The constant difference is how the sequence changes every term, so Troy would have to divide -24 by 2 since there is a term between 25 and 1.**

7 Create a recursive definition for a sequence that makes the fourth term -40. Try to complete this challenge in different and interesting ways! *Responses vary.*

Sequence	First Term	Rule
-10, -20, -30, -40, -50	-10	Constant difference: -10
5, -10, 20, -40, 80	5	Constant ratio: -2
320, -160, 80, -40, 20	320	Constant ratio: -0.5 (or equivalent)

Activity 3

Name: .... Date: .... Period: ....

Challenge Creator

8 Now it's your turn to design your own sequence challenge.

Challenges and responses vary.

a **Make it!**

- Write up to three terms anywhere in the sequence.
- Write a recursive definition for your sequence.

My Sequence	First Term	Rule
____, ____, ____, ____, ____		

b **Swap it!**

- Share your three terms and where they are in the sequence with a partner. Keep your recursive definition a secret!
- Write a recursive definition that completes your partner's sequence.

Partner's Sequence	First Term	Rule
____, ____, ____, ____, ____		

Partner's Sequence	First Term	Rule
____, ____, ____, ____, ____		

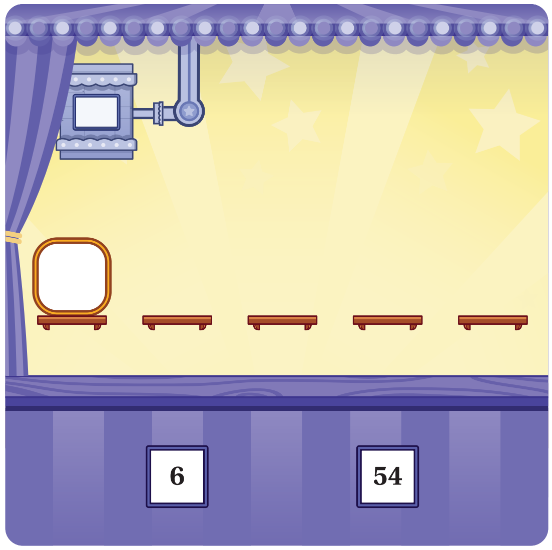
Partner's Sequence	First Term	Rule
____, ____, ____, ____, ____		

9 Synthesis

We learned that a recursive definition of a sequence includes a first term and a rule.

Describe a strategy for determining a recursive definition of a sequence that meets certain requirements.

Use the example if it helps with your thinking.



Responses vary. **First, decide whether to write a rule with a constant difference or a constant ratio. In this case, either will work, so I'll choose a constant difference. To get from 6 to 54 with a term in between, I can do this:** $\frac{54-6}{2}=24$**. To determine the first term, I just subtract:** $6-24=-18$**.**

Things to Remember:

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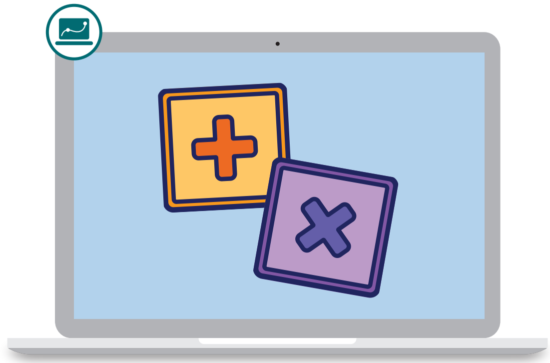

See the Sequence

Let's compare sequences using tables and graphs.

Warm-Up

1 Here are two sequences.

a Let's watch an animation to see how the machines create the first four terms of each sequence.

b What do you notice? What do you wonder?

I notice:

Responses vary.

- I notice that the top machine has a + on it, and the bottom machine has an ×.
- I notice that the terms in the top sequence are larger than the terms in the bottom sequence.
- I notice that the sequences are kind of in a race. The top sequence starts ahead (50 to 3) and pulls even further ahead with each term.

I wonder:

Responses vary.

- I wonder if these sequences have only 5 terms.
- I wonder if the sequences will have any of the same numbers in them.
- I wonder if the bottom sequence will catch up with the top sequence if they keep going.

Activity 1

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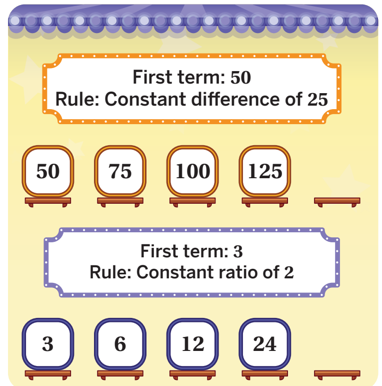
Sequence Types

2 The top sequence changes by a constant difference. We call that an **arithmetic sequence**. The bottom sequence changes by a constant ratio. We call that a **geometric sequence**. Which do you think will have a greater 10th term?



A. The arithmetic sequence **B.** The geometric sequence

C. They will be the same **D.** Not enough information

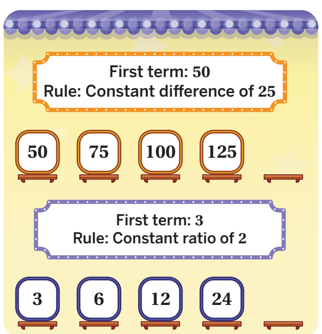
Explain your thinking.

Responses and explanations vary.

- **The geometric sequence. The arithmetic sequence starts ahead but adds only 25 every time. The geometric sequence multiplies each term by 2, so the differences between the terms keep getting larger. This may be enough to pass the arithmetic sequence.**
- **The arithmetic sequence. Comparing the first terms, the top sequence is greater by $50 - 3 = 47$. By the fourth term, the top sequence is further ahead: $125 - 24 = 101$. I expect this pattern to continue.**

3 Sequences can be represented in multiple ways.

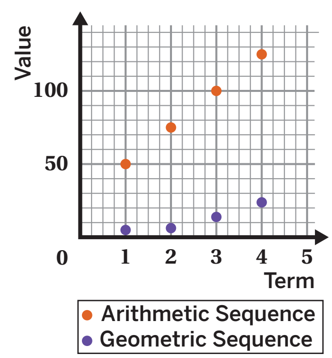
Recursive Definition



Table

Term	Arithmetic Sequence	Geometric Sequence
1	50	3
2	75	6
3	100	12
4	125	24

Graph



a **Discuss** What are the advantages and disadvantages of each representation?

***Responses vary.* See Teacher's Edition or digital Sample Responses for more detail.**

b Choose one representation. Explain how it could be used to help determine which sequence has the greater 10th term.

***Responses vary.* I could extend the graphs of each sequence and compare the y-values when the x-value is 10.**

Name: .... Date: .... Period: ....

Sequence Types (continued)

4 **a** Let's see which sequence has the greater 10th term.

b **Discuss** What do you notice about the graphs of the two sequences?

Responses vary.

- **I notice that the graph of the arithmetic sequence has points that form a line.**
- **I notice that the graph of the geometric sequence curves upward quickly.**

5 **a** Group together the cards that represent the same sequence.

Card A

Term	Value
1	3
2	1
3	-1
4	-3
5	-5

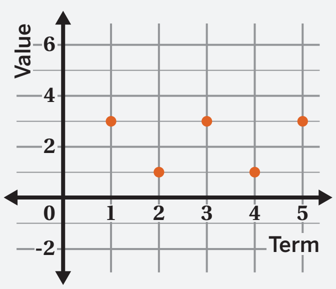
Card B

Term	Value
1	3
2	6
3	12
4	24
5	48

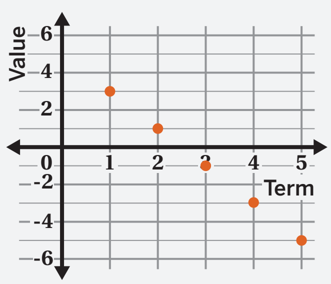
Card C

Term	Value
1	3
2	1
3	3
4	1
5	3

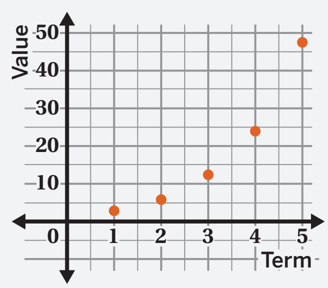
Card D



Card E



Card F



Sequence 1	Sequence 2	Sequence 3
First term: 3 **Rule:** Multiply the previous term by 2	**First term:** 3 **Rule:** Add -2 to the previous term	**First term:** 3 **Rule:** Alternate between 3 and 1
Card F Card B	Card E Card A	Card D Card C

b **Discuss:** Is each sequence arithmetic, geometric, or neither?

- **Sequence 1 is a geometric sequence.**
- **Sequence 2 is an arithmetic sequence.**
- **Sequence 3 is neither.**

Activity 2

Name: Date: Period:

Make Your Own Sequences

6 You will use Screen 6 to complete this activity.

a Create several different sequences and compare their tables and graphs. Then describe something that you found interesting or surprising.

Responses vary.

- **I found it interesting that the shape of an arithmetic sequence was always a line, no matter what numbers I entered.**
- **The shape of a geometric sequence was often a curve, but not always.**

b Decide whether each statement is *always, sometimes,* or *never* true. Explain your thinking. ***Explanations vary.***

Statement	Always, Sometimes, Never	Explanation
When you graph an arithmetic sequence, the points lie on a line.	**Always**	**Adding or subtracting the same number from the previous term will make equal-sized steps on a graph.**
The graph of a geometric sequence curves upward.	**Sometimes**	**If the constant ratio is a number between 0 and 1, like $\frac{1}{2}$, then the graph will curve downward.**
The graph of a geometric sequence with a positive first term will stay above the x-axis.	**Sometimes**	**The graph of a sequence with a first term of 3 and common ratio of -2 goes above and below the x-axis.**

c Write a recursive definition of a sequence that meets each set of criteria.

Criteria	Recursive Definition
Its graph lies on a horizontal line.	***Responses vary.*** **First term: 3 Constant difference: 0**
It approaches 0 but never reaches it.	***Responses vary.*** **First term: 100 Constant ratio: $\frac{1}{2}$**
Its 6th term is negative and 7th term is positive.	***Responses vary.*** **First term: 5 Constant ratio: -3**

Name: .... Date: .... Period: ....

Make Your Own Sequences (continued)

Explore More

7 Malik notices he can make an arithmetic sequence and a geometric sequence that have the same first two terms.

a In the table, continue the sequence in two ways: assuming it is *arithmetic* and assuming it is *geometric*.

Responses shown in table.

Term	Arithmetic Sequence	Geometric Sequence
1	8	8
2	12	12
3	**16**	**18**
4	**20**	**27**
5	**24**	**40.5**

b How long do you think it would take each sequence to reach 1,000?

Responses vary.

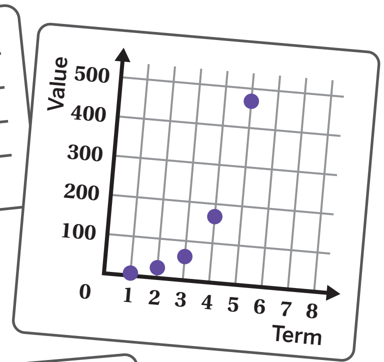
- **Arithmetic: This sequence starts at 8 and adds 4 to each term. I can use the equation $8 + 4x = 1000$ to help me think about getting from 8 to 1,000 by adding 4 repeatedly. If I solve it, $x = 248$, which means the 249th term will be 1,000.**
- **Geometric: This sequence multiplies by 1.5 each time. I used a calculator to try different values of x in the expression $8 \cdot 1.5^x$. I found that $8 \cdot 1.5^{12}$ was greater than 1,000, so I think it will only take about 13 terms for this sequence to reach 1,000.**

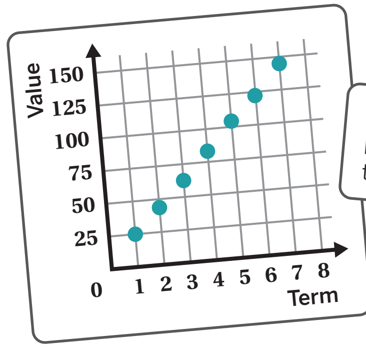
8 Synthesis

Sequences can be represented in multiple ways. What are some clues that a sequence might be arithmetic? Geometric?

Responses vary

Term	Value
1	6
2	18
3	54
4	162





First Term: 25
Rule: Add 20 to the previous term.

Arithmetic:

- **The rule in the recursive definition involves adding or subtracting each term by the same amount.**
- **The table has a constant difference.**
- **The points on the graph lie on a line.**

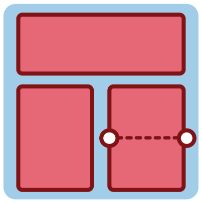
Geometric:

- **The rule in the recursive definition involves multiplying or dividing each term by the same amount.**
- **The table has a constant ratio.**
- **The graphs of geometric sequences look different depending on the constant ratio, but most will either curve up or curve down.**

Things to Remember:

Name: ________ Date: ________ Period: ________

Paper Patterns



Let's represent situations in different ways.

Warm-Up

1. Match each *expression* with an equivalent one.

a. $10 + 3 + 3 + 3 + 3 + 3$ b $10 \cdot 3^5$

b. $10 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$ d $10 \cdot \left(\frac{1}{3}\right)^5$

c. $10 - 3 - 3 - 3 - 3 - 3$ c $10 + 5(-3)$

d. $10 \div 3 \div 3 \div 3 \div 3 \div 3$ a $10 + 5 \cdot 3$

2. **Discuss:** Why might it be useful to have multiple forms of an expression?

Responses vary.

- The longer expressions show a repeated operation.
- The shorter expressions are more efficient to write.

Name: .... Date: .... Period: ....

Paper Cutting

Each of these patterns starts with a piece of paper that measures 8-by-10 inches.

3. Complete each representation.

Pattern 1	Pattern 2
Situation Cut off half of the paper and discard. Repeat.	**Situation** Cut a 1-by-8-inch strip off the paper and discard. Repeat.

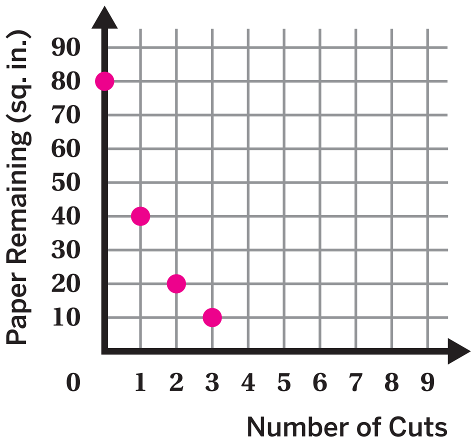
Pattern 1 — Table

Number of Cuts	Paper Remaining (sq. in.)
0	80
1	**40**
2	**20**
3	**10**

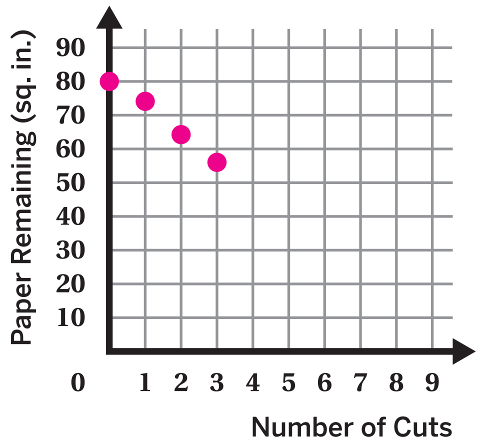
Pattern 2 — Table

Number of Cuts	Paper Remaining (sq. in.)
0	80
1	**72**
2	**64**
3	**56**

Pattern 1 — Graph



Pattern 2 — Graph



4. For each pattern, what is the area of the paper remaining after 9 cuts?

Pattern 1: 0.15625 square inches (or equivalent)

Pattern 2: 8 square inches

Activity
2

Name: ______ Date: ______ Period: ______

Explicit Expressions

5. Let's look at Wohali's and Kadeem's strategies for Pattern 1.

Kadeem wrote the expression $80 \cdot \left(\frac{1}{2}\right)^4$ to calculate the area of paper remaining after 4 cuts.

a What does each part of his expression represent in this situation?

Responses vary.

80 represents . . . **the area of the paper before any cuts have been made.**

$\frac{1}{2}$ represents . . . **that for each cut, there is half of the previous area remaining.**

4 represents . . . **the number of cuts.**

b Write an expression that represents the area of the paper remaining after n cuts. This expression is an example of an **explicit definition**.

$80 \cdot \left(\frac{1}{2}\right)^n$ (or equivalent)

6. Let's think about Pattern 2.

a Select *all* the expressions that could be used to determine the area of the paper remaining after 9 cuts.

- [x] **A.** $80 - 8 - 8 - 8 - 8 - 8 - 8 - 8 - 8 - 8$
- [ ] **B.** $80 + 9(8)$
- [ ] **C.** $80 \cdot (\text{-}8)^9$
- [x] **D.** $80 - 9(8)$
- [ ] **E.** $80 \cdot 0.5^9$

b Write an explicit expression to determine the area of the paper remaining after n cuts.

$80 - 8n$ (or equivalent)

c What does each part of your expression represent in this situation?

Responses vary.

- **80 represents the area of the original piece of paper.**
- **-8 represents the amount of area removed with each cut.**
- **$-8n$ represents the total area removed with n cuts.**
- **n represents the number of cuts.**

7. How are the explicit expressions for the two patterns alike? How are they different?

***Responses vary.* They both have 80 representing the starting area of the paper, but one of them represents repeated multiplication with an exponent and the other represents repeated subtraction with multiplication.**

Activity 3

Name: .... Date: .... Period: ....

More Representations

8. You will use a set of cards to complete this activity. Group the cards based on whether they have a constant difference, a constant ratio, or neither.

Constant Difference	Constant Ratio	Neither
A, C	B, D, E	F

9. Choose one card with a constant difference.

a Create the matching representations. *Responses vary.*

Situation

Kyrie has 120 stickers. Kyrie gives 10 stickers each to n people.

Explicit Expression

$120 - 10n$

Table

# of People, n	# of Remaining Stickers
0	120
1	110
2	100
3	90
...	...
10	20

b What does each part of your expression represent in this situation?

Responses vary.

- 120 represents the number of stickers Kyrie had before giving any away.
- -10 represents the number of stickers Kyrie gave to each person.
- $-10n$ represents the total number of stickers Kyrie gave away.
- n represents the number of people Kyrie gave stickers to.

Name: .... Date: .... Period: ....

More Representations (continued)

10. Choose one card with a constant ratio.

a Create the matching representations. ***Responses vary.***

Situation

A piece of paper starts with an area of 160 square inches. Each cut leaves $\frac{1}{4}$ of the paper remaining for n cuts.

Explicit Expression

$160 \cdot \left(\frac{1}{4}\right)^n$ **(or equivalent)**

Table

# of Cuts, n	**Remaining Paper (sq. in.)**
0	**160**
1	**40**
2	**10**
3	**2.5**
...	...
10	**≈0.00015**

b What does each part of your expression represent in this situation?

Responses vary.

- **160 represents the area of the paper before any cuts were made.**
- **$\frac{1}{4}$ represents the fraction of the paper left after each cut.**
- **$\left(\frac{1}{4}\right)^n$ represents the fraction of the paper remaining.**
- **n represents the number of cuts.**

Synthesis

11. What information from a situation can help you write an explicit expression?

Use the cards if they help with your thinking.

***Responses vary.* It can be helpful to determine the starting value, whether it's increasing or decreasing, and if it has a constant difference or constant ratio.**

Card A

A flag starts 4 feet from the ground. Kyrie raises it 2 feet every second for n seconds.

Card E

Number of Cuts	Paper Remaining (sq. in.)
0	160
1	40
2	10
3	2.5

Things to Remember:

More Representations

Directions: Make one copy per pair of students. Then pre-cut the the cards and give each pair of students one set.

Card A

A flag starts 4 feet from the ground. Kyrie raises it 2 feet every second for n seconds.

Card B

The number of fish in a lake doubles each year for n years. The lake starts with 4 fish.

Card C

$$120 - 10n$$

Card D

$$5 \cdot 3^n$$

Card E

Number of Cuts, n	Paper Remaining (sq. in.)
0	160
1	40
2	10
3	2.5

Card F

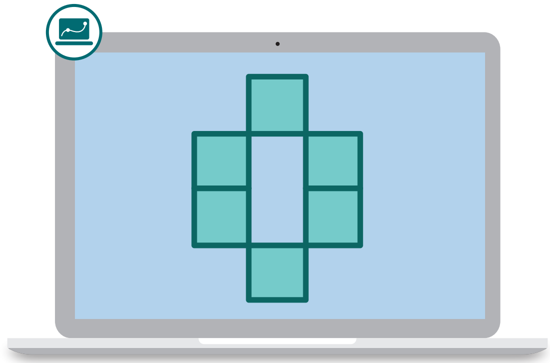
Years, n	Number of Fish
0	15
1	18
2	22
3	27

Unit 1

Lesson

6

Name: Date: Period:



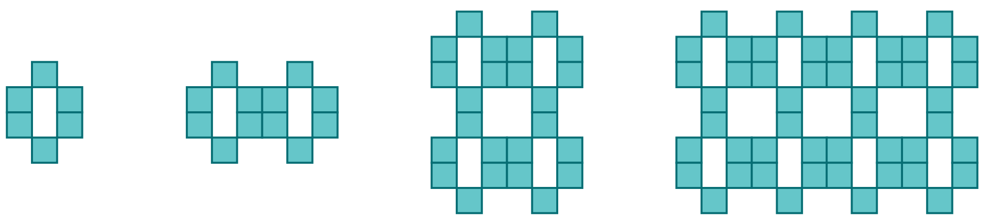
More Visual Patterns

Let's write explicit expressions for arithmetic and geometric sequences.

Warm-up

1 How do you see this pattern growing?

Figure 1 **Figure 2** **Figure 3** **Figure 4**



Responses vary.

- I see the pattern doubling from one figure to the next.
- I see the pattern mirroring. From Figure 1 to 2, it mirrors horizontally. From Figure 2 to 3, it mirrors vertically. From Figure 3 to 4, it mirrors horizontally again.

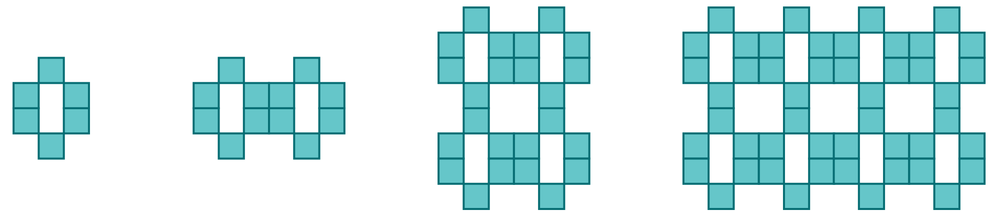
Activity 1

Name: .... Date: .... Period: ....

How Many Tiles?

2 Here is the pattern from the Warm-Up.

Figure 1 **Figure 2** **Figure 3** **Figure 4**



a How many tiles will there be in Figure 7?

384 tiles

b **Discuss** How could you determine the number of tiles in Figure 15?

Responses vary.

- **I could keep multiplying the number of tiles by 2 until I reach Figure 15.**
- **I could figure out how many times I need to multiply by 2 to get from one of the given figures to Figure 15. For instance, to go from Figure 1 to Figure 15, I would need to multiply the 6 tiles by 2 fourteen times, or $6 \cdot 2^{14}$.**

3 Here is Zoe's strategy for determining the number of tiles in Figure 15.

Figure 1 **Figure 2** **Figure 3** **Figure 4**

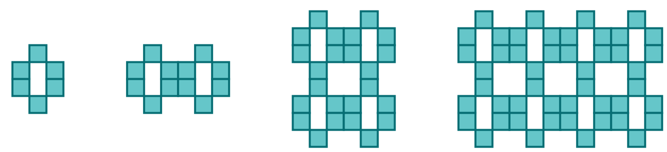


Figure 1	Figure 2	Figure 3	Figure 4
6 tiles	12 tiles	24 tiles	48 tiles
6	6·2	6·2·2	6·2·2·2

Figure 15: $6 \cdot 2^{15}$ tiles

a What did Zoe do well?

***Responses vary.* Zoe identified the rule correctly. She found that the relationship has a constant ratio of 2. She correctly represented Figures 1–4 with expressions.**

b What was Zoe's mistake?

***Responses vary.* Zoe's expression suggests that Figure 15 requires applying the constant ratio (2) fifteen times. Actually, Figure 15 requires applying the constant ratio 14 times.**

4 Amir and Maria want to write an explicit expression for the number of tiles in Figure n.

Amir writes: $6 \cdot 2^{(n-1)}$ Maria writes: $3 \cdot 2^n$

Which explicit expression is correct? Explain your thinking.

A. $6 \cdot 2^{(n-1)}$ **B.** $3 \cdot 2^n$ **C.** Both **D.** Neither

Explanations vary.

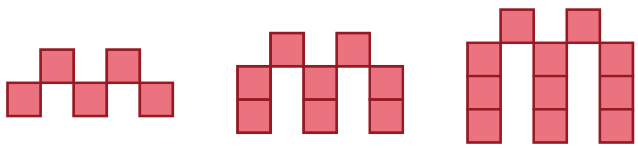
- **These are equivalent expressions. I can tell because both expressions work for all of the given figures. For instance, Figure 3 has 24 tiles. $6 \cdot 2^{(3-1)} = 3 \cdot 2^3 = 24$.**
- **The expressions are equivalent. The exponent tells you how many times to multiply by 2, which means Maria's expression will always multiply by 2 one more time than Amir's expression. But the 6 in Amir's expression makes up for this because $6 = 3 \cdot 2$.**

Activity 2

Name: .... Date: .... Period: ....

A New Pattern

5 Here is a new pattern.



How many tiles will there be in Figure 4 and Figure 15?

Responses shown in table.

Figure	Number of Tiles
1	5
2	8
3	11
4	**14**
...	...
15	**47**

6 Here are Omari's and Ivory's responses for Figure 15.

Figure	Number of Tiles (Omari)	Number of Tiles (Ivory)
15	$5 + 3(14)$	$2 + 3(15)$

Discuss:

- Where did the numbers in Omari's and Ivory's work come from?

  ***Responses vary.* Omari is starting from Figure 1 and applying the constant difference 14 times to get to Figure 15. I think Ivory is imagining a Figure 0 that has 2 tiles, and then the constant difference would be applied 15 times to get to Figure 15.**

- How might each student determine the number of tiles in Figure 50?

  ***Responses vary.* Omari might write $5 + 3(49)$. Ivory might write $2 + 3(50)$.**

7 Write an explicit expression for the number of tiles in Figure n.

Expressions vary.

- $5 + 3(n - 1)$
- $2 + 3n$

Activity 3

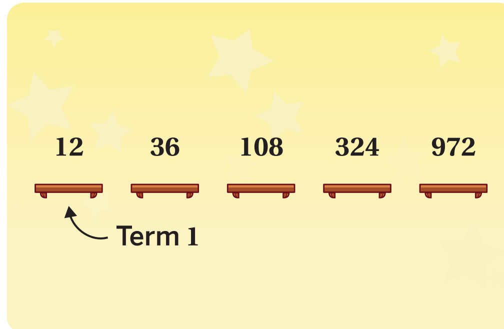
Name: ____________ Date: ________ Period: ________

Sequences Only

8 Let's look at a sequence *without* a visual pattern.

Here are the first five terms of the sequence.

Write an explicit expression for Term n.



Responses vary.

- $12 \cdot 3^{(n-1)}$
- $4 \cdot 3^{n}$

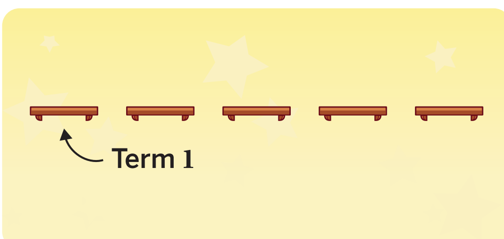
9 You will be designing a challenge for your classmates to solve.

Challenges and responses vary.

a **Make it!**

- Write the first five terms of your arithmetic or geometric sequence.
- Write an explicit expression for Term n of your sequence.

My Challenge | **Explicit Expression**



b **Solve it!**

- Share your sequence with a classmate. Keep your explicit expression a secret!
- Write an explicit expression for Term n of their sequence.

________'s Challenge | **Explicit Expression**

______, ______, ______, ______, ______

________'s Challenge | **Explicit Expression**

______, ______, ______, ______, ______

________'s Challenge | **Explicit Expression**

______, ______, ______, ______, ______

10 Synthesis

Describe one strategy for writing an explicit expression for Term n of a sequence.

27, 34, 41, 48, ...

Term 1

Use the example if it helps with your thinking.

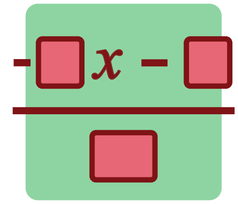
Responses vary.

- First, determine whether the sequence has a constant difference or constant ratio. If it has a constant difference, the explicit expression will be the first term plus the constant difference times $(n-1)$. For instance, an explicit expression for this sequence would be: $27+7(n-1)$. If the sequence has a constant ratio, the explicit expression will be the first term times the constant ratio to the power of $(n-1)$.
- You can think of the expression as two parts: one part is Term 0; the other part represents the change. Determine the constant difference or constant ratio and use it to write a value for Term 0. In this sequence that has a constant difference of 7, Term 0 would be $27-7=20$. To complete the expression, you also need to write the part that represents the change, using n and the constant difference or ratio. For this sequence, $+7n$ represents the change. Altogether, the explicit expression is $20+7n$.

Things to Remember:

Name: .... Date: .... Period: ....

Solving Strategies



Let's explore strategies for solving equations.

Warm-Up

Equivalent equations have the exact same solution.

1. **Discuss:** Which of these equations are equivalent to each other? How do you know?

Equation A $-2(2x + 3) = 10$	Equation B $2x + 3 = -5$
Equation C $2x + 3 = 12$	Equation D $-4x - 6 = 10$

Responses vary.

- **Equations A, B, and D are equivalent. I solved equation A by balancing and got the solution $x = -4$. When I substituted the solution for x in equations B and D, the equations were true. But when I substituted the solution into equation C, the equation was false. That means equation C is not equivalent to the others.**
- **Equations A, B, and D are equivalent because I can use the concept of balance to rearrange equations B and D to look the same as equation A.**

Activity 1

Name: Date: Period:

Equivalent or Not?

For each pair of equations, determine whether they are equivalent.

- If they're equivalent, explain how to get from one equation to the other.
- If they're not equivalent, explain how you know.

2. $5x = 24 + 2x$

$3x = 24$

Are the equations equivalent? **Yes**

Explanation: ***Explanations vary.***

- **The equations are equivalent because if you subtract $2x$ from both sides of the first equation, you get the second equation.**
- **If you solve both equations, you get the same solution: $x = 8$.**

3. $-3(2x + 9) = 12$

$-4 = 2x + 9$

Are the equations equivalent? **Yes**

Explanation: ***Explanations vary.***

- **The equations are equivalent because if you divide both sides of the first equation by -3, you get $2x + 9 = -4$, which is equivalent to the second equation.**
- **If you solve both equations, you get the same solution: $x = -\frac{13}{2}$.**

4. $\frac{1}{2}x - 8 = 9$

$x - 8 = 18$

Are the equations equivalent? **No**

Explanation: ***Explanations vary.***

- **The equations are not equivalent because if you multiply both sides of the first equation by 2 using the distributive property, you get $x - 16 = 18$.**
- **If you solve the equations, you get $x = 34$ for the first equation and $x = 26$ for the second equation.**

5. **a** Write one equation that is equivalent to $12 = 5x - 30 + x$ and one equation that is not.

Responses vary.

b Trade papers with a classmate. Then circle the equation they wrote that is equivalent to $12 = 5x - 30 + x$. Show or explain your thinking on their paper.

Responses vary.

Activity 2

Name: .... Date: .... Period: ....

Step It Up

Here are Sadia's and Amir's steps for correctly solving the same equation.

Sadia

$$6 - 7x = \frac{-15x - 12}{3}$$
$$6 - 7x = -5x - 4$$
$$10 - 7x = -5x$$
$$10 = 2x$$
$$5 = x$$

Amir

$$6 - 7x = \frac{-15x - 12}{3}$$
$$18 - 21x = -15x - 12$$
$$18 - 6x = -12$$
$$-6x = -30$$
$$x = 5$$

6. **Discuss:** How did each student solve the equation?

Responses vary. Sadia chose to simplify the expression $\frac{(-15x - 12)}{3}$ in her first step, and Amir chose to balance the equation by multiplying both sides by 3. From there, they each balanced both sides of the equation: Sadia added 4, then added $7x$, and then divided by 2; Amir added $15x$, then subtracted 18, then divided by -6.

7. How did Amir and Sadia take different first steps but have the same solution?

Responses vary. Every step that Sadia and Amir took, including simplifying and balancing both sides, produced equivalent equations. Equivalent equations have the same exact solution, and all of Sadia's and Amir's equations have the same solution: $x = 5$. This means that even though the details of their solving processes were different, both solving strategies resulted in the same value for x.

8. Caleb and Roberto also tried to solve the equation but made some errors. For each student's work:

a. **Discuss:** What is correct? What is incorrect?

b. Write a question to help each student see how they could revise their work.

Caleb

$$6 - 7x = \frac{-15x - 12}{3}$$
$$-1x = -5x - 4$$
$$4x = -4$$
$$x = -1$$

Roberto

$$6 - 7x = \frac{-15x - 12}{3}$$
$$6 - 7x = -5x - 4$$
$$2 - 7x = -5x$$
$$2 = 2x$$
$$1 = x$$

Responses vary.

a. Caleb and Roberto both correctly wrote an equivalent expression for $\frac{-15x - 12}{3}$: $-5x - 4$. Caleb thought $6 - 7x = -1x$, which is not true because 6 and $-7x$ are not like terms. Roberto subtracted 4 from both sides but that would not get rid of the -4.

b. I could ask Caleb, "Are 6 and $-7x$ like terms? How do you know?" I could ask Roberto, "What inverse operation will move -4 but keep the equation balanced?"

Name: ______ Date: ______ Period: ______

The Choice Is Yours

9. Examine these equations. Organize the equations into two or three groups based on patterns you notice. ***Responses vary.***

Equation A	Equation B	Equation C
$2(2a + 1.5) = 17 - 3a$	$-\frac{1}{2}(b + 3) - 5 = -\frac{7}{2}$	$\frac{-6 + 4c}{2} = 3(2c + 1)$

Equation D	Equation E
$4d + 1 = -2(d - 5)$	$\frac{x}{4} - 2 = 2x + 5$

Equation F	Equation G	Equation H
$9f + 3 - (f - 1) = 2(3f + 1)$	$g - 4 = -\frac{8 + 4g}{8}$	$h + 5h + 20 = h - 6 + h$

Group A	Group B	Group C
Equation A	Equation B	Equation C
Equation D	Equation E	Equation G
Equation F		
Equation H		

10. **Discuss:** How did you group the equations?

Responses vary. Group A contains equations without fractions that need to be distributed. Group B contains equations with fractions that have either a single value or variable in the numerator. Group C contains equations with fractions in which the numerator has two terms.

11. Choose *three* equations to solve. (Choose at least one equation from each group.) Show your thinking. ***Work varies.***

Equation A	Equation B	Equation C	Equation D
$a = 2$	$b = -6$	$c = -\frac{3}{2}$	$d = \frac{3}{2}$

Equation E	Equation F	Equation G	Equation H
$x = -4$	$f = -1$	$g = 2$	$h = -\frac{13}{2}$

Explore More

12. Two of the equations in this activity are equivalent. Identify the two equivalent equations and explain your thinking.

Equation A and Equation G. *Explanations vary.* They both have 2 as their solutions.

Synthesis

13. **a** Write an equation you think is challenging to solve.

Responses vary.

$$\frac{-6 + 4c}{2} = -3(2c + 1)$$

b What makes your equation challenging to solve?

Responses vary. **This equation is challenging to solve because it has negatives, a fraction, and you need to use the distributive property.**

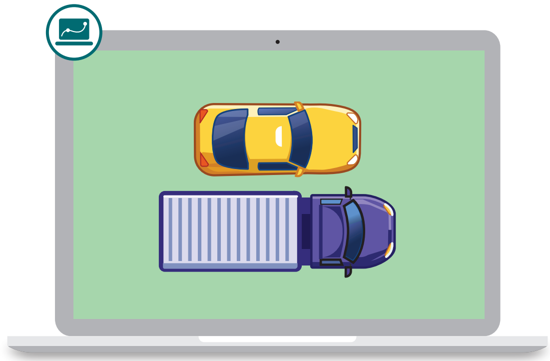
c What are some strategies or tips for solving equations like this?

Responses vary. **Just work on one step at a time. On the left, you can divide each number in the numerator by the denominator to eliminate the fractions. On the right, you can distribute, and then you have an easier problem.**

Things to Remember:

Unit 2
Lesson
3

Name: .... Date: .... Period: ....



Same Position

Let's explore how many solutions are possible for a one-variable equation.

Warm-Up

1 Let's watch the animation.

Write a story about what you see.

Responses vary. There is a taxi, a car, and a truck on a road. The car and the truck are rolling along at the same speed. The taxi is going so fast! It passes the truck and then ends up in the same position as the car. Maybe it will get a speeding ticket.

Activity 1

Name: ______ Date: ______ Period: ______

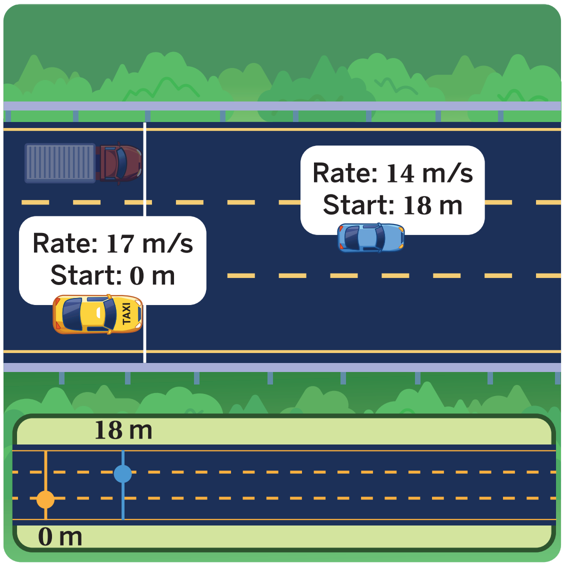
Same Position

2 The car and the taxi are moving at constant speeds.

Car Position Expression	Taxi Position Expression
$14t + 18$	$17t$

At what time, t, will the car and the taxi be in the same position?

6 seconds



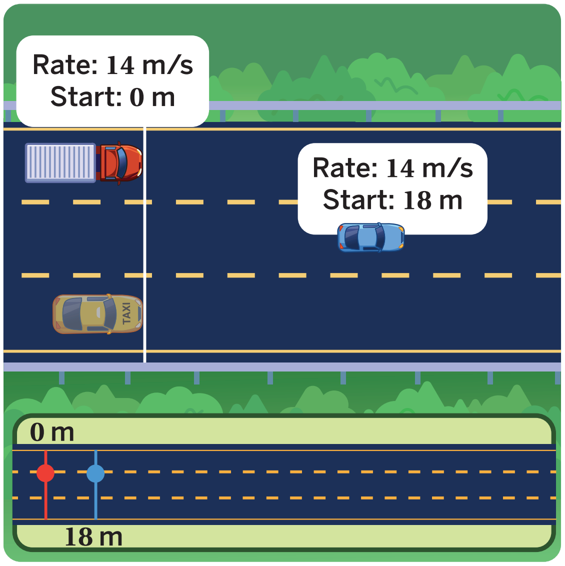
3 The truck and the car are moving at constant speeds.

Truck Position Expression	Car Position Expression
$14t$	$14t + 18$

a Let's see what happens at different times.

b **Discuss:** What do you know about when the truck and the car will be in the same position?

Responses vary. **The truck and the car will never be in the same position because they are going at the same rate. I can see this in their expressions, which both include $14t$.**



4 Here is Antwon's work on the previous problem.

What does his work say about the time, t, when the truck and the car will be in the same position?

Responses vary. **Antwon got an equation that doesn't make sense. $0 = 18$ will never be true, which means the car and the truck will never meet.**

Antwon

$$\begin{array}{rcl} 14t & = & 14t + 18 \\ -14t & & -14t \\ \hline 0 & = & 0 + 18 \end{array}$$

Name: .... Date: .... Period: ....

Same Position (continued)

5 The following equations represent when these vehicles will be in the same position.

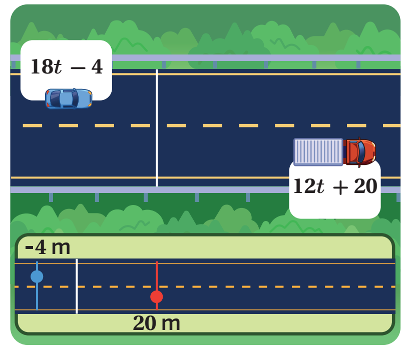
a $18t - 4 = 12t + 20$

How often will they be in the same position? Circle one.

Once Never Always

Once

If once, then after how many seconds? 4



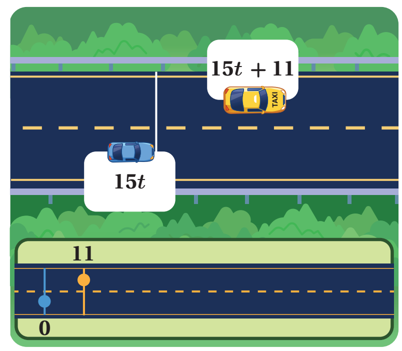
b $15t + 11 = 15t$

How often will they be in the same position? Circle one.

Once Never Always

Never

If once, then after how many seconds? ....



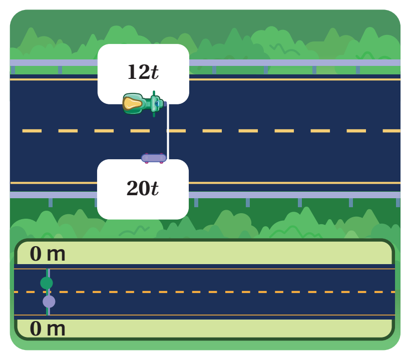
c $12t = 20t$

How often will they be in the same position? Circle one.

Once Never Always

Once

If once, then after how many seconds? 0



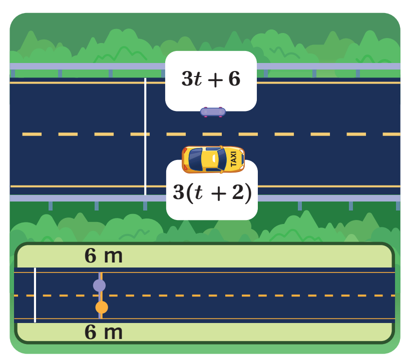
d $3t + 6 = 3(t + 2)$

How often will they be in the same position? Circle one.

Once Never Always

Always

If once, then after how many seconds? ....



Activity 2

Name: ______ Date: ______ Period: ______

Once, Never, Always

6 Here are Jazz's and Nikhil's strategies for solving a challenge from the previous activity.

Jazz

$$\begin{aligned} 12t &= 20t \\ -12t \quad & \;\; -12t \\ \hline 0 &= 8t \\ 0 &= t \end{aligned}$$

they will meet when 0 = t

Nikhil

$$\frac{12t}{t} = \frac{20t}{t}$$

$$12 = 20$$

they will never meet

Discuss: Is each strategy correct?

Jazz's strategy is correct. Nikhil's strategy is not correct. *Explanations vary*. Jazz's strategy is correct because when I plugged 0 back in for t, the equation was true. Nikhil's strategy is not correct because 12 is not equal to 20. Something went wrong when he divided both sides by t.

7 Each equation represents the time, t, when two vehicles will meet.

$12 - t = t - 12$	$t + 1 = t + 1$	$t = t + 2$
$2t + 6 = 2(t + 3)$	$2t = 8t$	$8(t + 1) = 8t - 8$

Sort the six equations based on how often the vehicles will be in the same position.

Once	Never	Always
$12 - t = t - 12$ $2t = 8t$	$t = t + 2$ $8(t + 1) = 8t - 8$	$t + 1 = t + 1$ $2t + 6 = 2(t + 3)$

Activity 2

Name: .... Date: .... Period: ....

Once, Never, Always (continued)

8 Darryl and Jasmine solved $t + 1 = t + 1$ and got $0 = 0$.

- Darryl says the vehicles will never be in the same position.
- Jasmine says the vehicles will always be in the same position.

Who is correct? Explain your thinking.

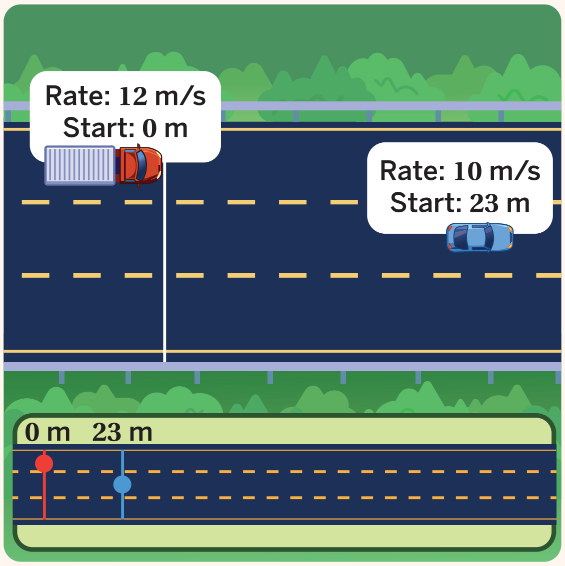
Jasmine. *Explanations vary*. No matter what value for t you use, the equation will always be true. This means the cars will always be in the same position.

Explore More

9 The truck and the car are moving at constant speeds.

Write an expression, in terms of t, for the position of a taxi that makes *both* statements true:

- The taxi and the truck will *never* be in the same position.
- The taxi and the car will be in the same position when $t = 8$.



Vehicle	Position Expression
Truck	$12t$
Car	$10t + 23$
Taxi	$\mathbf{12t + 7}$

10 Synthesis

How can you tell whether an equation will have:

- No solution?

  Responses vary. An equation will have no solution if I get an untrue statement after solving it. For example, $t = t + 2$ is $0 = 2$, which isn't ever true.

- Infinitely many solutions?

  Responses vary. An equation will have infinitely many solutions if the left and right side of the equation are equivalent. For example, $2t + 6$ and $2(t + 3)$ are equivalent, so no matter what value of t I use, the equation will always be true.

$t + 1 = t + 1$

$8(t + 1) = 8t - 8$

$2t = 8t$

$t = t + 2$

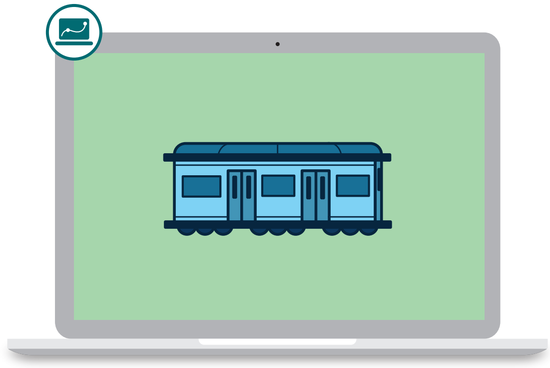
$12 - t = t - 12$

$2t + 6 = 2(t + 3)$

Things to Remember:

Unit 2
Lesson
4

Name: .... Date: .... Period: ....



Subway Seats

Let's explore what different forms of linear equations reveal about a situation.

Warm-Up

1 Which one doesn't belong?

Equation A	Equation B	Equation C	Equation D
$x + y = 5$	$x + y - 5 = 0$	$x = 5 - y$	$5 + x = y$

Explain your thinking.

Responses and explanations vary.

- Equation A doesn't belong because it's the only equation that has a number on one side and no numbers on the other side.
- Equation B doesn't belong because it's the only equation that is equal to 0.
- Equation C doesn't belong because it's the only equation with a negative y.
- Equation D doesn't belong because it's the only equation where y is by itself.

Activity 1

Name: .... Date: .... Period: ....

Crowded Subways

2 Some subway cars can be crowded.

In order to fix this, the transit authority decided to remove seats to fit more people.

Some residents feel that this isn't fair. Why might they feel this way?

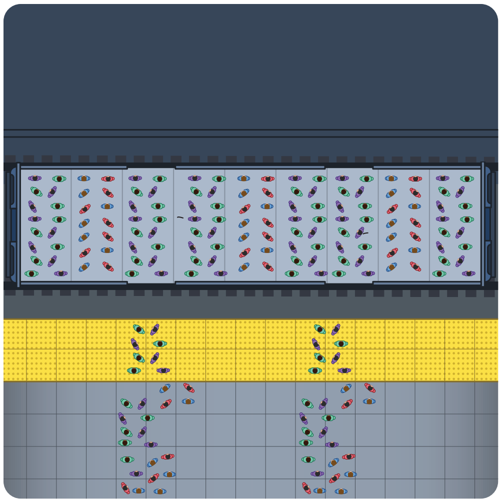
***Responses vary.* Not all people can stand, especially for long periods of time.**
For example, some people with disabilities, people with injuries, some senior citizens, or some young kids might not be able to use a subway without seats.

3 Some subways removed all of the seats to allow more room for people to stand.

- A subway car has about 600 square feet.
- A standing passenger requires 2 square feet.

What is the *standing capacity* on this subway car with no seats?

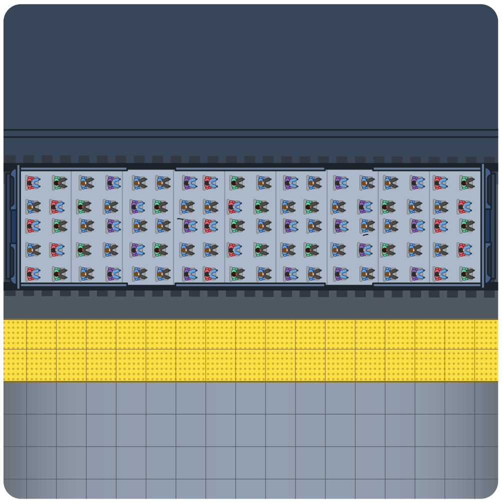
300 people



4
- A subway car has 600 square feet of floor space.
- A seat requires 6 square feet.

What is the *seating capacity* on this subway car with no room to stand?

100 people

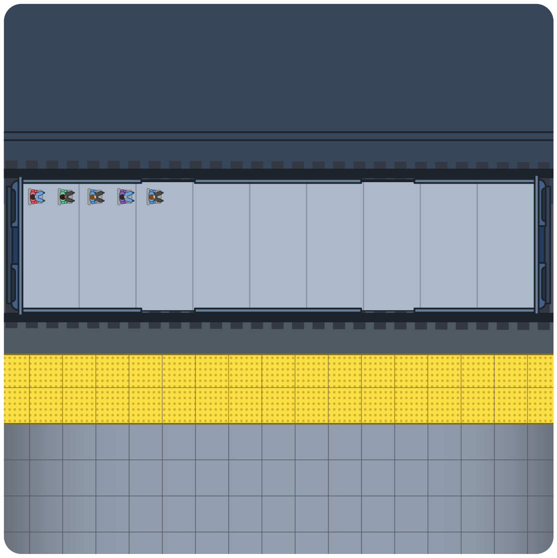


Name: ______ Date: ______ Period: ______

Crowded Subways (continued)

5 The capacity of this subway car is modeled by $6t + 2d = 600$, where t is the seating capacity and d is the standing capacity.

For each number of seats, determine how many standing passengers can fit.



Seating Capacity, t	Standing Capacity, d
5	**285**
10	**270**
15	**255**

6 Here is Tiam's strategy for determining the number of standing passengers that can fit when you know the number of seats.

- t is the seating capacity.
- d is the standing capacity.

Tiam

$$6t + 2d = 600$$
$$-6t \quad\quad -6t$$
$$\frac{2d}{2} = \frac{600 - 6t}{2}$$
$$d = 300 - 3t$$

What do 300 and -3 mean in this situation?

Responses vary.

300: **300 people can stand in a subway car when there are no seats.**

-3: **When a seat is added to the subway car, 3 fewer people can fit.**

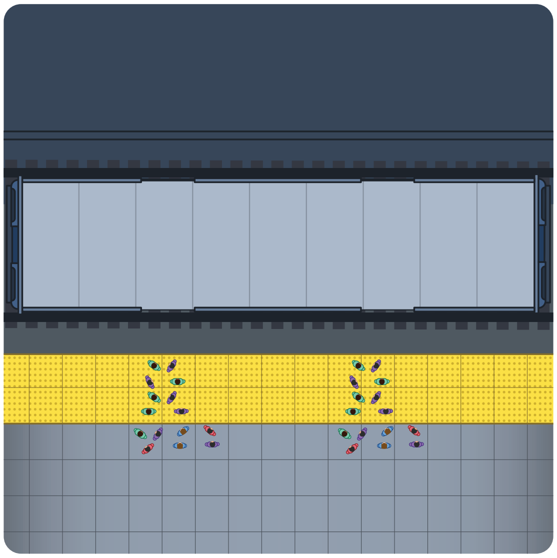
Activity 2

Name: ______ Date: ______ Period: ______

Standing and Sitting

7 The capacity of this subway car is modeled by $6t + 2d = 600$, where t is the seating capacity and d is the standing capacity.

For each number of standing passengers, determine the seating capacity.



Seating Capacity, t	Standing Capacity, d
90	30
50	150
20	240

8 Solve for t so that the transit authority can calculate the seating capacity for any standing capacity.

$t = \frac{600 - 2d}{6}$ **(or equivalent)**

9 Group the equations that represent the same relationship. One equation will have no match.

$x = \frac{50 - 3y}{2}$ $\quad x = \frac{50 - 2y}{3}$ $\quad y = \frac{50 - 3x}{2}$ $\quad y = \frac{50 - 2x}{2}$

$2x + 3y = 50$	$3x + 2y = 50$
$x = \frac{50 - 3y}{2}$	$x = \frac{50 - 2y}{3}$ $y = \frac{50 - 3x}{2}$

Equation with no match: $y = \frac{50 - 2x}{2}$

10 Synthesis

Here are two equations that we considered in this lesson about subway capacity.

$$6t + 2d = 600$$
$$t = 100 - \frac{1}{3}d$$

- t is the seating capacity.
- d is the standing capacity.

Pick *two* numbers and explain what they mean in this situation. ***Responses vary.***

6: **Each seated passenger takes up 6 square feet of floor space.**

2: **Each standing passenger takes up 2 square feet of floor space.**

600: **The total amount of floor space in a subway car is 600 square feet.**

100: **100 seated passengers can fit in a subway car when there are only seats.**

$-\frac{1}{3}$: **When a seat is removed from the subway car, 3 more people can fit.**

Things to Remember:

Unit 2
Lesson
6

Name: Date: Period:

Shelley the Snail

Let's connect graphs, tables, and equations to the situations they represent.

Warm-Up

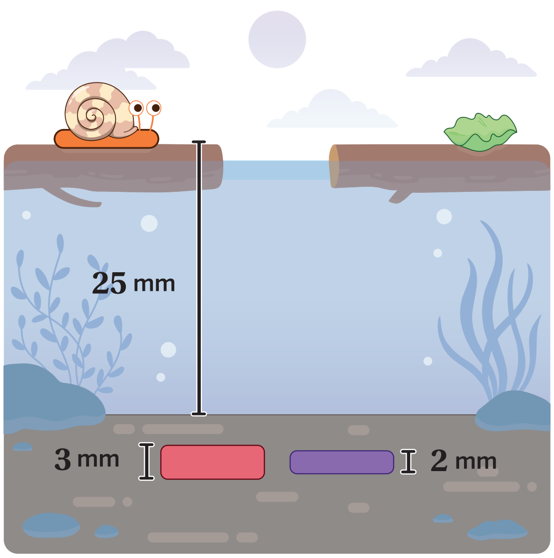
1 Shelley the Snail loves eating lettuce leaves.

She needs to cross the gap to get them.

Write different combinations of blocks that fill the gap.

Responses vary.

Number of 3 mm Blocks	Number of 2 mm Blocks
5	5
7	2
1	11



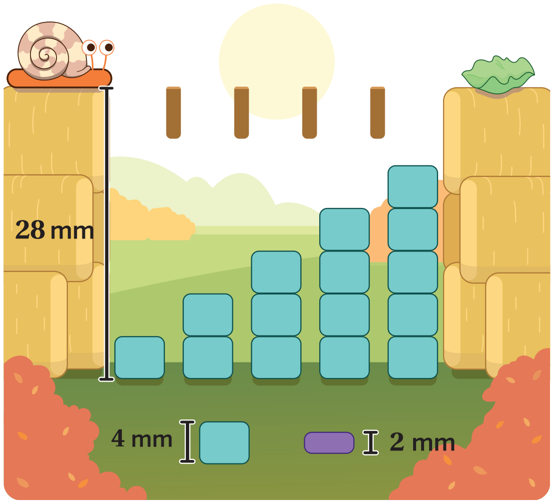
Activity 1

Name: .... Date: .... Period: ....

Mind the Gap

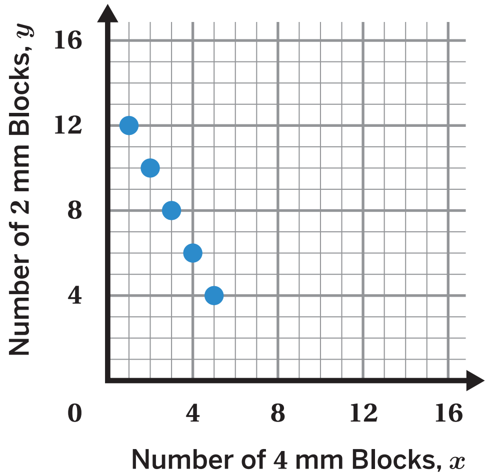
2 Complete the table to help Shelley get to the lettuce.

Number of 4 mm Blocks, x	Number of 2 mm Blocks, y
1	12
2	10
3	8
4	6
5	4



3 Shelley has to cross a gap that is 28 mm deep using 4 mm and 2 mm blocks. This graph shows some of the possible combinations of blocks. Complete the table.

Number of 4 mm Blocks, x	Number of 2 mm Blocks, y
0	14
6	2
7	0



4 Here is Tyrone's work from the previous problem. Write an equation for the number of 2 mm blocks, y, needed for any number of 4 mm blocks, x.

$y = 14 - 2x$ (or equivalent)

Number of 4 mm Blocks, x	Number of 2 mm Blocks, y
0	14 — 2(0)
6	14 — 2(6)
7	14 — 2(7)
x	?

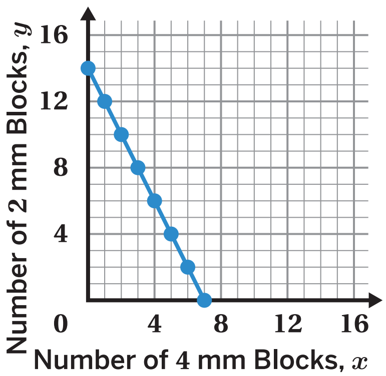
Activity 1

Name: ______ Date: ______ Period: ______

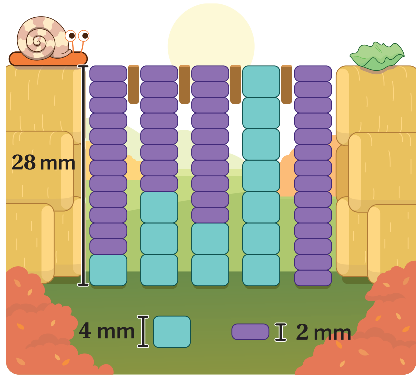
Mind the Gap (continued)

5 The x-*intercept* of the graph is $(7, 0)$. The y-*intercept* of the graph is $(0, 14)$.

Graph



Situation



Equation

$y = 14 - 2x$

a) Select *one* representation.

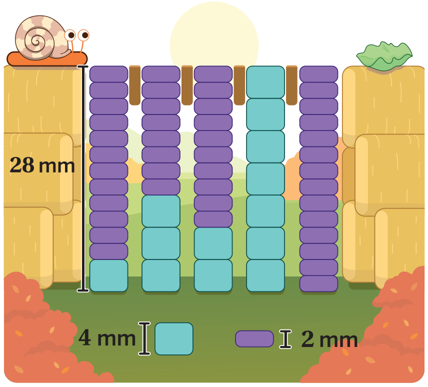
b) Show or explain where you see the intercepts.

x-intercept: ***Responses vary.***
- **Graph: I can see the x-intercept where the graph crosses the x-axis.**
- **Situation: I can see the x-intercept in the stack that only has 4 mm blocks.**
- **Equation: I can calculate the x-intercept when I input 0 for y and solve for x.**

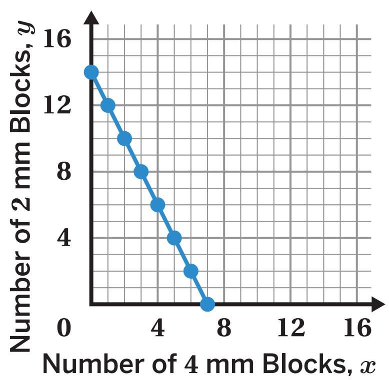
y-intercept: ***Responses vary.***
- **Graph: I can see the y-intercept where the graph crosses the y-axis.**
- **Situation: I can see the y-intercept in the stack that only has 2 mm blocks.**
- **Equation: I can calculate the y-intercept when I input 0 for x and solve for y. I can also see it as the constant in the equation.**

6 Here is the same relationship represented in two different ways.

$4x + 2y = 28$



$y = 14 - 2x$



Discuss:
- How do you see the equation in each representation?
- Are these equations equivalent? Why or why not?

Responses vary.
- **In the diagram, there are different combinations of 4 mm blocks and 2 mm blocks that total 28 millimeters. In the graph, the y-intercept is at $(0, 14)$, and as the number of 4 mm blocks increases by 1, the number of 2 mm blocks decreases by 2.**
- **These equations are equivalent because they represent the same situation.**

Activity
2

Name: ____________ Date: ________ Period: ________

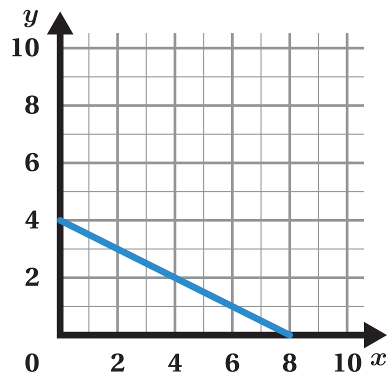
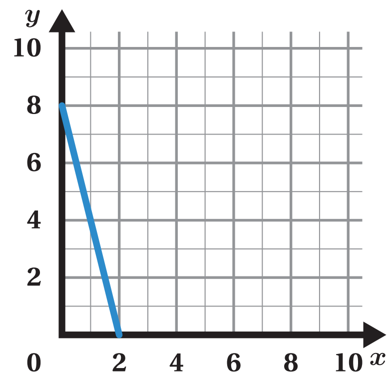
Rearrange It

7 Solve the equation $4x + 2y = 28$ for y to show that it is equivalent to $y = 14 - 2x$. Show or explain your thinking. ***Responses vary.***

$$4x + 2y = 28$$
$$2y = 28 - 4x$$
$$y = \frac{28 - 4x}{2}$$
$$y = 14 - 2x$$

8 Match each graph with two equations. Two equations will have no match.

$2x + 8y = 24$	$y = 8 - 2x$	$2x + 4y = 16$
$y = 4 - \frac{1}{2}x$	$8x + 2y = 16$	$y = 8 - 4x$

Graph		
Equations	$2x + 4y = 16$ $y = 4 - \frac{1}{2}x$	$8x + 2y = 16$ $y = 8 - 4x$

9 Rearrange each equation to solve for y.

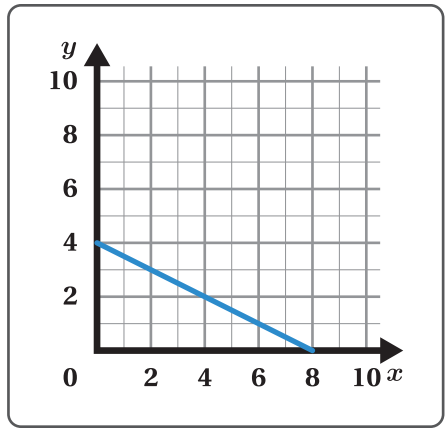
$6x + 2y = 34$ $y =$ **$17 - 3x$ (or equivalent)**	$5x + 2y = 46$ $y =$ **$23 - \frac{5}{2}x$ (or equivalent)**
$2x + 4y = 26$ $y =$ **$\frac{26}{4} - \frac{2}{4}x$ (or equivalent)**	$3x + 4y = 40$ $y =$ **$10 - \frac{3}{4}x$ (or equivalent)**

10 Synthesis

How would you convince a classmate that these three cards represent the same situation?

$2x + 4y = 16$

$y = 4 - \frac{1}{2}x$



Responses vary. **I can see that the graph has a y-intercept of $(0, 4)$ and a slope of $-\frac{1}{2}$, which matches the equation $y = 4 - \frac{1}{2}x$. I can solve the equation $2x + 4y = 16$ for y to show that it's equivalent to the other equation.**

Things to Remember:

Unit 2
Lesson
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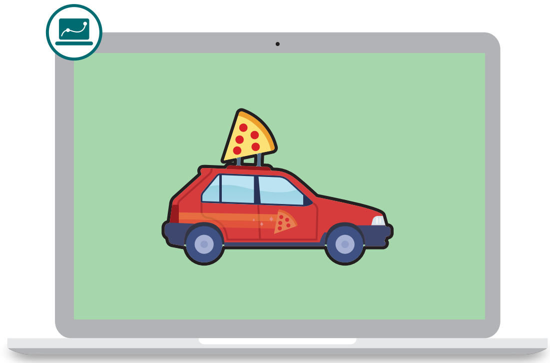
Name: ______ Date: ______ Period: ______



Pizza Delivery

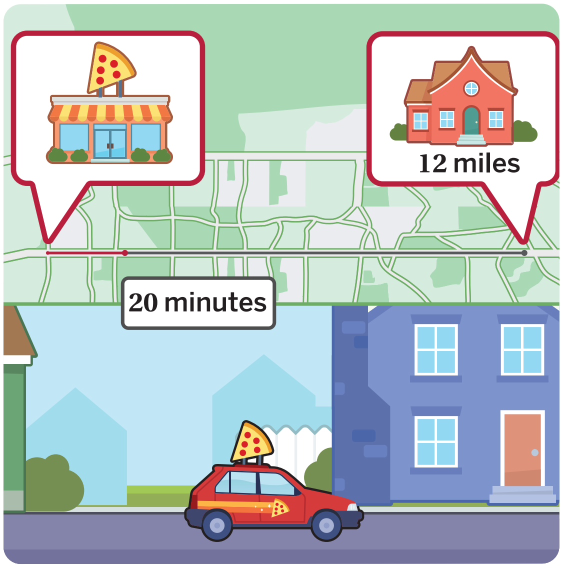
Let's write inequalities to represent constraints.

Warm-Up

1 Let's watch an animation.

Write a story about what you see.

Responses vary. My friend and I ordered a pepperoni pizza. It took them about 15 minutes to prepare our order, but driving the 12 miles to my house took forever! The pizza finally arrived 51 minutes after I put in the order.



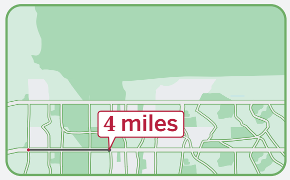
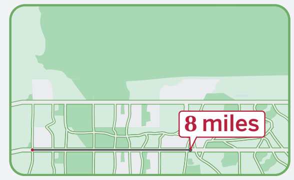
Name: ____________ Date: ________ Period: ________

Order Up!

2 Desmos Pizza is now offering delivery!

It takes 15 minutes to prepare an order and 3 minutes to drive each mile.

How long would it take to deliver each order?

Distance (mi)			
Time (min)	21	27	39

3 Here is Mariana's work from the previous problem.

Write an expression for the number of minutes it would take to deliver a pizza x miles away.

$15 + 3x$ (or equivalent)

Distance (mi)	Time (min)
2	15 + 6
4	15 + 12
8	15 + 3(8)
x	?

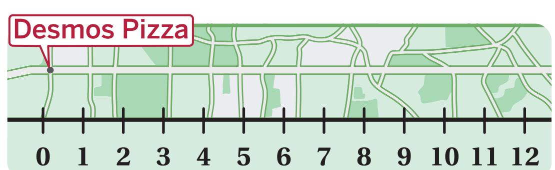
4 Desmos Pizza wants to deliver orders in 30 minutes or less.

a Which inequality represents this situation?

A. $15 + 3x \leq 30$ B. $15 + 3x \geq 30$ C. $15 + 3x > 30$ D. $15 + 3x < 30$

b Show or describe all the distances they can deliver to in 30 minutes or less.

Responses showing or describing 5 miles or less are considered correct.



Name: ______ Date: ______ Period: ______

Order Up! (continued)

5 Here is how Mariana and Camila figured out all of the distances in which pizza could be delivered in 30 minutes or less.

Mariana

Miles, x	Minutes
1	18
2	21
3	24
4	27
5	30
6	33

$x \le 5$

You have to Live 5 miles away or Less.

Camila

$15 + 3x = 30$

$-15 \qquad -15$

$3x = 15$

$x = 5$

5 miles is the farthest you can live from Desmos Pizza.

Discuss: Why is each strategy helpful? Identify a weakness of each strategy.

Responses vary.

Mariana
- It was helpful to see how distances less than 5 miles could have pizza delivered under 30 minutes, but 6 miles took longer than 30 minutes.
- This strategy worked, but it looks like it would take a long time to get to an answer.

Camila
- The equation was helpful because it could be solved in only two steps.
- This strategy only answered what the *farthest* distance is that you can live from Desmos Pizza.

6 A large pizza is \$12 and toppings are \$2 each. Mariana can spend as much as \$20 on a pizza.

a If x is the number of toppings, which of these inequalities represents the situation?

A. $12 + 2x \ge 20$ B. $12x + 2 \ge 20$ C. $12 + 2 \le 20x$ D. $12 + 2x \le 20$

b Show or describe all the numbers of toppings, x, that Mariana can get for \$20 or less.

Responses vary.
- Mariana can get 0, 1, 2, 3, or 4 toppings.
- Mariana can get 4 toppings or less. The pizza place might also allow her to get a $\frac{1}{2}$ order of a topping.

7 A **constraint** is a limitation on possible values in a model. Here are some examples of constraints:
- Desmos Pizza wants to deliver in 30 minutes or less.
- Mariana can spend as much as \$20 on a pizza.
- Desmos Pizza wants to sell more than 50 pizzas per day.

What is a constraint in *your* life?

Responses vary.
- I'm allowed to play a maximum of 5 hours of video games each week.
- I have to do at least 2 chores every day.
- I have to practice clarinet for at least 20 minutes per day.
- I need to be 16 years old or older to get my driver's license.

Activity 2

Name: ______ Date: ______ Period: ______

Trampoline World

8 Jamir is planning to host a party at Trampoline World.

Discuss: What constraints might Jamir think about when planning this party?

Responses vary.

- How many people are invited.
- How many people fit in each room.
- How long the party will last.
- How much the party will cost.

9 Hosting a party at Trampoline World costs a flat fee of \$80, plus \$30 per hour for the small room or \$50 per hour for the large room.

Match each constraint to an inequality, where x represents the number of hours for the party. One inequality will have no match.

$30 + 80x \geq 140$	$80 + 30x \leq 140$	$80 + 50x \geq 140$	$80 + 80x \leq 140$

Mariana's party in the small room costs at most $140.	The owner wants to earn at least $140 for a party in the large room.	Amoli can spend up to $140 for a party that uses both the big and small rooms.
$80 + 30x \leq 140$	$80 + 50x \geq 140$	$80 + 80x \leq 140$

Name: .... Date: .... Period: ....

Trampoline World (continued)

10 Jamir can spend as much as \$155 for a party in the large room.

Write an inequality to match this new constraint, where x represents the number of hours for the party.

$80 + 50x \leq 155$ (or equivalent)

11 Select *all* the possible numbers of hours, x, that could work with Jamir's constraints.

- [x] **A.** 0.75 hours
- [x] **B.** 1 hour
- [x] **C.** 1.5 hours
- [ ] **D.** 2.25 hours
- [ ] **E.** 3 hours

10 Synthesis

What are some suggestions you have for writing a constraint as an inequality?

Responses vary.

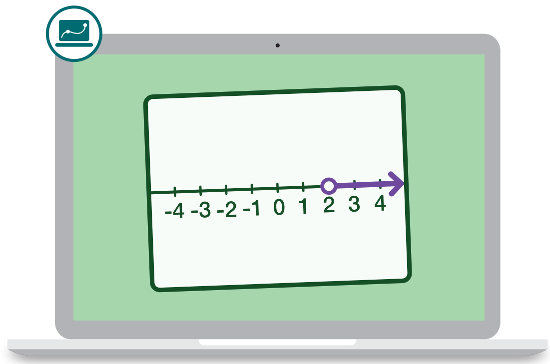
- Identify keywords, like *less*, *more*, and *at least*, that can tell you which inequality to use.
- The variable represents the unknown value or the value that changes.
- Reread your inequality to make sure it makes sense.

Things to Remember:

Name: ____________ Date: ________ Period: ________

Graphing Inequalities

Let's represent solutions to inequalities on a number line.

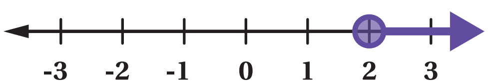


Warm-Up

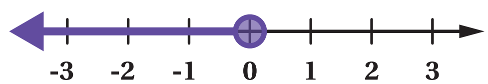
1 Here are four different inequalities.

Discuss: What do you notice?

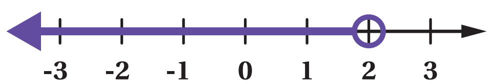
$x \geq 2$



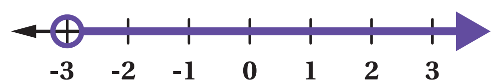
$x \leq 0$



$x < 2$



$x > -3$

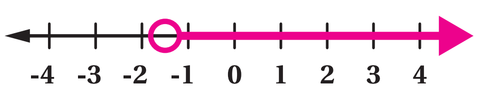


Responses vary.

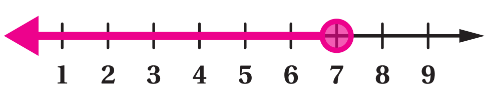
- I notice that when the circle is filled in, the symbol is $\leq$ or $\geq$, and when the circle is empty, the symbol is $<$ or $>$.
- I notice that when the arrow points to the right, the inequality says $x \geq$ or $>$, and when the arrow points to the left, the inequality says $x \leq$ or $<$.

2 Make a graph of *all* the solutions to each inequality.

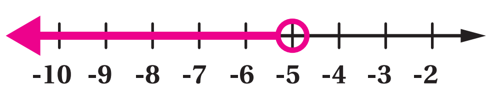
$x > -1.5$



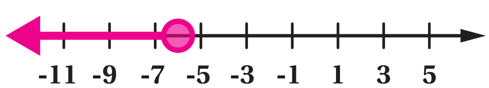
$x \leq 7$



$x < -5$



$x \leq -6$



Activity 1

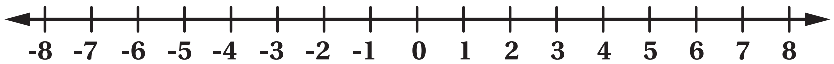
Name: .... Date: .... Period: ....

Show a Solution

3 **a** Plot a solution to this inequality.

Responses greater than 4 are considered correct.

$$2x - 8 > 0$$



Share your response with your classmates.

b **Discuss:** Are any of the points incorrect? How do you know?

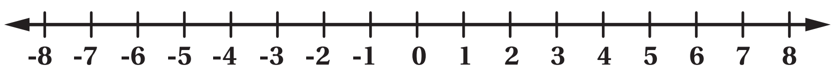
Responses vary.

- **No. All of the points are numbers greater than 4.**
- **Yes. I plugged in 4 and $2(4) - 8$ is equal to 0, not greater than 0.**

4 **a** Plot a solution to this inequality.

Responses less than or equal to 0 are considered correct.

$$6x \leq 3x$$



b Explain how you know that your point is a solution (or why there is no solution).

Explanations vary.

- **I know -5 is a solution because when I substitute -5 into the inequality for x, I get a true statement.**
- **0 times anything is 0, so since the inequality symbol is $\leq$, 0 is a solution.**

5 Lan explains that $6x \leq 3x$ does not have a solution: *6 of something is always more than 3 of the same thing*.

Is Lan's statement correct? Circle one.

Yes No I'm not sure

Show or explain your thinking.

Explanations vary.

- **You can make big numbers less than little numbers if they're multiplied by a negative.**
- **6 of 0 is equal to 3 of 0.**

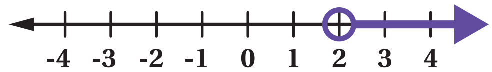
Activity 2

Name: .... Date: .... Period: ....

Solution Sets

6 How can you check that this inequality and graph represent the solutions to $\frac{1}{4}x > \frac{1}{2}$?

$x > 2$

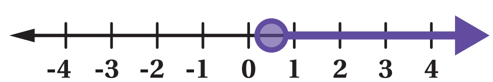


Responses vary.

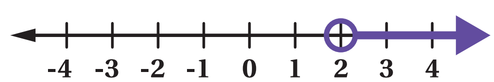
- **Values to the right of 2 make the inequality true, and values to the left of 2 make the inequality false. 2 also makes it false, which is why the circle is not filled in.**
- **The circle is not filled in on the graph, and the inequality symbol doesn't include the equal sign. Substituting 2 for x in the original inequality shows that 2 is the correct boundary point because $\frac{1}{4}(2) = \frac{1}{2}$.**
- **If I multiply everything in the inequality $\frac{1}{4}x > \frac{1}{2}$ by 4, I get $x > 2$.**

7 Match each inequality to a number line that represents its solutions.

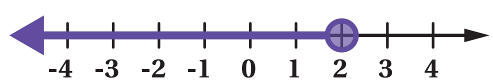
a. $5x + 4 \geq 7x$ C



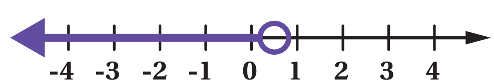
b. $3 - x < 1$ ..... b



c. $2(x + 3) \geq 7$ ..... a



d. $8x - 2 < 4x$ ..... d

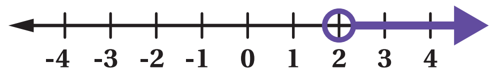


Name: Date: Period:

Solution Sets (continued)

8 Lan matched this inequality and number line.

$$5x + 4 \geq 7x$$

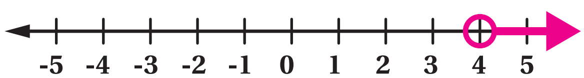


How could you convince Lan that this inequality and number line don't match?

Responses vary.

- **The inequality symbol is $\geq$, so the circle should be filled in. The circle on the graph is not filled in.**
- **The graph says that 3 is a solution, but when I test 3 in the inequality, I get $19 \geq 21$ which is a false statement.**

9 Create a graph of the solutions to the inequality $15 - x < 11$.



Explain your thinking.

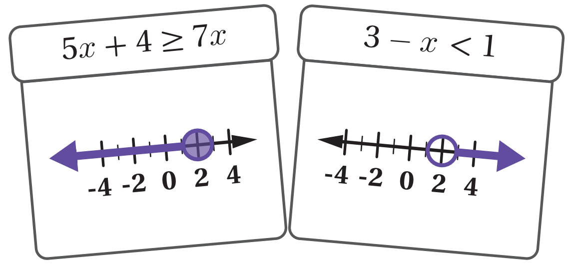
Explanations vary.

- **$15 - 4 = 11$, but 4 is not included in the solution since the symbol is $<$. $15 - 5 = 10$, so 5 is included in the solution. $15 - 3 = 12$, so 3 is not included in the solution.**
- **If I start at 15 and need to get below 11, I need to drop more than 4.**

10 Synthesis

Explain how you can determine whether a number line represents the solutions to an inequality.

Use these examples if they help with your thinking.



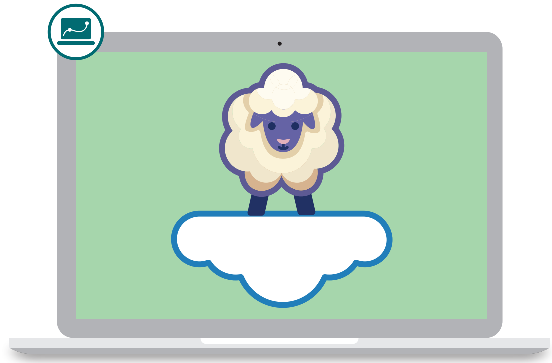
Explanations vary.

- **If the inequality includes the equal sign, the boundary point should be filled in. If the inequality doesn't include the equal sign, the boundary point should be open.**
- **I can find the location of the boundary point by pretending that the inequality is an equation and finding the value of the variable that makes the equation true.**
- **Think carefully about what kinds of numbers will make the inequality true. For example, to make $3 - x < 1$ true, I need to subtract enough from 3. 2 isn't quite enough, but anything bigger than 2 will work.**

Things to Remember:

Unit 2
Lesson
10

Name: ______________________ Date: __________ Period: __________



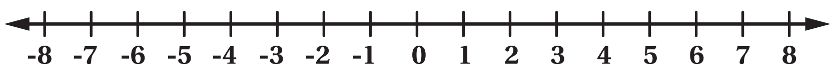
Solutions and Sheep

Let's make connections between solving one-variable equations and solving inequalities.

Warm-Up

1 **a** Plot *three* different solutions to the inequality $10 - 5x < 0$.

Any number greater than 2 is considered correct.



Share your response with your classmates.

b **Discuss:** Are any of the points incorrect? How do you know?

***Responses vary.* $x = 0$ is incorrect because when I plug it into the inequality, I get $10 < 0$, which is false.**

2 Let's look at all the correct solutions to $10 - 5x < 0$.

Kayleen and Leo are discussing how to write the solutions to this inequality.

Kayleen says the solutions are $x < 2$.
Leo says the solutions are $x > 2$.

Whose claim is correct? Circle one.

Kayleen's Leo's Both Neither

Explain your thinking.

***Explanations vary.* All of the blue points on the graph are on numbers that are greater than 2. Leo's inequality says "x is greater than 2," so it matches the solutions.**

Activity 1

Name: .... Date: .... Period: ....

Feed the Sheep

3 Shira the Sheep loves eating grass. She does *not* like water.

a Let's watch what happens when we try out different inequalities.

b **Discuss:** What do you notice?

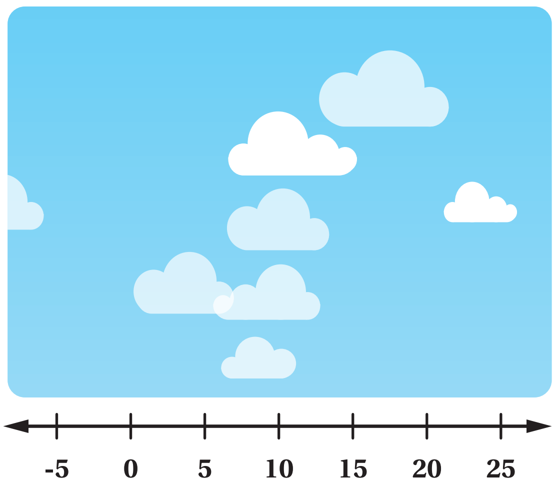
Responses vary.

- **Shira is happy when she eats all of the grass.**
- **When Shira eats just some of the grass, she doesn't get wet but is still unhappy.**

4 Here is an inequality: $\frac{x}{3} \geq 5$.

Solve the inequality to help Shira eat all the grass.

$x \geq 15$ or $15 \leq x$



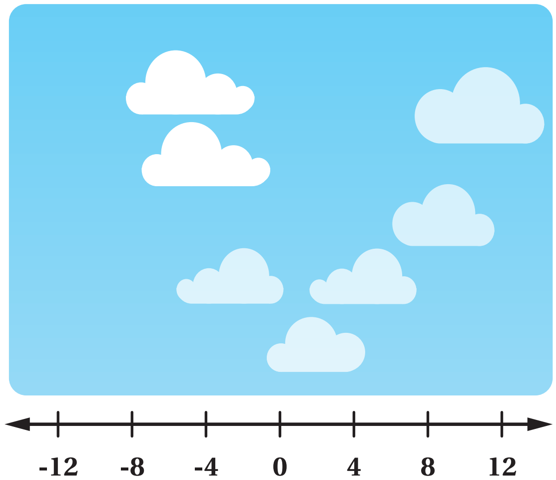
Name: ______ Date: ______ Period: ______

Feed the Sheep (continued)

5 Here is a new inequality: $9 > 2x - 4$.

Solve the inequality to help Shira eat all the grass.

$x < 6.5$ or $6.5 > x$



6 Here is some of Kayleen's work from the previous problem.

$\boxed{9 > 2x - 4}$

$9 = 2x - 4$

$13 = 2x$

$6.5 = x$

Test

$x = 6$	$x = 7$
$9 > 2(6) - 4$	$9 > 2(7) - 4$
$9 > 8$	$9 > 10$
	False!

a **Discuss:** What do you notice and wonder about Kayleen's strategy?

Responses vary.

- **I notice that Kayleen turned the inequality into an equation.**
- **I notice that Kayleen solved the equation.**
- **I notice that Kayleen tested two points in the original inequality.**
- **I wonder why Kayleen chose to test those two points.**
- **I wonder if you always need to test two points.**
- **I wonder if Kayleen shaded to the left or right of $x = 6.5$.**

b Describe how Kayleen's work can help her decide where to shade the solutions on the number line.

Responses vary. **Kayleen tested a value on either side of the boundary point. She should shade from the boundary point toward the value that made the inequality true because that's where all the solutions will be.**

Activity 2

Name: .... Date: .... Period: ....

Solving for Sheep

7 Here is a new inequality: $8x - 4 < 10x + 2$.

Solve the inequality to help Shira eat all the grass.

-3 < x or x > -3

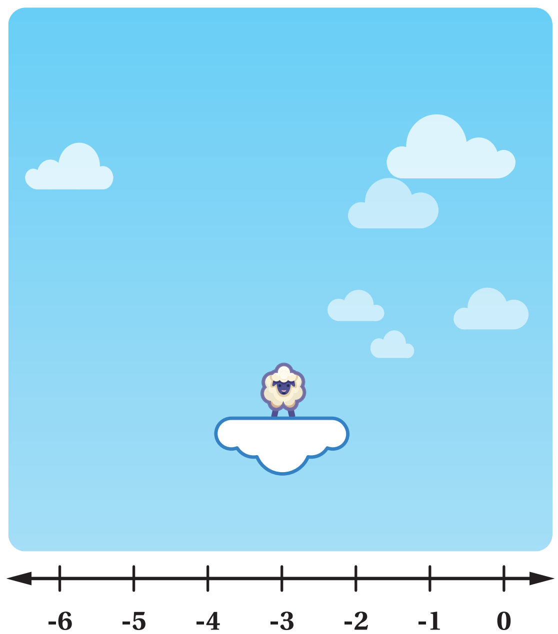
8 Leo was trying to solve the previous inequality: $8x - 4 < 10x + 2$.

He knew the sheep needed to land at -3, but didn't know if the grass was to the *right* or the *left*.

He wrote $8(0) - 4 < 10(0) + 2$.

How might that help Leo decide where the grass is?

***Responses vary.* If Leo gets a true statement when he evaluates each side of the inequality, he'll shade the side where 0 is located. If the inequality is false, he'll shade the side that does not include 0. He might want to test a point on the side he doesn't shade to make sure.**



Name: ______ Date: ______ Period: ______

Solving for Sheep (continued)

9 Here is a new inequality: $-2(x + 2) \leq -23$

Solve the inequality to help Shira eat all the grass.

$x \geq 9.5$ or $9.5 \leq x$

10 For each of the challenges:

- Decide with your partner who will complete Column A and who will complete Column B.
- Solve as many inequalities as you have time for.
- The solutions in each row should be the same. Compare your solutions, then discuss and resolve any differences.

Column A	Column B
$2x + 10 > 20$ $x > 5$	$\frac{x}{5} + 9 > 10$ $x > 5$
$\frac{x}{5} + 10 \leq 12$ $x \leq 10$	$5x - 10 \leq 40$ $x \leq 10$
$-10 \geq 4x + 4$ $-3.5 \geq x$ (or equivalent)	$14 \leq -2x + 7$ $-3.5 \geq x$ (or equivalent)
$3 - 2x > 3$ $x < 0$	$8 > 4x + 8$ $x < 0$
$\frac{x+3}{5} < 2$ $x < 7$	$\frac{x+8}{5} < 3$ $x < 7$
$5x + 10 \geq 3x + 12$ $x \geq 1$	$2x - 4 \geq x - 3$ $x \geq 1$

11 Synthesis

Describe a strategy for solving any inequality.

Use the examples if they help with your thinking.

$10 - 5x < 0$

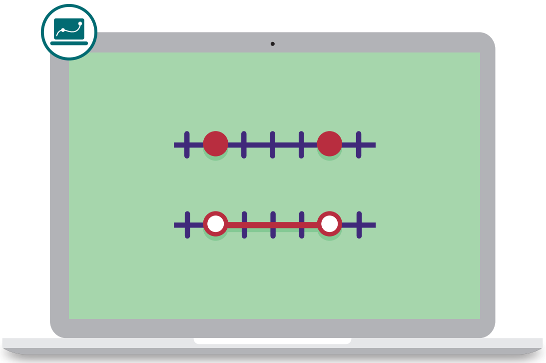
$9 > 2x - 4$

$8x - 4 < 10x + 2$

Responses vary. **Take your inequality and rewrite it as an equation. Solve the equation to find the boundary point. Pick a value on the left or the right of the boundary and test it. If the value makes the inequality true, shade toward that number. If the value makes the inequality false, shade away from that number. Then decide whether to shade the boundary point (yes if the inequality is $\le$ or $\ge$; no if the inequality is $<$ or $>$).**

Things to Remember:

Name: .... Date: .... Period: ....



Absolute Value Solutions

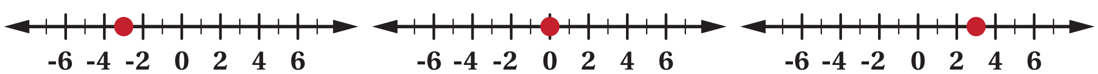
Let's solve absolute value equations and inequalities.

Warm-Up

1 $|x|$ is pronounced "the *absolute value* of x."

a Here are three different values of $|x|$.

$|-3| = 3$ $|0| = 0$ $|3| = 3$



b How would you explain to someone else what $|x|$ means?

Responses vary.

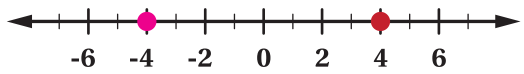
- **$|x|$ means how far away x is from 0.**
- **The absolute value of a number is that number without its sign.**
- **Finding the absolute value of x means taking all the negative values of x and making them positive.**

Activity 1

Name: ______ Date: ______ Period: ______

Showing Solutions

2 Here is one solution to $|x| = 4$.



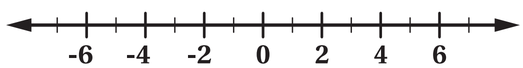
Show another solution to $|x| = 4$.

Explain your thinking.

$x =$ -4. Explanations vary.

- **The absolute value of a negative number is its positive value.**
- **-4 is also 4 units away from 0.**

3 **a** Show a solution to $|x| < 4$.



Responses between -4 and 4 are considered correct.

Share your response with your classmates.

b **Discuss:**

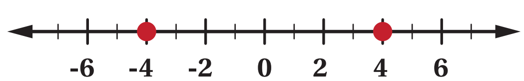
- Are any points incorrect?
- Are any solutions missing?

Responses vary.

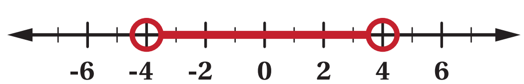
- **Any values that are not between -4 and 4 on the number line are incorrect.**
- **There are many values missing, such as decimals and irrational numbers!**
- **Any number between -4 and 4 that's not listed is missing.**

4 Here are all the solutions to $|x| = 4$ and $|x| < 4$.

$| x | = 4$



$| x | < 4$



How are the solutions alike? How are they different?

Alike:

***Responses vary.* The two solutions have the same boundary points at -4 and 4.**

Different:

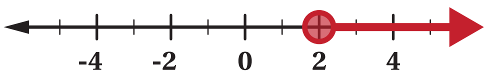
***Responses vary.* The solutions to the equation are 4 and -4, while the solutions to the inequality are all the points between -4 and 4.**

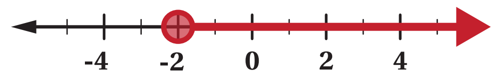
Activity 1

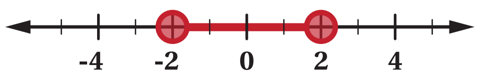
Name: ________ Date: ________ Period: ________

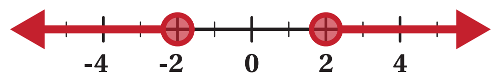
Showing Solutions (continued)

5 Which number line represents the solutions to $|x| \geq 2$?

A. 

B. 

C. 

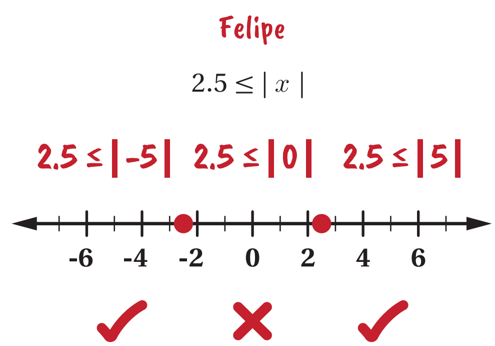
D. 

6 Felipe is graphing the solutions to $2.5 \leq |x|$.

Here is some of his work.

Describe how Felipe's work can help him decide where to shade the solution set on the number line.

***Responses vary.* Felipe tested a point in each section of the number line. He should shade the regions that contain solutions that make the inequality true.**



Activity 2

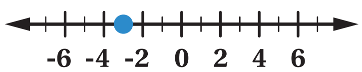
Name: .... Date: .... Period: ....

Solving Strategies

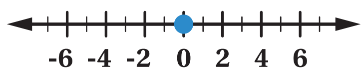
7 Polina says the value of $|x-2|$ is the distance between any number and 2.

a Here are three different values of $|x-2|$.

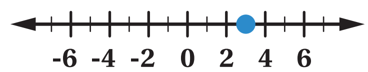
$|-3-2|=5$



$|0-2|=2$



$|3-2|=1$



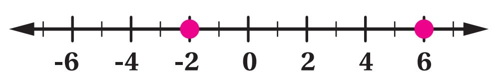
b **Discuss:** Do you agree with Polina?

Responses vary. **I agree with Polina because when you subtract, you're finding the difference between x and 2, which is the same as the distance.**

8 **a** Show or describe all the solutions to $|x-2|=4$.

Responses vary. **The solutions are all the points that are 4 units away from 2.**

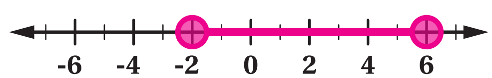
$|x-2|=4$



b Show or describe all the solutions to $|x-2|\le 4$.

Responses vary. **The solutions are all the numbers from -2 to 6.**

$|x-2|\le 4$



Activity
2

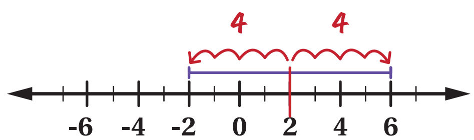
Name: .... Date: .... Period: ....

Solving Strategies (continued)

9 Two students graphed the solutions to $|x-2| \le 4$.

Esi

$$|x-2| \le 4$$

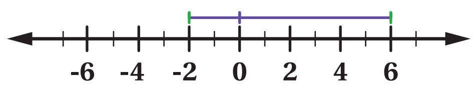


Deja

$$|x-2| \le 4$$

$(x - 2) = 4$ $-(x - 2) = 4$

$x = 6$ $x = -2$



$|0 - 2| \le 4$

✔

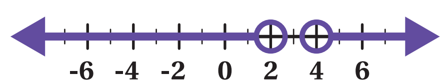
Discuss: How would you describe each students' strategy?

Responses vary.

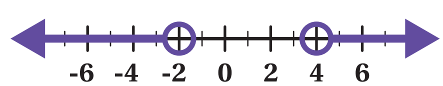
- Esi made a mark at 2 and counted 4 units on both sides of 2. Then she shaded in the numbers between those points.
- Deja solved the equation $|x-2| = 4$ to find the boundary points. She knew that ≤ 4 meant the solutions would be between those points, so she shaded in that region.

10 Match each inequality to the number line that represents its solutions.

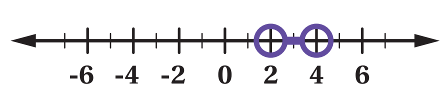
a. $|x-3| > 1$ ...a...



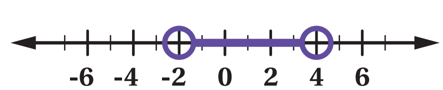
b. $|x-3| < 1$ ...c...



c. $|x-1| > 3$ ...b...



d. $|x-1| < 3$ ...d...



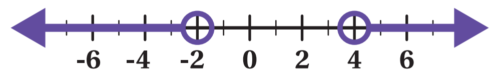
Activity 3

Name: Date: Period:

Repeated Challenges

11 Raven wants to use the graph of $|x - 1| > 3$ to help her make a graph of $|x + 1| > 3$.

$|x - 1| > 3$



a **Discuss:**

- How would the solutions be alike?
- How would they be different?

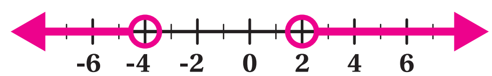
Responses vary.

- **The solutions to both inequalities would be on the outside of the boundary points.**
- **Instead of being 3 units away from 1, the solutions would be 3 units away from -1.**

b Graph all the solutions to $|x + 1| > 3$.

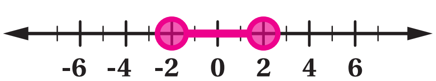
Response shown on the graph.

$|x + 1| > 3$

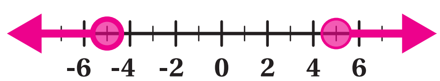


12 Graph all the solutions to each inequality or equation.

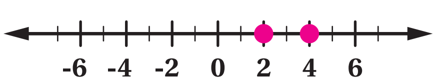
$|x| \leq 2$



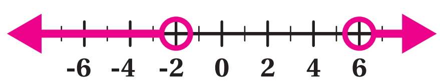
$|x| \geq 5$



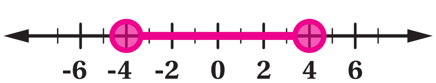
$|x - 3| = 1$



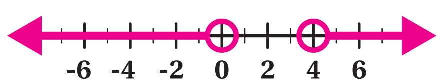
$|x - 2| > 4$



$4 \geq |x|$



$|x - 2| > 2$

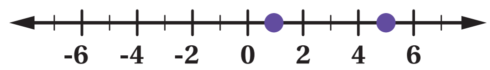


13 Synthesis

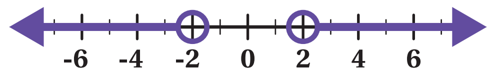
Describe a strategy you would use to graph the solutions to absolute value equations and inequalities.

Responses vary. I would think about the absolute value of a number as the distance. So the solutions to $|x - 3| = 2$ are the numbers that are exactly 2 units away from 3. The solutions to $|x| > 2$ are the numbers that are farther than 2 units away from 0.

$|x - 3| = 2$



$|x| > 2$



Things to Remember:

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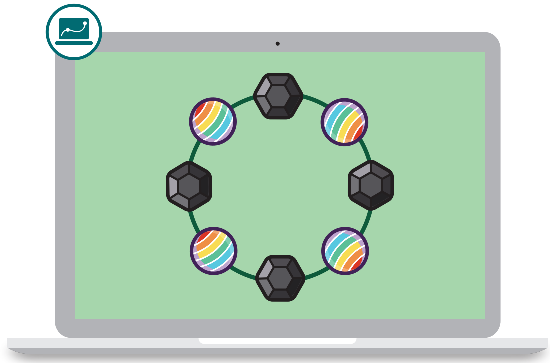


Bracelet Budgets

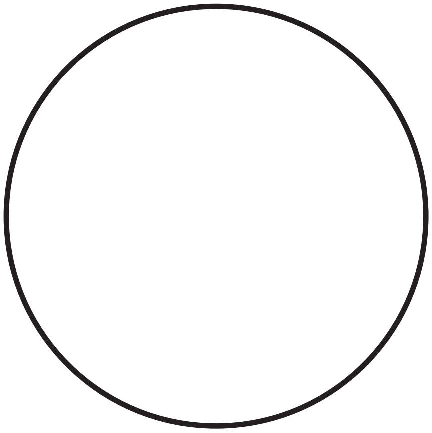
Let's explore solutions to two-variable inequalities graphically and symbolically.

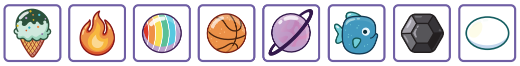
Warm-Up

1 Draw or describe a bracelet. You can use any combination of the beads shown.

Tell us about your bracelet design.

Bracelets vary.





Activity 1

Name: ______ Date: ______ Period: ______

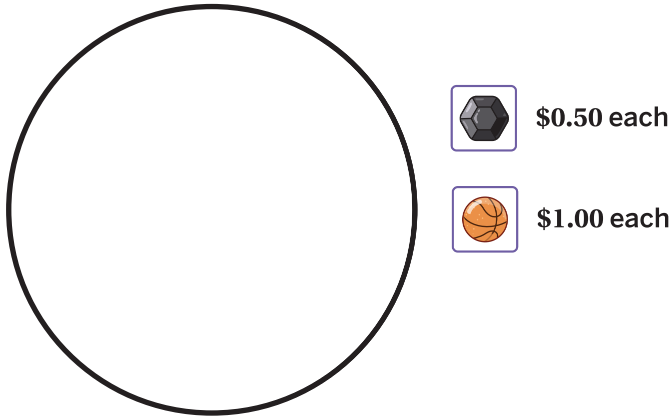
Modeling with Inequalities

2 Here are two types of beads:

- Black beads are $0.50 each.
- Basketball beads are $1 each.

Draw or describe a $5 bracelet.

Responses vary.
- **4 $0.50 beads and 3 $1 beads**
- **6 $0.50 beads and 2 $1 beads**



3 Each of these points represents a $5 bracelet.

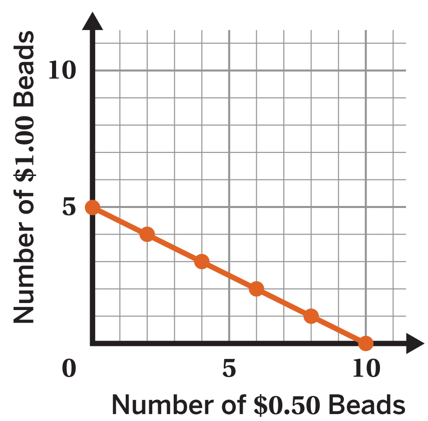
- x is the number of $0.50 beads.
- y is the number of $1.00 beads.

Which equation represents all the $5 bracelets? Circle one.

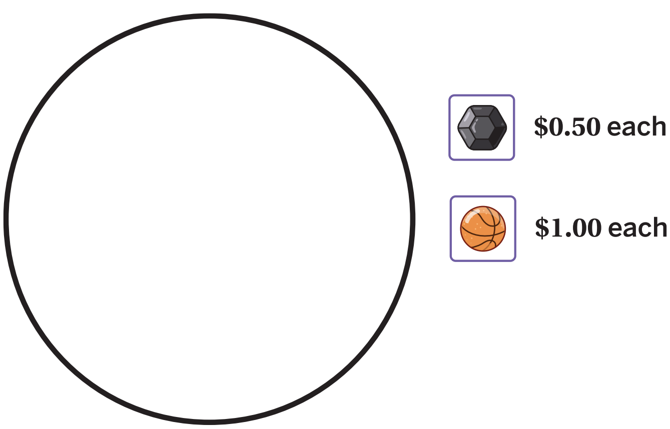
$x + y = 5$ $\qquad$ $0.5x + y = 5$ $\qquad$ $y = 0.5x + 5$

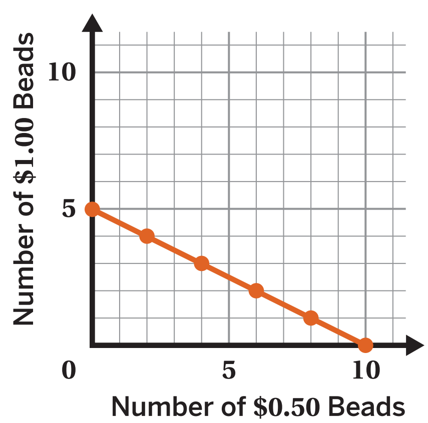
Explain your thinking.

***Explanations vary.* $0.5x$ is the cost of the $0.50 beads. y is the cost of the $1 beads. $5 is the total cost.**



4 Binta can spend $5 or less on a bracelet. Graph some bracelets that Binta could buy. Draw or describe them if it helps with your thinking.





Points that are solutions to $0.5x + 1y \leq 5$ are considered correct.

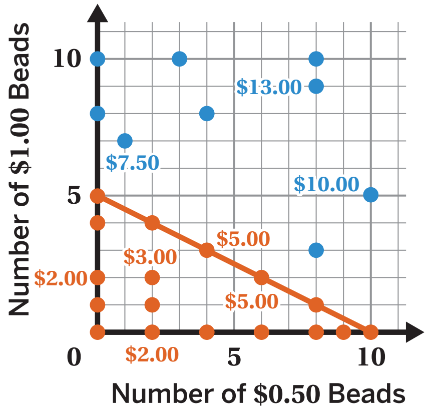
Activity 1

Name: .... Date: .... Period: ....

Modeling with Inequalities (continued)

5 Here are some bracelets that Binta could buy and some she could not buy for \$5 or less.

What do you notice? What do you wonder?



I notice:

Responses vary.
- **I notice all the bracelets below the line are less than \$5.**
- **I notice the closer a bracelet is to the line, the closer it is to \$5.**
- **I notice that some of the bracelets cost the same amount.**

I wonder:

Responses vary.
- **I wonder which bracelet Binta will make.**
- **I wonder what the biggest bracelet Binta can make is.**
- **I wonder if changing the order of the beads changes the price.**

6 Binta can spend \$5 or less on a bracelet.

- x is the number of \$0.50 beads.
- y is the number of \$1.00 beads.

Which statement describes all the bracelets that Binta can buy? Circle one.

$0.5x + y \leq 5$ $\qquad$ $0.5x + y \geq 5$ $\qquad$ $0.5x + y = 5$

Explain your thinking.

Explanations vary. **$0.5x + y$ represents the cost of the bracelet, since there are some number of \$0.50 beads and some number of \$1 beads. We know that Binta needs the bracelet to cost \$5 or less, so that means the total cost needs to be ≤ 5.**

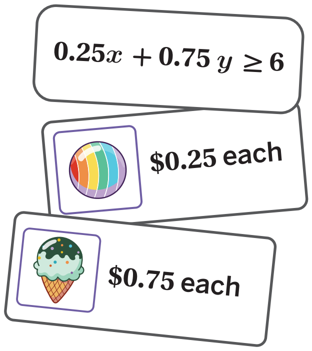
Activity 2

Name: ______ Date: ______ Period: ______

Solutions to Inequalities

7 Caleb loves Binta's bracelet and wants to make his own.

The inequality represents all the bracelets he can make.

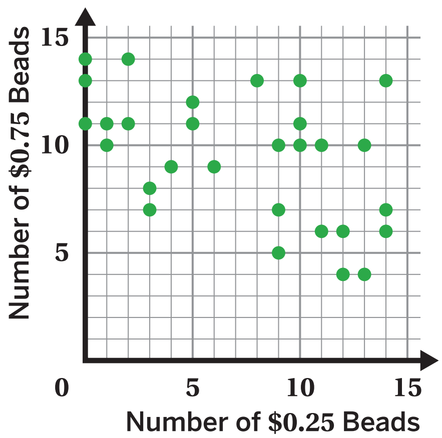


Discuss: What does each number and variable in the inequality represent about Caleb's bracelet?

Responses vary.

- **The 0.25 is the cost of the rainbow beads and the 0.75 is the cost of the ice cream beads.**
- **The x represents the number of rainbow beads and y represents the number of ice cream beads.**
- **All together, the cost has to be $6 or more.**

8 Each of these points is a solution to $0.25x + 0.75y \geq 6$.



a Let's look at several points to see what this means.

b In your own words, what is a solution to an inequality with two variables?

***Responses vary.* When you substitute the x- and y-values of a solution into an inequality with two variables, they make the inequality true.**

9 Is $(2, 10)$ also a solution to $0.25x + 0.75y \geq 6$? Circle one.

Solution   Not a solution   I'm not sure

Explain your thinking.

Explanations vary.

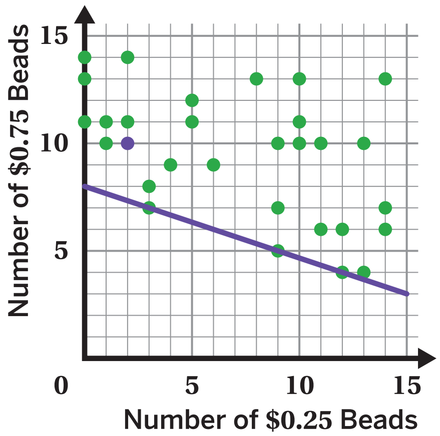
- **$(2, 10)$ is very close to three other points that are solutions, so it's probably a solution, too.**
- **I substituted 2 for x and 10 for y, and the total was more than 6, so it has to be a solution.**

Name: .... Date: .... Period: ....

Solutions to Inequalities (continued)

10 Here is how two students determined that $(2, 10)$ is a solution to $0.25x + 0.75y \geq 6$.

Marco



Jada

$0.25x + 0.75y \geq 6$

$0.25(2) + 0.75(10) \geq 6$

$0.50 + 7.50 \geq 6$

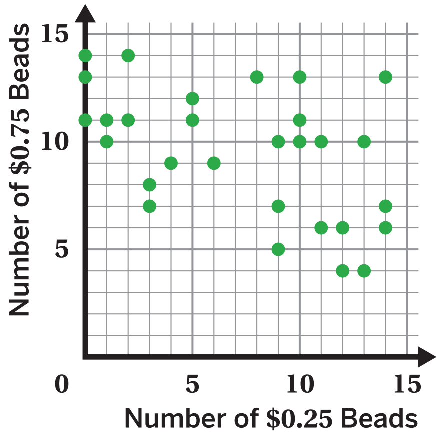
$8 \geq 6$

Explain each person's strategy to a partner.

Explanations vary.

- **Marco graphed the equation $0.25x + 0.75y = 6$ and plotted the point. Since (2, 10) is on the same side of the line as the other solutions, he thought it was a solution.**
- **Jada tested the point in the inequality. She showed that when $x = 2$ and $y = 10$, the inequality is true, which means (2, 10) is a solution.**

11 The graph shows some solutions to $0.25x + 0.75y \geq 6$.



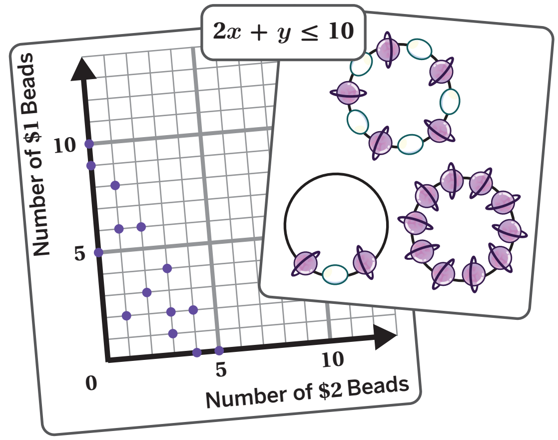
Select *all* of the other points that are also solutions.

- [x] A. $(14, 8)$
- [ ] B. $(13, 1)$
- [ ] C. $(7, 5)$
- [x] D. $(1, 13)$
- [x] E. $(0, 8)$

13 Synthesis

How can you tell if a point is a solution to a two-variable inequality?

Use the example if it helps with your explanation.



***Responses vary.* If a point is a solution to a two-variable inequality, then it will make a true statement when I substitute the point's x-value and y-value into the inequality.**

Things to Remember:

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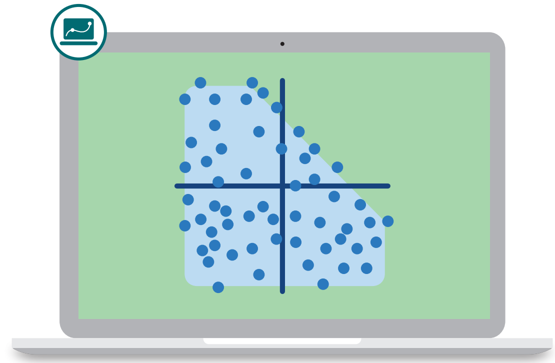


All of the Solutions

Let's represent all of the solutions to two-variable inequalities graphically.

Warm-Up

1 Two students created graphs of the *solutions* to $x \leq 7$.

How are their graphs alike? How are they different?

Responses vary.

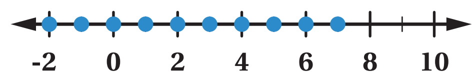
Alike:

- **Both graphs have a closed circle on 7.**
- **8 is not a solution on either graph.**

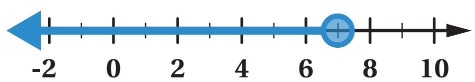
Different:

- **Student A graphed only the integer solutions.**
- **Student B graphed all the solutions.**

Student A



Student B



Activity 1

Name: ______ Date: ______ Period: ______

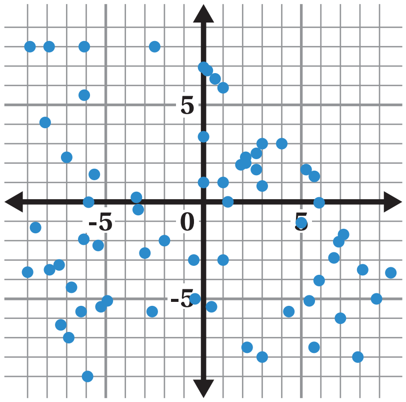
Some to All

2 Write three solutions to $x + y \le 7$. Try thinking of x- and y-values that no one else will!

Responses vary.

Solution 1 (x, y)	Solution 2 (x, y)	Solution 3 (x, y)
(1, 6)	(5.2, -1.5)	(0, 3.4)

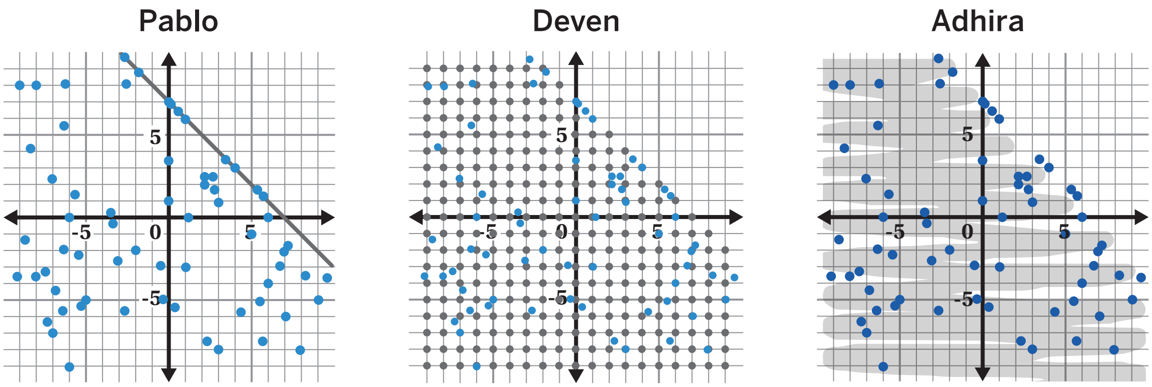
3 Here is a graph of some of the solutions to $x + y \le 7$.



Sketch what you think the graph of *all* the solutions to $x + y \le 7$ looks like.

Responses vary. **See Problem 4 for three examples.**

4 Here are Pablo's, Deven's, and Adhira's sketches of *all* the solutions to $x + y \le 7$.



a Select *one* sketch.

b **Discuss:** What do you like about this sketch? What would you change?

Responses vary.

- I like that Pablo included a line that creates a boundary between the points that are a solution and the points that are not a solution.
- I like that Deven added many more integer solutions.
- I would add solutions with decimals to Deven's sketch.
- I would add a boundary line to Adhira's sketch, like Pablo did. That would help make her shading more precise.

Activity 2

Name: ______ Date: ______ Period: ______

Shading the Solutions

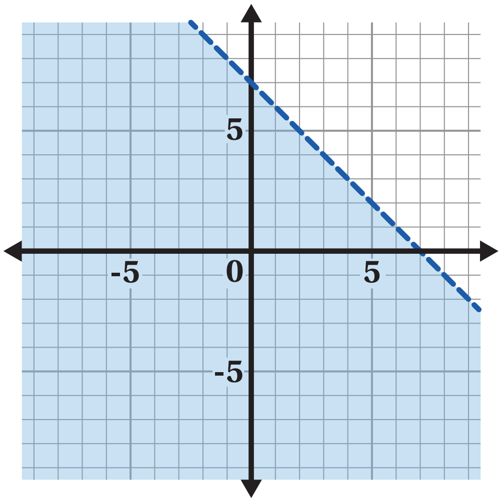
5 **a** Let's watch an animation to see what a graph of all solutions looks like.

b How does this graph represent all the solutions to $x + y \leq 7$?

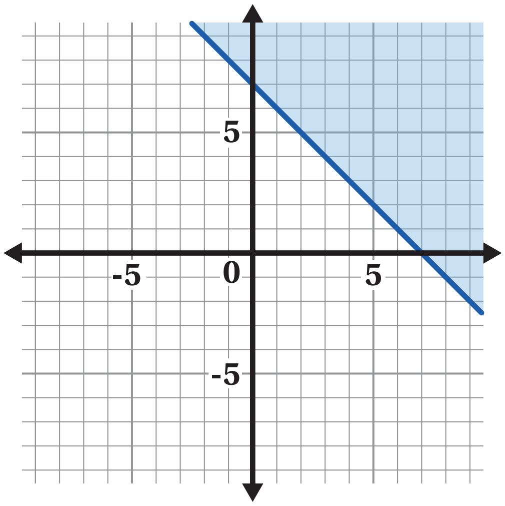
***Responses vary.* The shaded part represents all of the points that make the inequality $x + y < 7$ true. The points on the line are points that make the equation $x + y = 7$ true.**

6 Here are the graphs of two other inequalities.

$x + y < 7$



$x + y \geq 7$



Discuss:

- How are the graphs of the two inequalities different?
- What does the **boundary line** represent?

Responses vary.

- **The graphs are different because one line is solid and the other line is dashed. Each graph also has a different shaded region .**
- **I notice that when the line is solid, the symbol is $\leq$ or $\geq$, and when the boundary line is dashed, the symbol is $<$ or $>$. The boundary line represents the points along the line $x + y = 7$. When the inequality has the symbol $\leq$ or $\geq$, the points along the boundary line make the inequality true.**

Activity 3

Name: Date: Period:

Which Region?

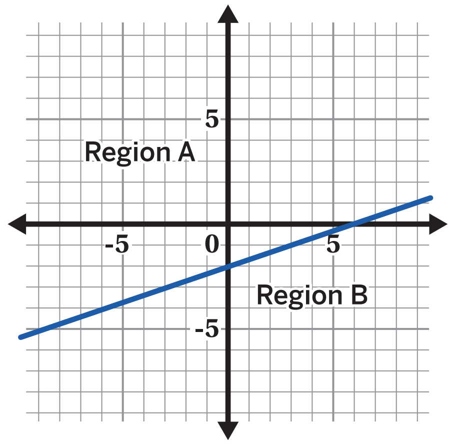
7 Here is the graph of $x - 3y = 6$.

Where will the solutions to $x - 3y \geq 6$ be? Circle one.

Region A Region B I'm not sure

Explain your thinking.

***Explanations vary.* When I plugged $(0, 0)$ into the inequality, I got a false statement. Since $(0, 0)$ is in Region A, I think Region B is where the solutions will be.**

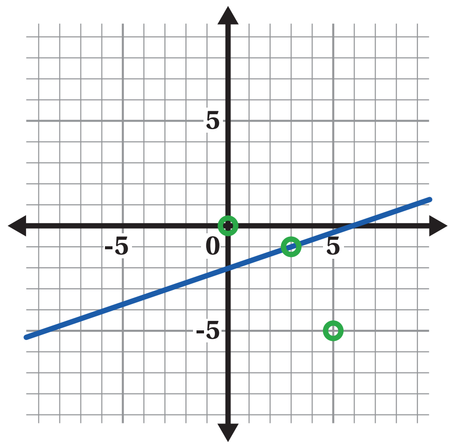


8 Rebecca thought that checking points would help her decide how to graph the solutions to $x - 3y \geq 6$.

She chose the points $(0, 0)$, $(3, -1)$, and $(5, -5)$.

a **Discuss:** Why do you think Rebecca chose these points?

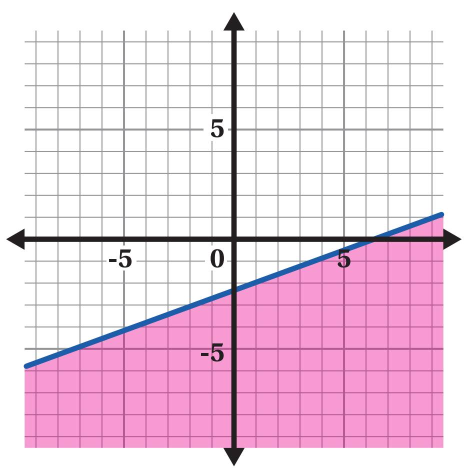
***Responses vary.* She chose one point above the line, one below the line, and one on the line.**



b Select *all* the points that are solutions to $x - 3y \geq 6$.

- ☐ **A.** $(0, 0)$
- ☑ **B.** $(3, -1)$
- ☑ **C.** $(5, -5)$

9 Graph *all* the solutions to $x - 3y \geq 6$.



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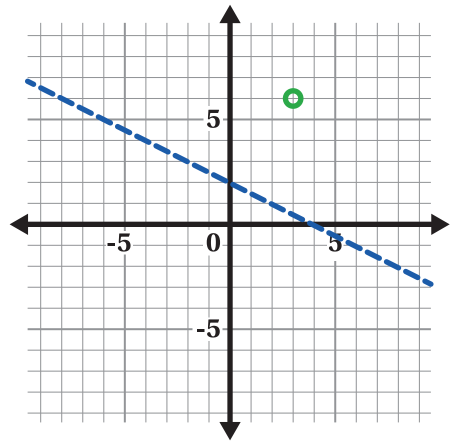
Which Region? (continued)

10 Nathan is graphing the solutions to $x + 2y < 4$.



a **Discuss:** Why is his line dashed?

***Responses vary.* The line is dashed to show that points on the line are not included in the solution. They aren't included because the symbol is $<$, not $\leq$.**

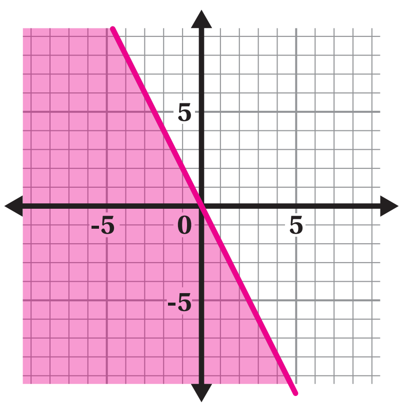
b Nathan determined that $(3, 6)$ is *not* a solution.

Does he have enough information to graph all the solutions?

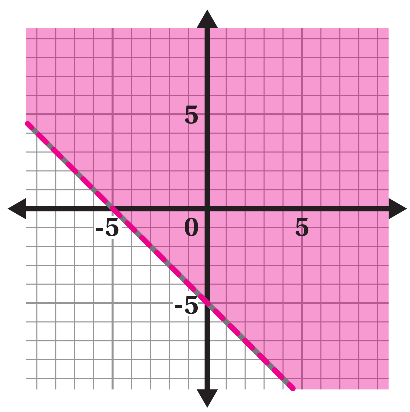
Yes. *Explanations vary.* $(3, 6)$ is not a solution to $x + 2y < 4$ because it makes $x + 2y$ greater than 4. This means all the points on the same side as $(3, 6)$ will make $x + 2y$ greater than 4. The line is where $x + 2y$ is equal to 4, so all of the points on the other side must make $x + 2y$ less than 4.

11 Graph all the solutions to the following inequalities. The graph of each corresponding equation has been given to you.

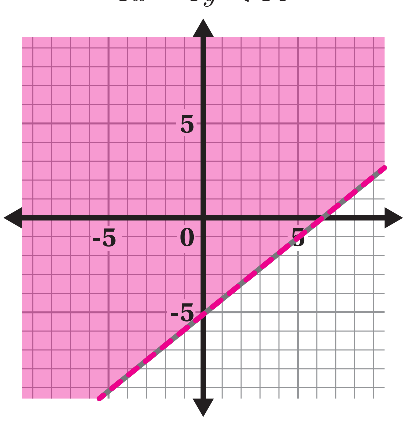
$2x + y \leq 0$



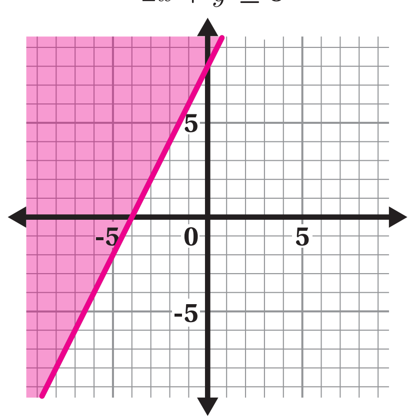
$x + y > -5$



$5x - 6y < 30$

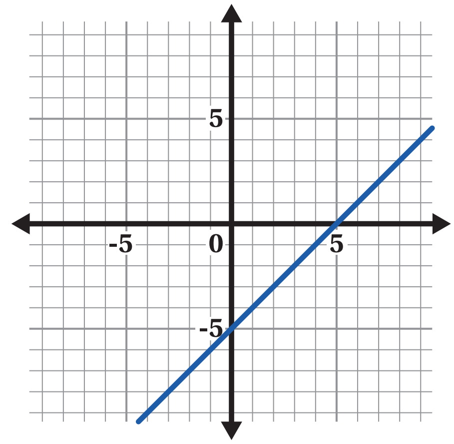


$-2x + y \geq 8$



13 Synthesis

Here is the graph of $x - y = 5$



Describe a strategy for graphing all the solutions to $x - y < 5$, also called the **solution region**.

Responses vary. **I would test a point like $(0, 0)$ because I can calculate if it's a solution to the inequality quickly. If it is, I would shade that side. If it's not, I would shade the other side. I'd make the line dashed because the inequality is less than, not less than or equal to.**

Things to Remember:

Unit 3
Lesson
1

Name: ________ Date: ________ Period: ________

Survey Says

Let's make sense of the kinds of data that can be collected and write questions to get to know each other better.

Warm-Up

1 Every year, the U.S. government sends the American Community Survey (A.C.S) to thousands of people.

The goal is to gather "vital information about our nation and its people."

What would you like to know about our nation and its people? List at least three questions.

Responses vary.

- **What are the ages of the people who live in the United States?**
- **Where do most people live in the United States?**
- **How much are people paying for rent in the United States?**

Activity 1

Name: .... Date: .... Period: ....

Types of Data

2 A local group conducted a survey similar to the ACS to gather more information about their community.

a Take a look at this list that includes the initials of seven survey participants and their responses.

Anonymous People	What is your age?	Do you have health insurance?	What is your main occupation?	How did you get to work or school last week?	What is the monthly rent where you live?
M.A.	19	No	Cook	Bicycle	$1,200
P.Y.	25	Yes	Construction worker	Car	$900
Z.M.	53	Yes	Nurse	Walked	$1,700
B.H.	62	Yes	I don't work	–	–
J.S.	44	Yes	Accountant	Worked from home	$1,250
O.L.	12	I'm not sure	Student	Car	–
A.B.	29	No	Carpenter	Bus	$1,050

b What do you notice about the responses? What do you wonder? ***Responses vary.***

I notice:

- **I notice that some responses are words, some are numbers, and some have blanks.**
- **I notice that not every person responded to every question.**
- **I notice that the youngest age is 12 and the oldest age is 62.**

I wonder:

- **I wonder why people gave their initials.**
- **I wonder why two people didn't respond about the rent question.**
- **I wonder how people answer the transportation question if they use multiple modes of transport.**

Activity
1

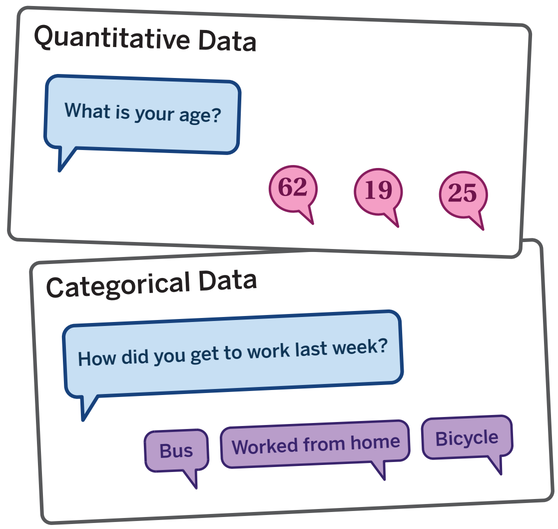
Name: .... Date: .... Period: ....

Types of Data (continued)

3 Asking different types of questions can produce different types of data.

What is your age? produces **quantitative data** (also called *numerical data*).

How did you get to work last week? produces *categorical data*.



Describe what you think these terms mean.
Responses vary.
Quantitative data: **The data values are numbers or measurements.**

Categorical data: **The data values are words.**

4 Which type of data would each of these questions produce?

Question	Categorical or Quantitative?
Do you live within two miles of a grocery store?	**Categorical**
What type of fuel is used to heat your home?	**Categorical**
What is the monthly rent where you live?	**Quantitative**
How old is the youngest person in your family?	**Quantitative**
How many minutes does it take you to get to work or school?	**Quantitative**
What type of building do you live in?	**Categorical**

5 Convince someone about which type of data this question would produce: *Do you live within two miles of a grocery store?*
Responses vary. **This question would produce categorical data because the responses would be yes, no, or I don't know. If the question was "How many miles do you live from a grocery store?" then it would be quantitative.**

Activity 2

Name: .... Date: .... Period: ....

Crafting and Asking Questions

6 Jalen is making a survey that includes a question about people's television-watching habits.

He wrote three different ways to ask the question.

- Do you watch a lot of television?
- How many hours of T.V. did you watch last night?
- How much T.V. did you watch last month?

a **Discuss:** What are some advantages and disadvantages of each question?

Responses vary.

- **First question: This question would collect categorical data that shows how people feel about their television-watching habits, but "a lot" could mean a different amount to different people.**
- **Second question: The amount of T.V. someone watched "last night" might not be a good measure of their habit of watching T.V.**
- **Third question: A month might be a long time to measure how much T.V. someone watched, and people might estimate instead of giving exact answers.**

b If you were making the survey, how would you write the question?

Responses vary.

- **On average, how many hours of T.V. do you watch each week?**
- **How many minutes of T.V. did you watch last week?**

7 Think of something you want to learn about your classmates.

a Write a question for your classmates to answer. Will the data you collect be quantitative or categorical? ***Responses vary.***

Question:

Circle one: Categorical Quantitative

b Collect responses to your question. ***Responses vary.***

Initials	Response

Initials	Response

Activity
2

Name: Date: Period:

Crafting and Asking Questions (continued)

8 Look at your classmates' responses to your question.

a) Summarize the data in some way.

Responses vary.

b) Now that you've seen the responses, how would you change your survey question?

Responses vary.

9 Synthesis

How can you decide whether a question will produce quantitative or categorical data?

Responses vary. A question will produce quantitative data when the responses are numbers or measurements, like number of minutes. A question will produce categorical data when the responses are words, like types of fuel or yes, no, maybe.

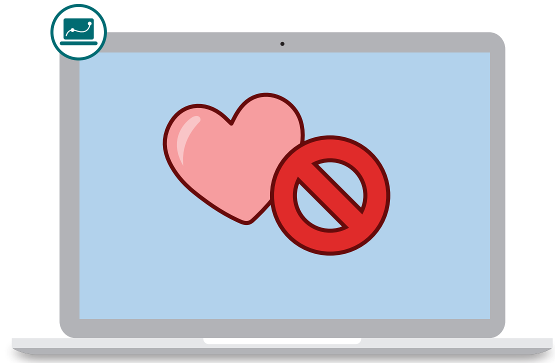
How many minutes does it take you to get to work or school?

What is your age?

What type of fuel is used to heat your home?

Things to Remember:

Name: Date: Period:



Love It or Leave It

Let's make sense of dot plots and histograms as ways to visualize one-variable data.

Warm-Up

1 This is a game called *Love It or Leave It*.

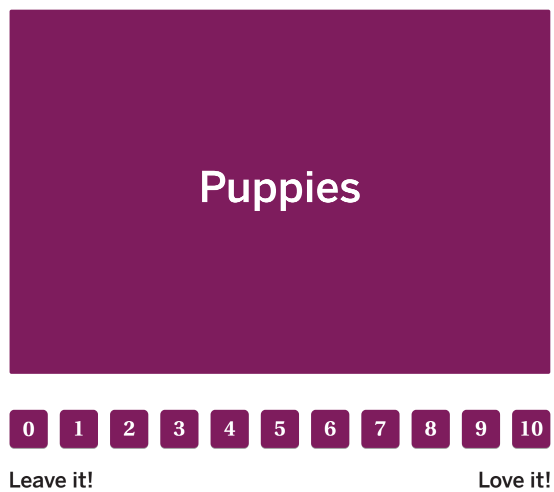
A word or phrase will appear on the screen.

Rate it on a scale of 0-10.

10 = Love it!
0 = Leave it!

Use the digital activity to play several rounds of this game. rounds of this game.

Responses vary.



Activity 1

Name: ________ Date: ________ Period: ________

Dot Plots

You'll use the digital activity for Problems 2–4.

2 Look at some ratings from *Love It or Leave It* in a *dot plot.*

What surprises you?

Responses vary.
- **People either love reality TV or really don't like it.**
- **More people like puppies than dislike puppies.**

3 Look at another dot plot showing ratings from your class.

Which of these topics could the dot plot represent? Circle one.

Hiking   The zoo   Traffic   Saturday

Explain your thinking.

Responses and explanations vary. **I think the dot plot represents ratings for traffic because most people don't like traffic and only 1 out of the 10 dots is above 3.**

4 **a** Replace P with C or H in dotplot(P) to see the dot plot for those data sets.

b **Discuss:** What ratings do you think data set H could represent?

Responses vary. **Data set H could represent the class ratings for chocolate, ice cream, beaches, or Saturday.**

Activity 2

Name: .... Date: .... Period: ....

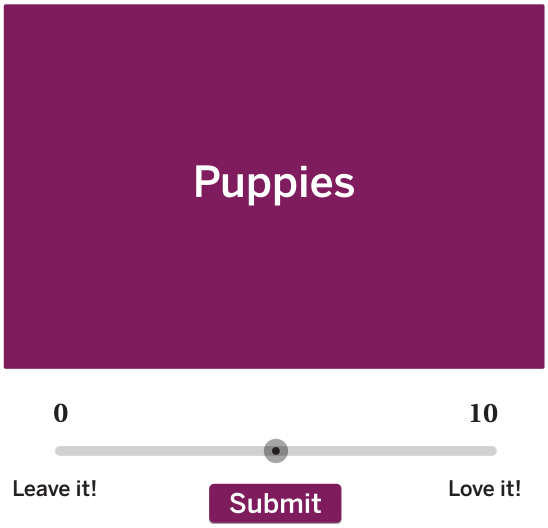
Histograms

5 The makers of *Love It or Leave It* are considering changing the way people rate words.

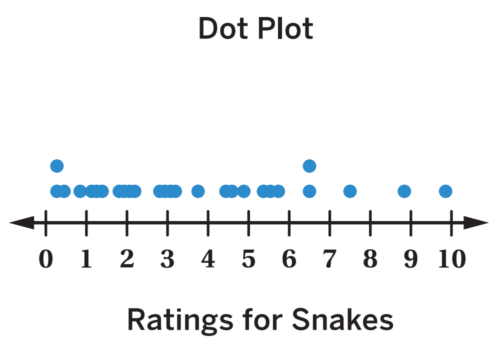
a Use the digital activity to play several rounds of the new version.

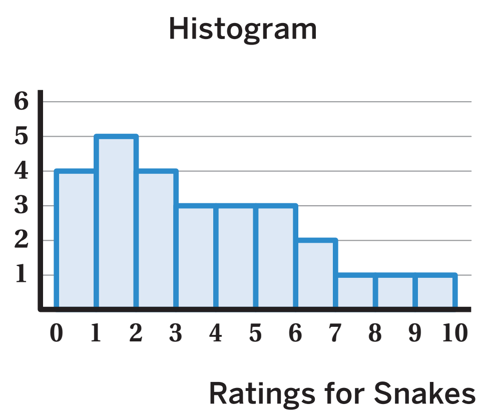
b How would the new version change the data that gets collected?

***Responses vary*. The new version changes the data that gets collected because answers could be decimals instead of just whole numbers.**



6 This dot plot and *histogram* show many students' ratings from the new version.





a Take a look at how a histogram is made in the digital activity.

b How many students gave snakes a rating between 2 and 3?

4 students

Activity 2

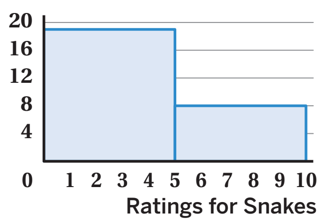
Name: .... Date: .... Period: ....

Histograms (continued)

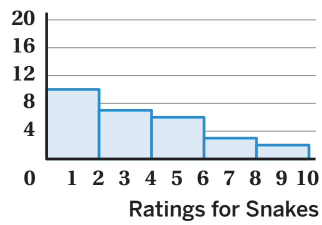
7 Here is the data for snakes from the previous problem.

a Take a look at how the bin width changes the histogram.

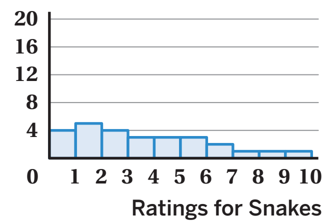
Bin Width: 5



Bin Width: 2



Bin Width: 1



b **Discuss:**

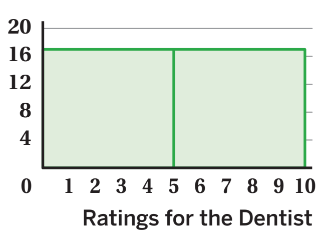
- What do you think *bin width* means?
- What stays the same when the bin width changes? What does not?

Responses vary.

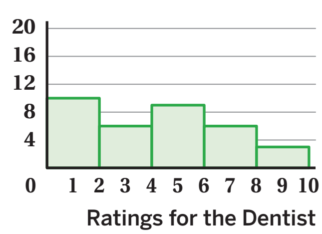
- **Bin width is how large of groups the data in the histogram is split into. For example, when the bin width is 5, the data is split into groups by every 5th number. The bins go from 0 to 5 and 5 to 10.**
- **The data stays the same; the bins change heights as their widths change.**

8 Here are ratings for the dentist from Kwame's class.

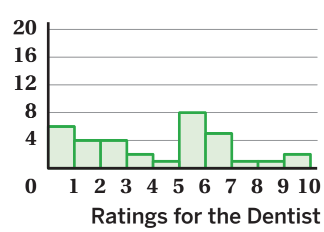
Bin Width: 5



Bin Width: 2



Bin Width: 1



Kwame says: *Based on the data, people seem to like and dislike the dentist equally.*

What is true about Kwame's claim? What is misleading?

It's true that . . . ***Responses vary.*** **It's true that a bin width of 5 splits all ratings into an upper and lower half, and there is an equal number of ratings in each half.**

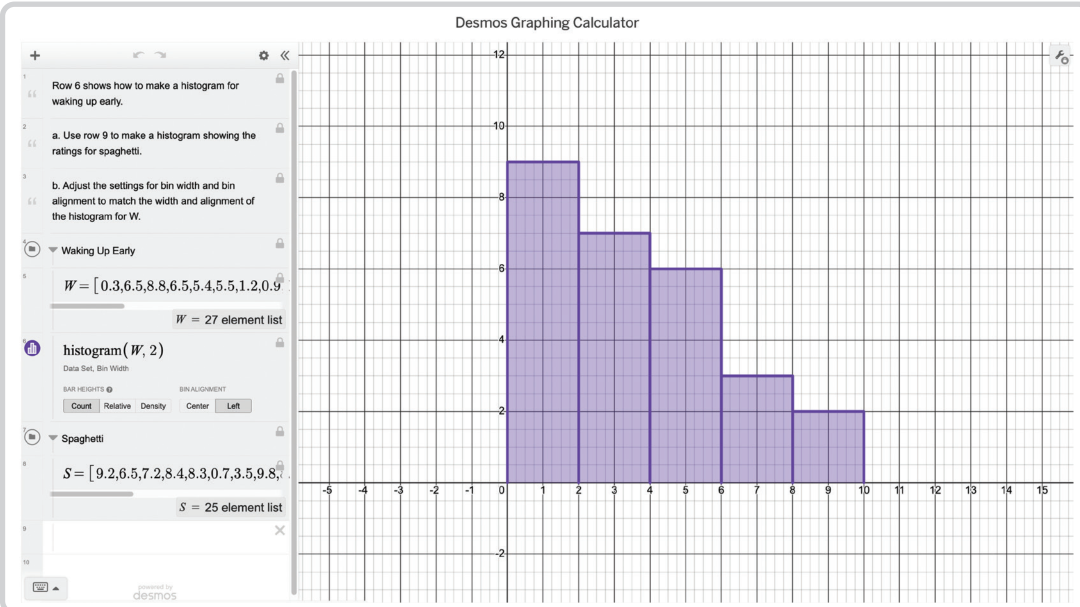
It's misleading that . . . ***Responses vary.*** **It's misleading that people like and dislike the dentist equally. It might be more accurate to say that some people strongly dislike the dentist and a lot of others feel indifferent. If you change the bin width, the most common rating is between a 5 and 6, which isn't a strong like. The second most common rating is between a 0 and 1, which is a very strong dislike.**

Activity 3

Name: ______ Date: ______ Period: ______

Making Histograms

9 **a** Use the digital activity to make a histogram showing the ratings for spaghetti.



b Adjust the settings for bin width and bin alignment to match the width and alignment of the histogram for W.

Explore More

10 **a** Ask people to rate something on a 0–10 scale. Collect your responses in the table.

Responses vary.

Initials	Rating	Initials	Rating	Initials	Rating

b Use the digital activity to make a histogram and/or dot plot of the data.

Graphs vary.

11 Synthesis

In this lesson, we examined two ways of visualizing data.

What are the advantages of a dot plot?

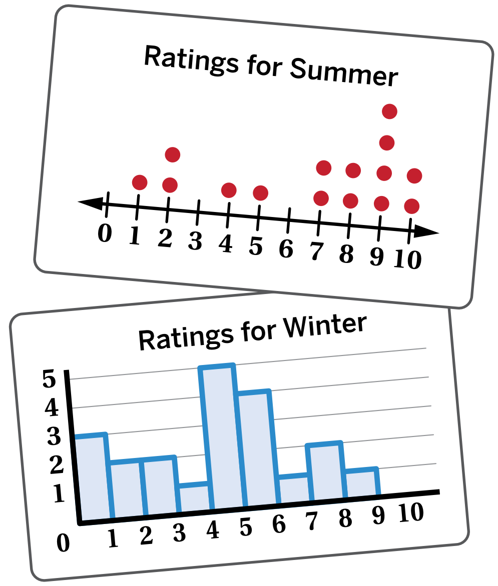
Responses vary.

- **The data is easy to visualize when there are whole-number data points and the number of points is small.**
- **You can see each individual data point and what it represents.**

What are the advantages of a histogram?

Responses vary.

- **It helps to bucket the data when there is a large range of possible responses, like if the data includes decimal values.**
- **A histogram is useful when there are a lot of data points.**



Things to Remember:

Name: .... Date: .... Period: ....

Better Weather?

Let's use box plots to visualize and compare weather data.

Warm-Up

1 Mia lives in Seattle, Washington. Her best friend Bao lives in Charleston, South Carolina.

Mia and Bao are debating whose city has better weather.

How could they compare the weather in Seattle and Charleston?

Responses vary.

- **Mia and Bao could compare the temperatures in their cities for the past week.**
- **They could see how many times it has rained recently in each city.**

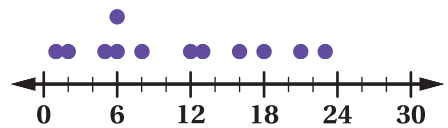
Activity 1

Name: ________ Date: ________ Period: ________

Rainy Days

2 Mia and Bao wondered: *Which city has rainier weather?*

Mia made a dot plot of the number of rainy days for each month in Seattle in 2021.



Source: *National Weather Service*

a What was the *greatest* number of rainy days in a month?

23 days

b What was the *fewest* number of rainy days in a month?

1 day

3 In the digital activity, look at some *statistics* of rainy days for each month in Seattle in 2021. A statistic is a single number that measures something about a data set.

Minimum	Q1	Median	Q3	Maximum
1	5.5	10	17	23

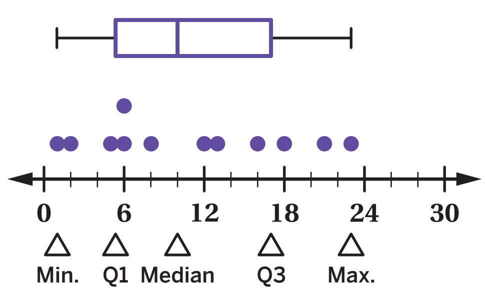
Discuss: What do you think each of these statistics tells us about the data?

Responses vary.

- **Minimum: The lowest number of rainy days.**
- **Q1: The number that splits the lower half of the data in half. Three months had between 0 and 5.5 rainy days and three months had between 5.5 and 10 rainy days.**
- **Median: The number that splits the data in half. Six months had less than 10 rainy days and six months had more than 10 rainy days.**
- **Q3: The number that splits the upper half of the data in half. Three months had between 10 and 17 rainy days and three months had between 17 and 23 rainy days.**
- **Maximum: The highest number of rainy days.**

4 The five statistics on the previous problem split the data into quarters, which are called *quartiles*.

A *box plot* is one way to represent quartiles.



a Use the digital activity to see how a box plot for the Seattle data gets made.

b **Discuss:**

- Why do you think this is called a box plot?
- What percentage of the data is inside the box? How do you know?

Responses vary.

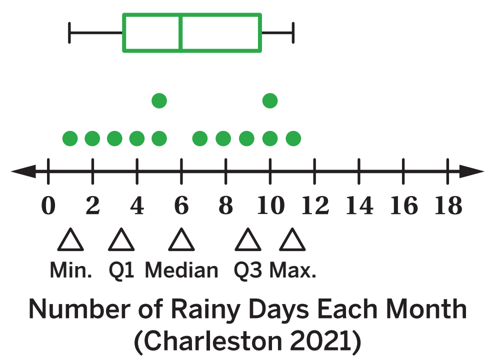
- **I think this is called a box plot because there is a box between the Q1 and Q3 lines.**
- **The middle 50% of the data is in the box.**

Activity 2

Name: .... Date: .... Period: ....

Comparing

5 Here is a dot plot and a box plot of the number of rainy days for each month in Charleston in 2021.



Bao says: *In half of the months of the year, Charleston had at least 6 rainy days.*

a **Discuss:** How can the box plot help you know that Bao's statement is true?

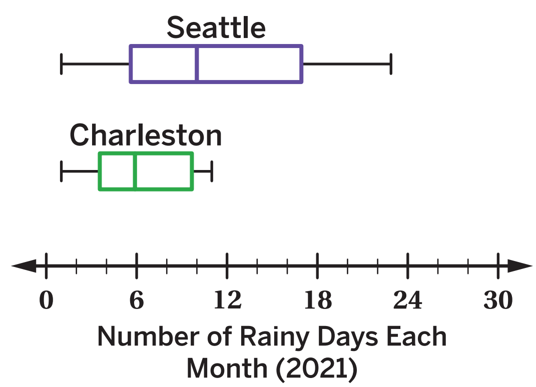
***Responses vary.* I can tell Bao's statement is true because 6 is the median. The median is the value that splits the data in half.**

b Write *two* more true statements that can be determined from the dot plot or box plot.

Responses vary.

- **The rainiest month had 11 days of rain.**
- **25% of months had more than 9.5 rainy days.**

6 Here are two box plots showing the number of rainy days for each month in Seattle and Charleston in 2021.



Use the box plots to help Bao convince Mia that Seattle is the rainier city.

Responses vary.

- **Seattle is the rainier city because the median is higher. In Seattle, the median is 10 rainy days per month, while in Charleston, the median is 6 rainy days per month.**
- **The maximum number of rainy days in Seattle (23) is more than twice the maximum number of rainy days in Charleston (11).**
- **The most number of rainy days in Charleston is only one day more than the median number of rainy days in Seattle.**
- **The number of rainy days in Seattle is more unpredictable. Both of the cities had a minimum number of 1 day, but Seattle's maximum number is much higher than Charleston's.**

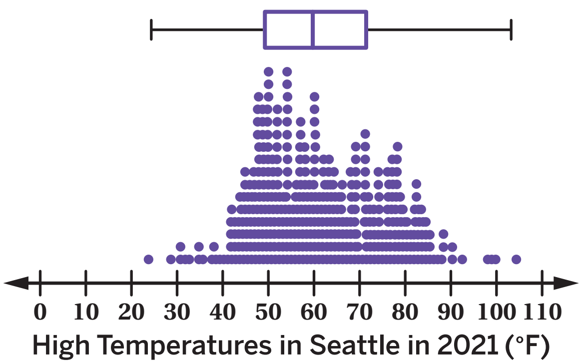
Activity 3

Name: .... Date: .... Period: ....

Temperature

7 Mia and Bao also wondered: *Which city has hotter weather?*

Here are the high temperatures in Seattle for each day of 2021.



Which representation do you prefer for making sense of the data? Circle one.

Dot plot Box plot

Explain your thinking.

Responses and explanations vary.

- **The box plot is better for me because there are too many dots to make sense of in the dot plot.**
- **I can see the shape of the data better in the dot plot.**

8 Use the digital activity to create a box plot for the Charleston data.

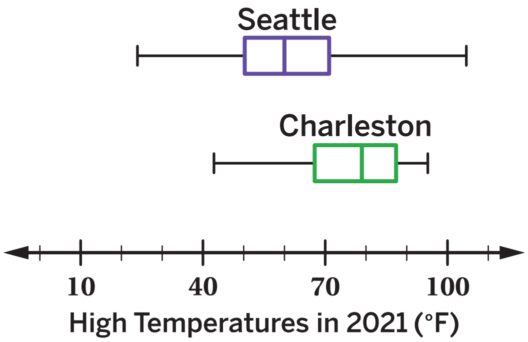
See sample response in the digital activity.

Discuss: How does the temperature in Seattle compare to the temperature in Charleston?

Responses vary.

- **The temperature in Charleston is generally higher than in Seattle because Charleston's median temperature is much higher.**
- **Seattle gets hotter than Charleston because the maximum temperature is much higher in Seattle than in Charleston.**
- **There is more variation in the weather in Seattle. The minimum temperature is lower and the maximum temperature is higher than in Charleston.**

9 Here are two box plots showing the high temperatures in Seattle and Charleston.



What is the *median* of each data set?

Seattle: **60°F**

Charleston: **79°F**

Name: ________________ Date: ________ Period: ________

Temperature (continued)

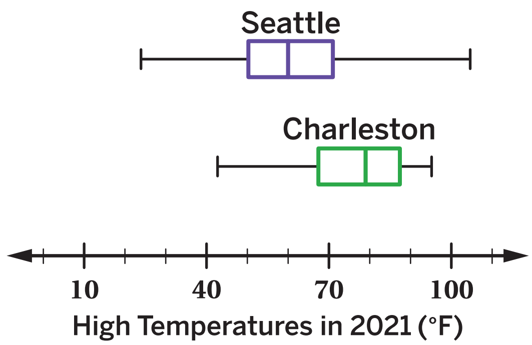
10 Here are the box plots for Seattle and Charleston.

Select *all* the statements that are true.

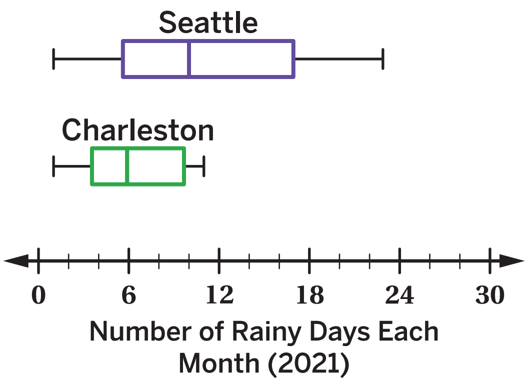
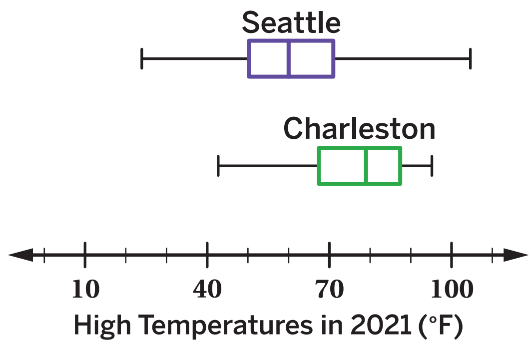
☑ **A.** Seattle had at least one day over $100°$.

☐ **B.** Charleston had exactly 25 days with a high temperature over $87°$.

☑ **C.** For about half of the days, Seattle's high temperature was between $50°$ and $72°$.



11 Here are two ways that we compared the weather in Seattle and Charleston in this lesson.

Rainy Days	High Temperatures
	

Which city's weather do you prefer? Circle one.

Responses vary.

Seattle Charleston Neither

Use information from the box plots to support your reasoning.

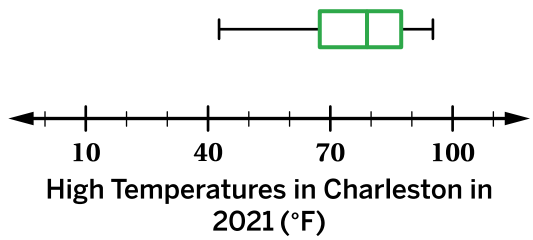
Responses vary.

- **I prefer the weather in Seattle because I like cooler weather, and a median temperature of $60°$ is more appealing to me than a median temperature of $79°$.**
- **I prefer the weather in Charleston because I hate the rain and I hate using umbrellas.**
- **I prefer neither because I want to live in a place that snows a lot and neither of these places have temperatures cold enough to snow most of the time.**

12 Synthesis

Describe what each part of a box plot tells you about a data set.

Use the box plot if it helps with your thinking.



Responses vary. **The box plot breaks the data up into four sections: the left section outside the box, two sections inside the box separated by the median, and the right section outside the box. Each section represents 25% of the data. The ends of the whiskers tell me the lowest and highest values in the data set, or the minimum and maximum.**

Things to Remember:

Name: ______________ Date: ________ Period: ________

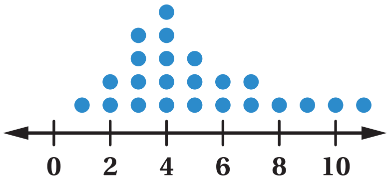
Shapes of Data

Let's describe different shapes of data.

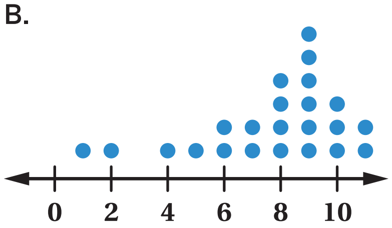
Warm-Up

1 Which one doesn't belong? Explain your thinking.

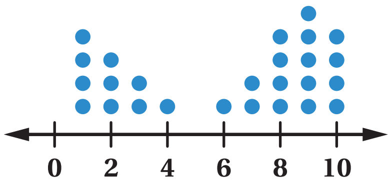
A.



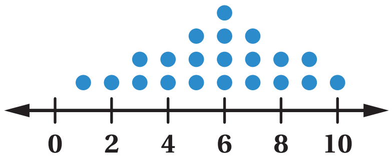
B.



C.



D.



Responses and explanations vary.

- A because it's the only one that has data points at every positive number.
- B because it's the only one where most of the data points are on the right.
- C because it looks very different from the others. It has two different peaks and no data in the middle.
- D because it's the only one that doesn't have something weird going on. It's also the only one with the most data at 6.

Activity 1

Name: .... Date: .... Period: ....

Polygraph

2 Play a few rounds of Polygraph with your classmates!

You will use the Activity 1 Sheet. For each round:

- You and your partner will take turns being the Picker and Guesser.
- Picker: Select a histogram from the Activity 1 Sheet. Keep it a secret!
- Guesser: Ask the Picker yes-or-no questions, eliminating histogram until you're ready to guess which histogram the Picker chose.

Record helpful questions from each round in this workspace:

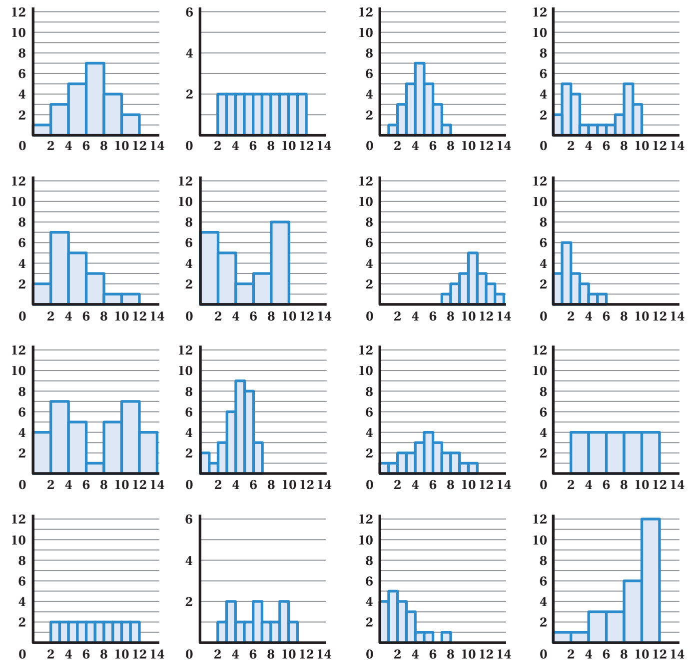
Responses vary.

Name: .... Date: .... Period: ....

Shapes of Data Sets

3 Here are some terms that describe the *shape* of a data set:

- Skewed
- Uniform
- Symmetric
- Bimodal
- Bell-shaped



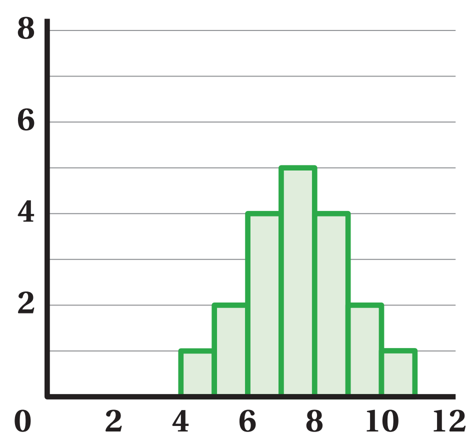
a Let's see which histograms each term describes.

b **Discuss:** What does each term mean?

Responses vary.

- Skewed: Most of the data is on the left or right.
- Uniform: The histogram bars are all the same height.
- Symmetric: The left side of the data is the same as the right side.
- Bimodal: The data looks like two mountains next to each other.
- Bell-shaped: The data looks like one mountain, or a bell.

4 Here is a histogram.



Select *all* the terms that describe the shape of this histogram.

☐ **A.** Skewed

☐ **B.** Uniform

☑ **C.** Symmetric

☐ **D.** Bimodal

☑ **E.** Bell-shaped

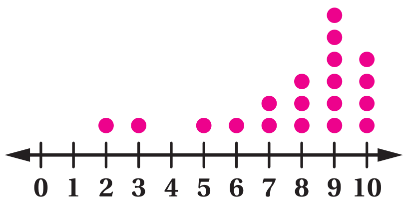
Name: ______ Date: ______ Period: ______

Shapes of Data Sets (continued)

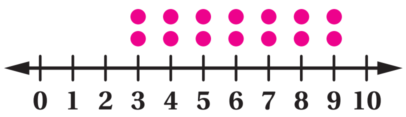
5 Make a dot plot that matches each shape.

Dot plots vary.

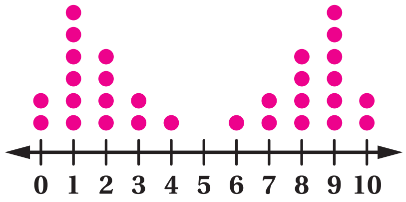
Skewed



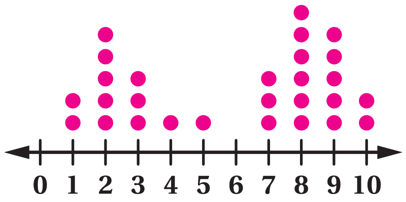
Uniform



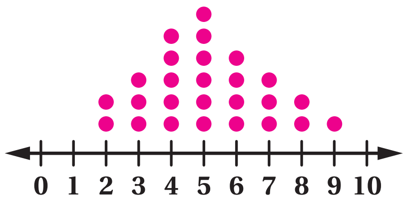
Symmetric



Bimodal



Bell-shaped



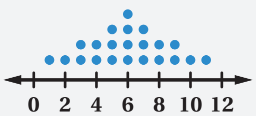
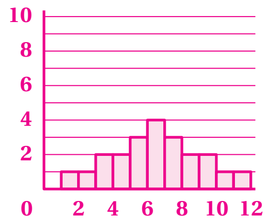
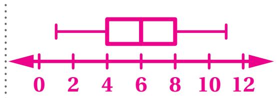
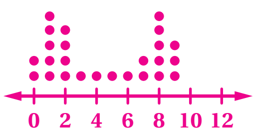
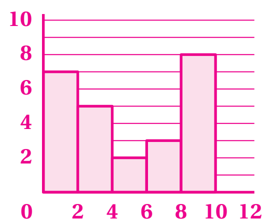
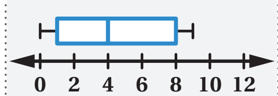
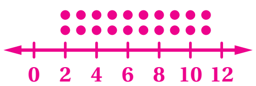
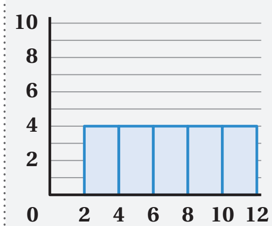
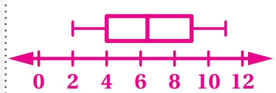
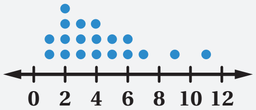
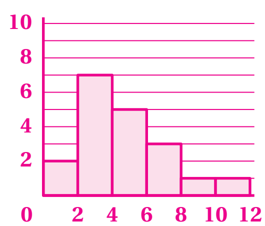
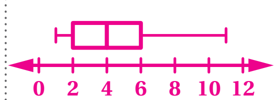
Activity 3

Name: ______________________ Date: __________ Period: __________

All the Representations

6 You will use a set of cards to complete this activity.

- Match each card with one of the data sets.
- Select the terms that describe the shape of each data set. Share your reasoning with a partner.

	Dot Plot	Histogram	Box Plot	Description
Data Set 1		Card H 	Card C 	☐ Skewed ☑ Symmetric ☐ Bimodal ☑ Bell-shaped ☐ Uniform
Data Set 2	Card E 	Card D 		☐ Skewed ☐ Symmetric ☑ Bimodal ☐ Bell-shaped ☐ Uniform
Data Set 3	Card G 		Card B 	☐ Skewed ☑ Symmetric ☐ Bimodal ☐ Bell-shaped ☑ Uniform
Data Set 4		Card A 	Card F 	☑ Skewed ☐ Symmetric ☐ Bimodal ☐ Bell-shaped ☐ Uniform

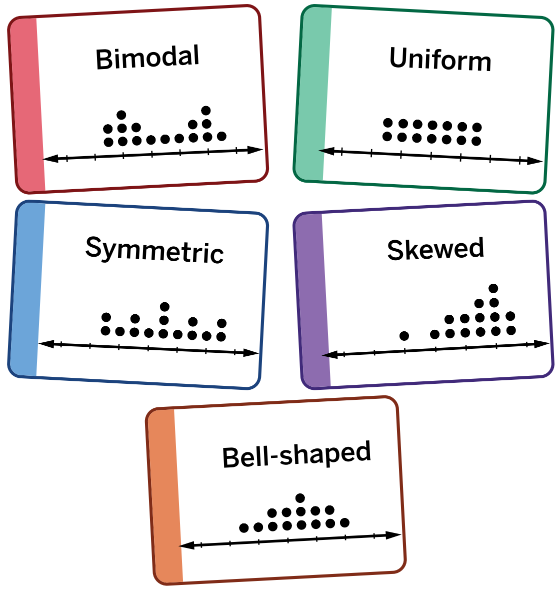
7 Synthesis

Here are some of the terms we used to describe data sets today.

Choose *three* terms. Describe what each term means in your own words.

Responses vary.

- Bimodal: There are two peaks in the data.
- Uniform: The data points are evenly distributed, so there are about the same number of data points at each value.
- Symmetric: The data has a line of symmetry.
- Skewed: There are more data points on one side of the dot plot than the other.
- Bell-shaped: Most of the data is at the center, with fewer points farther from the center.

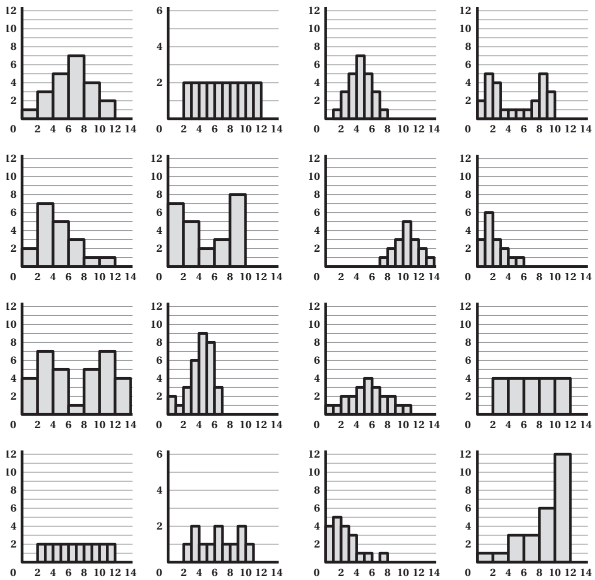


Things to Remember:

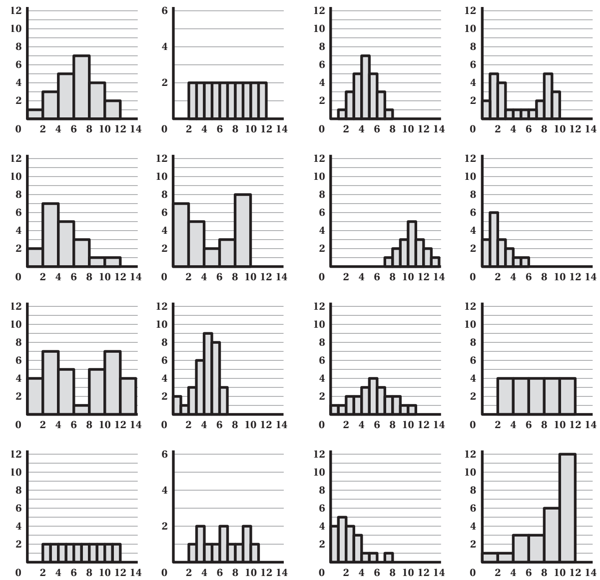
Name: ________________ Date: ________ Period: ________

Polygraph Set A

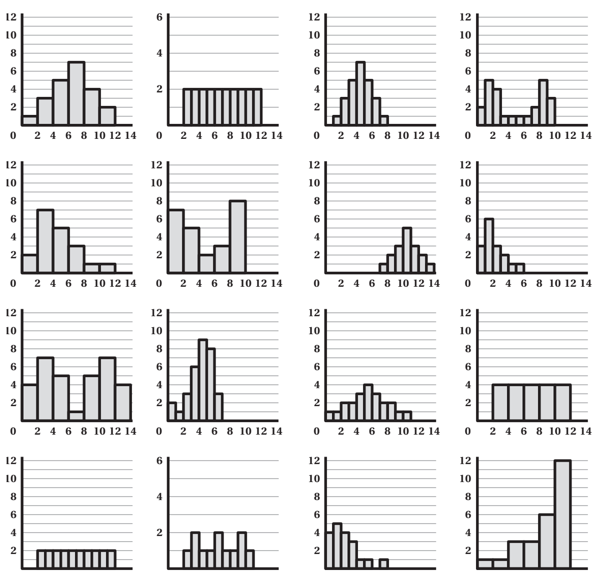
Round 1



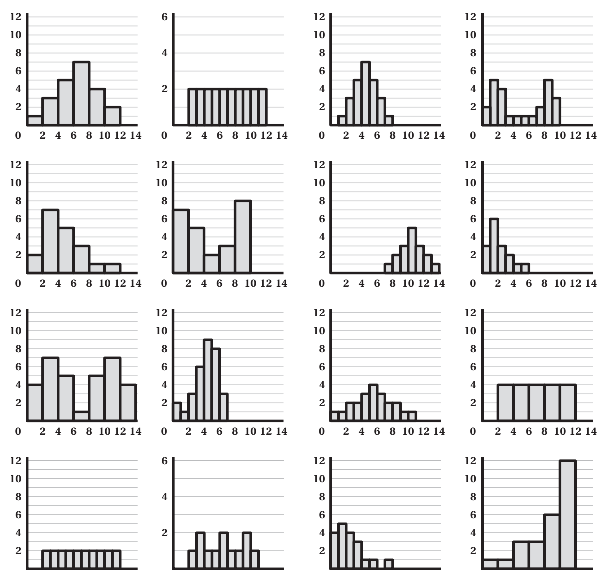
Round 2



Round 3



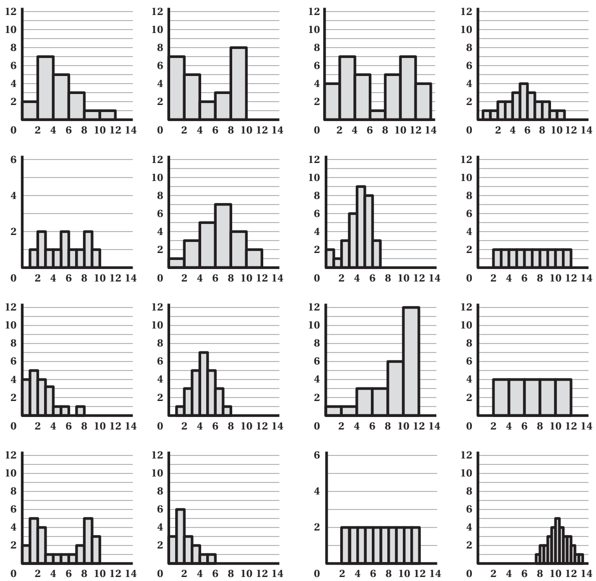
Round 4



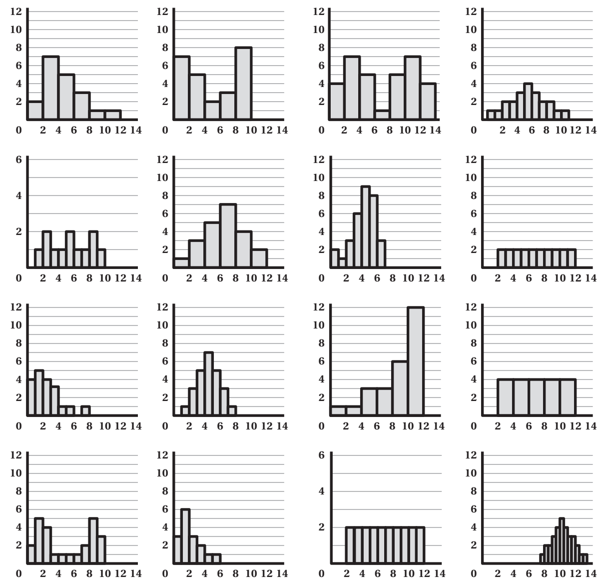
Name: .... Date: .... Period: ....

Polygraph Set B

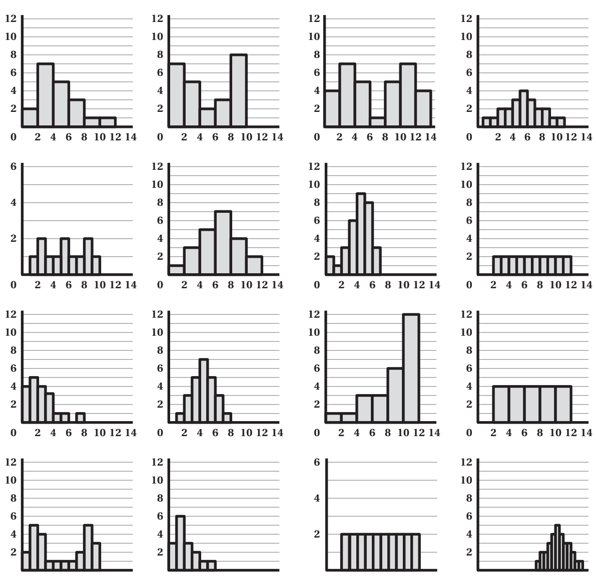
Round 1



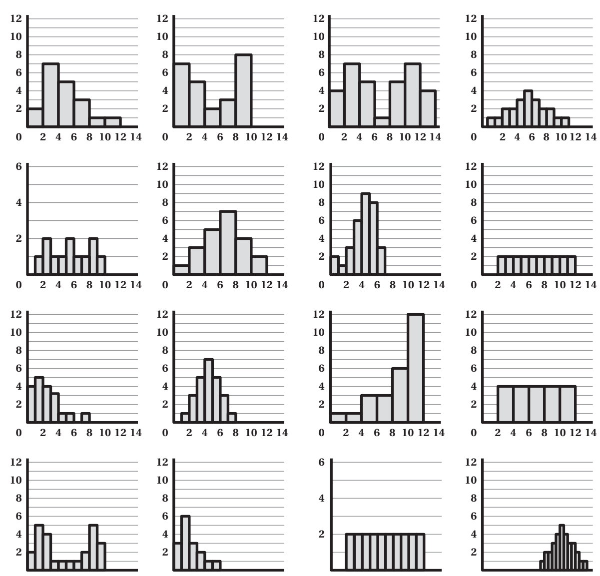
Round 2



Round 3



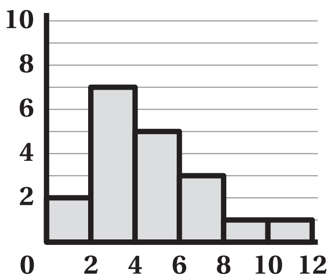
Round 4



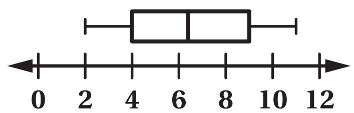
All the Representations

Directions: Make one copy per pair of students. Then pre-cut the cards and give each pair of students one set.

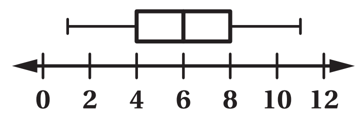
Card A



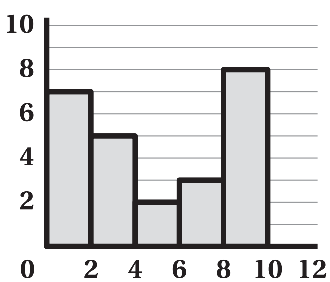
Card B



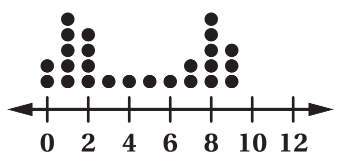
Card C



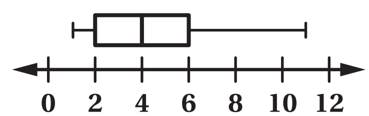
Card D



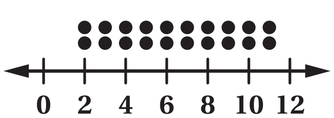
Card E



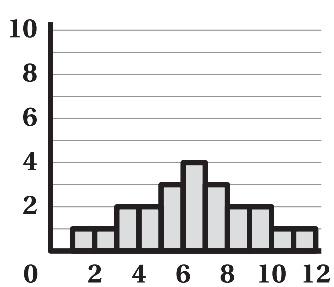
Card F



Card G

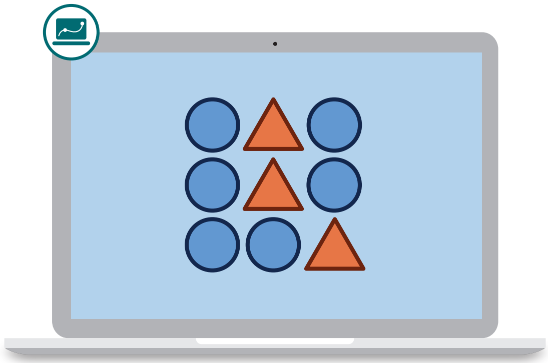


Card H



Unit 3
Lesson
7

Name: .... Date: .... Period: ....



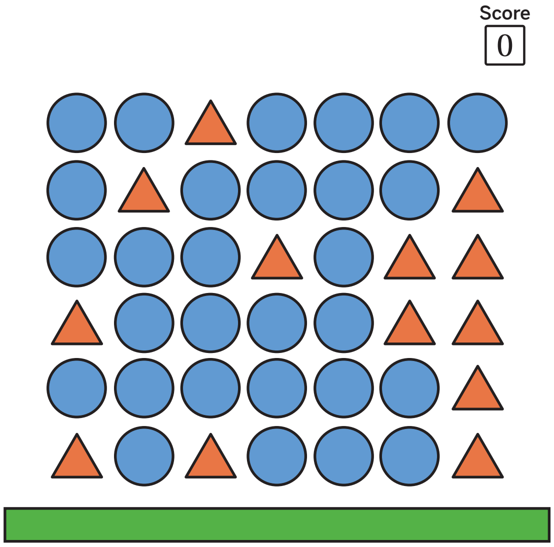
Quick Pick

Let's explore how extreme values impact mean and median.

Warm-Up

1 This game is called Quick Pick. Here's how it works:

- You pick different shapes to get points.
- Triangles are 3 points. Circles are 2 points.
- A five-second timer starts after your first pick.



Use the digital activity to play up to 15 times.

Scores vary.

Activity 1

Name: ________ Date: ________ Period: ________

Measures of Center

2 In the digital activity, look at your scores and the scores of three students from another class.

Choose a student and describe their scores.

Responses vary.

- Dhruv got the score 26 three times. He also had one game where he only got 2 points.
- Chloe played the game 12 times. Her data is pretty bell-shaped.
- Oscar had scores ranging from 15 to 31.

3 A **measure of center** is a single number that represents a central value in a data set.

Mean and *median* are two measures of center.

Here are the mean and median for Oscar's scores.

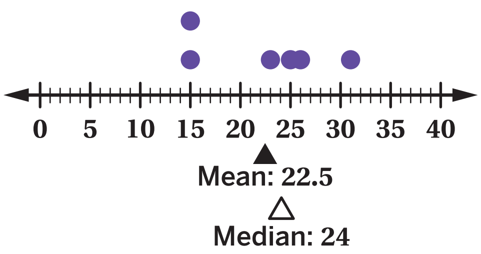
What are some things you know about calculating the mean and median?

Responses vary.

- The median is the value in the middle.
- To figure out the mean, add up all the numbers and divide by how many numbers there are.

Oscar's Scores

15	25	15	23	26	31

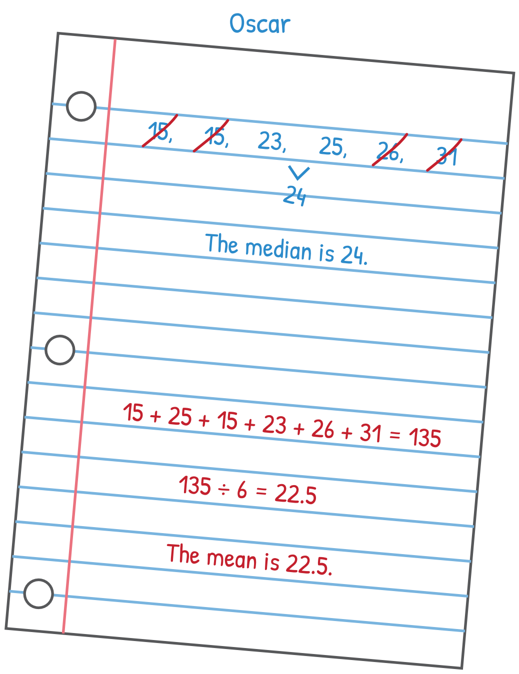


4 Here is the work Oscar did to calculate the median and the mean.

Discuss: How did Oscar calculate each measure of center?

Responses vary.

- To calculate the median, Oscar put the numbers in order from smallest to largest. He determined the numbers in the middle and then found the middle of those two numbers.
- To calculate the mean, Oscar added up all of the numbers and then divided by 6 because there are 6 numbers in his data set.



Activity 2

Name: .... Date: .... Period: ....

Shape and Center

5 Here are Dhruv's scores from the game. Determine the median and mean.

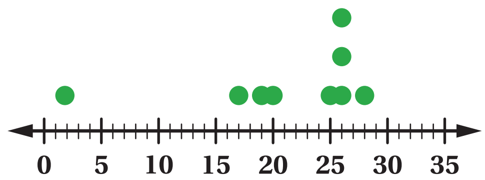
Use the Desmos Graphing Calculator if it helps with your thinking.

Median: 25

Mean: 21

Dhruv's Scores

2	17	26	26	20	28	25	19	26



6 In the digital activity, look at the mean and median for Dhruv's scores.

a **Discuss:** Why do you think the mean and median are far apart?

***Responses vary*. The mean is much less than the median because of the one game where Dhruv scored only 2 points.**

b Which measure of center would you use to represent Dhruv's typical score? Circle one.

Mean   Median

Explain your thinking.

***Responses and explanations vary*. I would use the median because the time Dhruv scored 2 impacts the mean but not the median. That score was probably a mistake and not really typical.**

7 Here are Chloe's scores from the game.

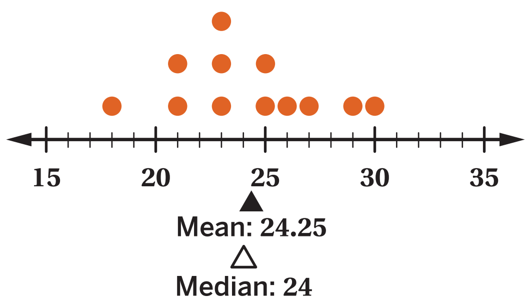
Imagine that her highest score was 300 instead of 30.

Which measure(s) of center would increase? Circle one.

Mean   Median   Both   Neither

Chloe's Scores

18	30	23	23	25	29	27	26	25	21	23	21

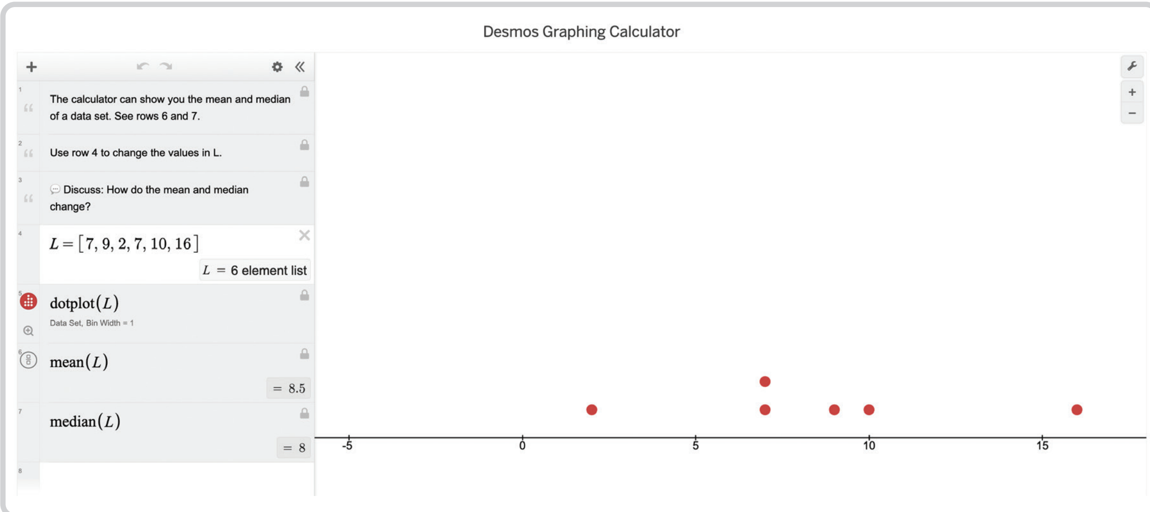


Activity
3

Name: .... Date: .... Period: ....

Calculate and Create

8 The Desmos Graphing Calculator can show you the mean and median of a data set.

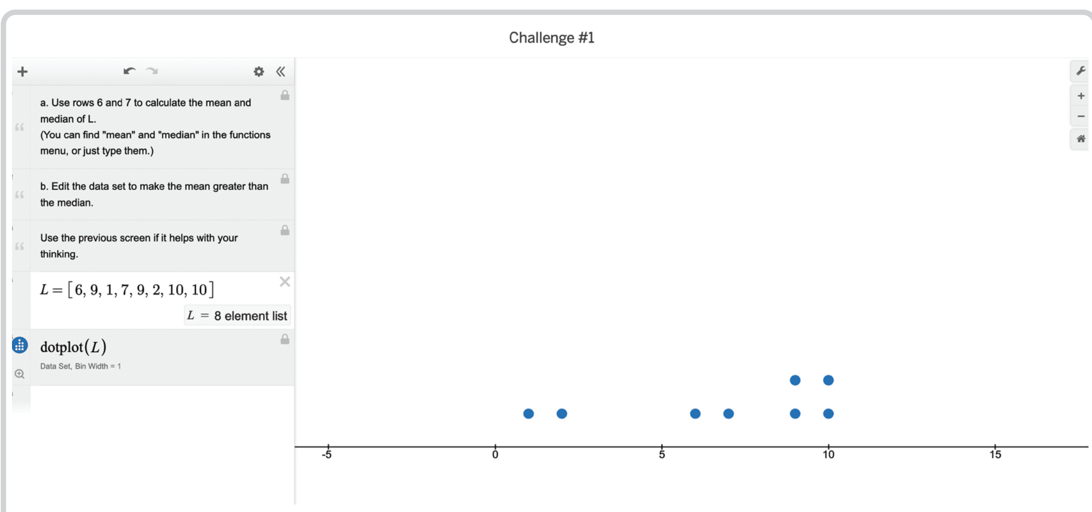


Use the digital activity to change the values in L.

Discuss: How do the mean and median change?

***Responses vary.* I noticed that the mean got a lot bigger when I added a large number, but the median didn't change much.**

9 **a** In the digital activity, calculate the mean and median of L. (You can find "mean" and "median" in the functions menu, or just type them.)



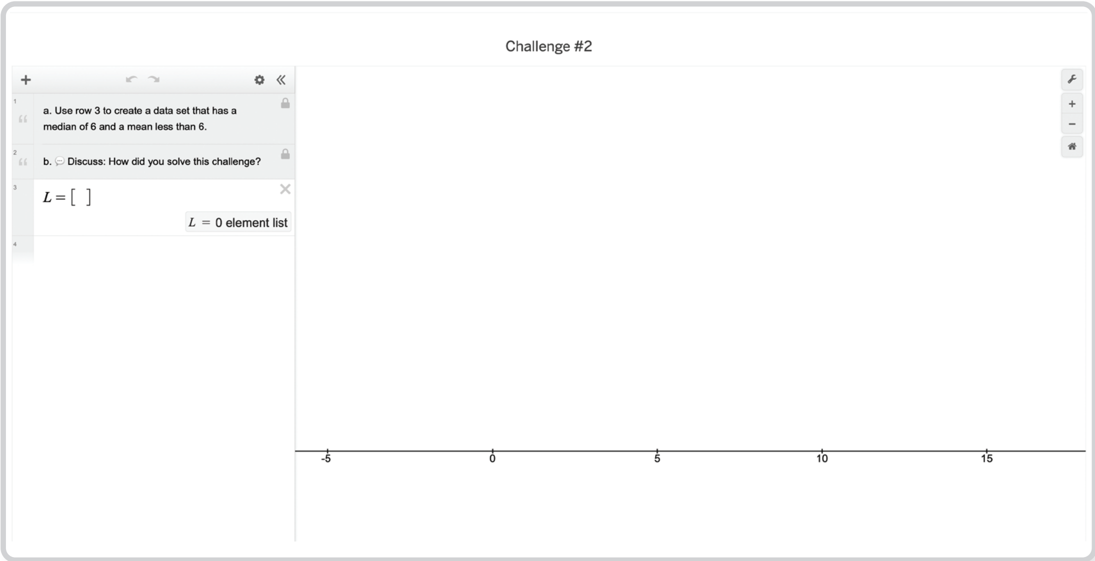
b Edit the data set to make the mean greater than the median.

Responses vary as students change the data set.

Name: ________ Date: ________ Period: ________

Calculate and Create (continued)

10 **a** In the digital activity, create a data set that has a median of 6 and a mean less than 6.



Responses vary. $L = [0, 0, 0, 6, 7, 7, 7]$

b **Discuss:** How did you solve this challenge?

Responses vary. **I made sure the median was 6 by keeping 6 in the middle of my data set. To make the mean less than 6, I made the numbers less than 6 really small and the numbers greater than 6 close to 6.**

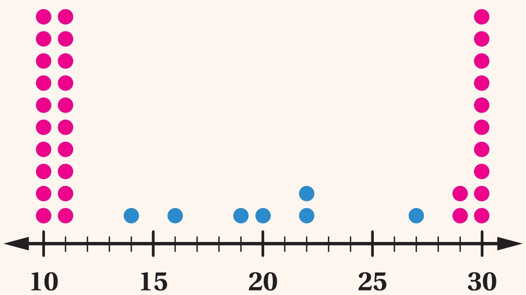
Explore More

11 Here is a dot plot.

The mean and median are currently equal.

Add data points to make the mean and median as far apart as you can.

Responses vary. **Here is an example of a dot plot with a mean and median 7.15 units apart.**



12 Synthesis

How are median and mean *alike*? How are they *different*?

Use the data set if it helps with your thinking.

Responses vary.

Alike:

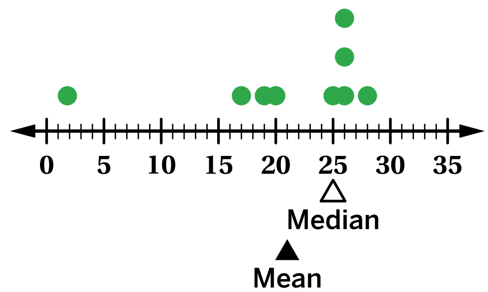
- **Mean and median are both measures of center.**
- **They're both one number that represents the typical value.**

Different:

- **Mean and median are different because they're calculated differently.**
- **The median is different from the mean because it's less likely to change when you add extreme values.**

Dhruv's Scores

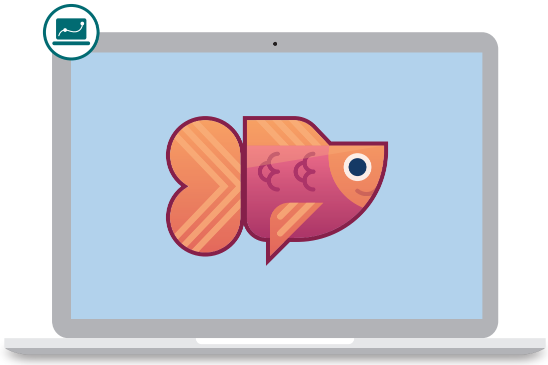
2	17	26	26	20	28	25	19	26



Things to Remember:

Unit 3
Lesson
8

Name: Date: Period:



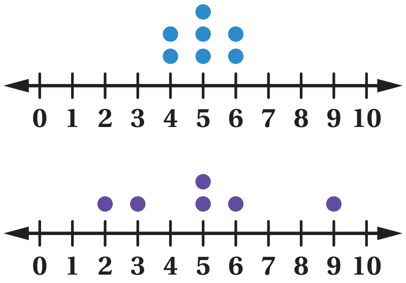
Finding Desmo

Let's explore what standard deviation describes about a data set.

Warm-Up

1 Here are two data sets.

How are they alike? How are they different?



Responses vary.

Alike:

- **It looks like the middle of each data set is at 5.**
- **Both data sets have a mean and median of 5.**

Different:

- **The points in the bottom data set are a lot more spread out.**
- **The MAD of the top data set is much smaller.**

Activity 1

Name: .... Date: .... Period: ....

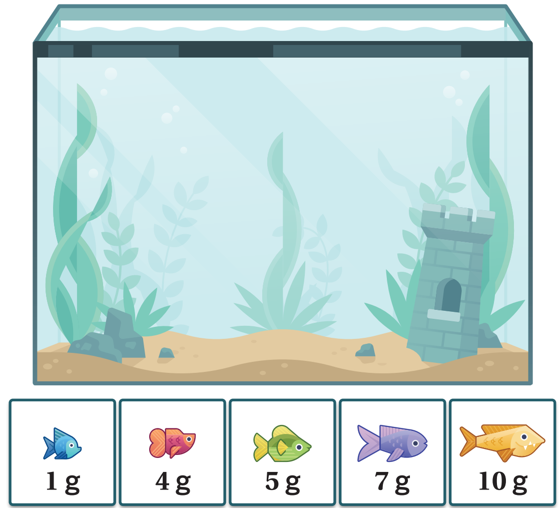
Introduction to Standard Deviation

2 You are getting a new fish tank.

a In the digital activity, add up to 10 fish to your tank. *Fish tanks vary.*

b Tell a partner how you decided which fish you wanted and how many.

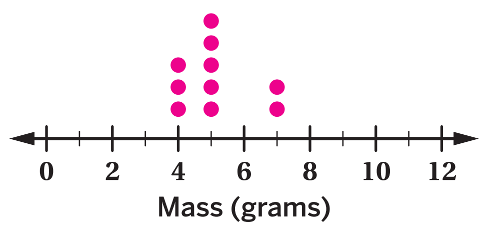
Responses vary.



3 A customer at the pet store says: *I would like fish that are close to the same size*.

a In the digital activity, build them a fish tank you think they would like.

Fish tanks vary.



b **Discuss:** What do you notice and wonder about the dot plot?

Responses vary.

- I notice the middle of the diamond shows the mean.
- I notice the standard deviation is the same as half the width of the diamond.
- I notice the standard deviation gets larger if the masses of the fish are spread out.
- I wonder what the standard deviation is measuring.
- I wonder how you can calculate the standard deviation.
- I wonder why the standard deviation is 0 when the fish are all the same size.

Activity 1

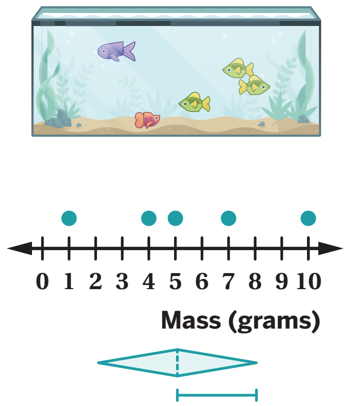
Name: ________ Date: ________ Period: ________

Introduction to Standard Deviation (continued)

4 One way to determine the consistency of data is to calculate the **standard deviation**, which is a **measure of spread**.

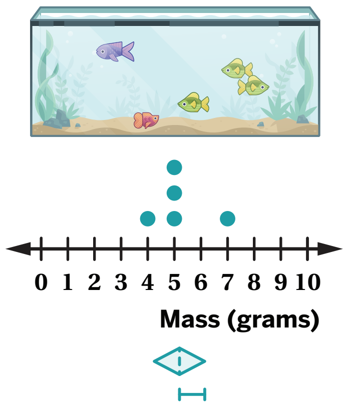
Here are three different fish tanks.

Tank A



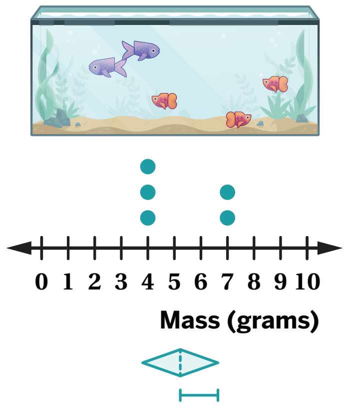
Standard deviation: 3.01 grams

Tank B



Standard deviation: 0.98 grams

Tank C



Standard deviation: 1.47 grams

Explain what you think standard deviation measures.

Explanations vary.

- Standard deviation measures how close or spread out the weights of the fish are. If the fish are all similar in weight, the standard deviation is small. If they're very different in weight, the standard deviation is large.
- Standard deviation measures the spread of the fish. It's similar to MAD because the more spread out the data is, the larger the standard deviation will be.

Activity 2

Name: ________ Date: ________ Period: ________

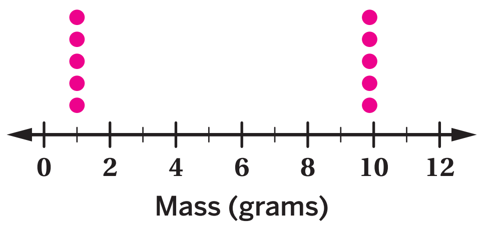
Standard Deviation Challenges

Use the digital activity for Problems 5–7.

5 Make a fish tank with a large standard deviation. Discuss your strategy with a partner.

Note: You can add up to 10 fish to your tank.

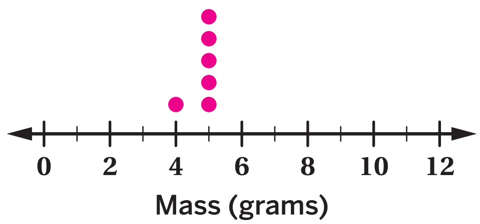
Fish tanks vary.



6 Make a fish tank with a small standard deviation that is greater than 0. Discuss your strategy with a partner.

Note: You can add up to 10 fish to your tank.

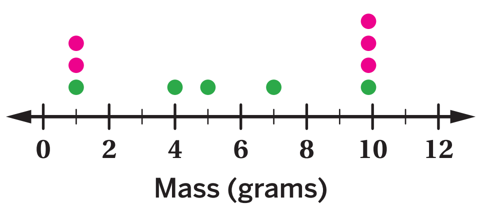
Fish tanks vary.



7 Add fish to this tank to increase the standard deviation.

Discuss: Is there anything you find surprising?

***Fish tanks and responses vary.* Adding three 10-gram fish and two 1-gram fish changed the standard deviation a lot.**



Original standard deviation: 3.01 g

Activity 3

Name: .... Date: .... Period: ....

Calculating Standard Deviation

8 Here are the masses, in grams, of the fish in three new tanks.

Tank D	13, 13, 13, 13, 14, 15
Tank E	3, 4, 6, 7, 7, 9
Tank F	7, 7, 7, 7, 7, 9

Which tank do you think has the largest standard deviation? Circle one.

Tank D   Tank E   Tank F   I'm not sure

Explain your thinking.

Responses and explanations vary.

- **I think Tank E has the largest standard deviation because the fish masses are the most spread out.**
- **I'm not sure which tank has the largest standard deviation because I don't know how to calculate it.**

Note: Students are not yet expected to calculate standard deviation.

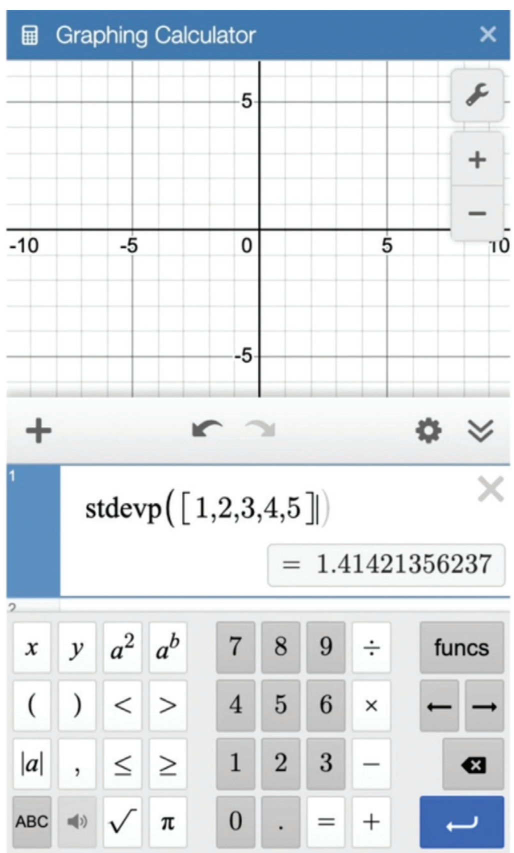
9 Calculators can help calculate standard deviation.

a In the digital activity, watch an animation to see how to calculate the standard deviation of [1, 2, 3, 4, 5].

b Use the Desmos Graphing Calculator to answer:

What is the standard deviation of [3, 4, 6, 7, 7, 9]? Circle one.

1.75   2   3.14   6



Name: .... Date: .... Period: ....

Calculating Standard Deviation (continued)

10 Order the fish tanks from smallest to largest standard deviation.

Use the Desmos Graphing Calculator to help with your thinking.

Tank A	13, 13, 13, 13, 14, 15
Tank B	3, 4, 6, 7, 7, 9
Tank C	7, 7, 7, 7, 7, 9

Tank C	Tank A	Tank B

Smallest Standard Deviation **Largest Standard Deviation**

11 Here is a data set: 1, 2, 3, 4, 5, 6, 7.

Select *all* the moves that would make the standard deviation *larger*.

Use the calculator to help you with your thinking.

- ☐ **A.** Removing the 7 from the data set.
- ☐ **B.** Adding a 0 to the data set.
- ☑ **C.** Removing the 3 from the data set.
- ☑ **D.** Adding a 4 to the data set.
- ☐ **E.** Increasing each value by 1.

7 Synthesis

Describe what the mean and standard deviation tell us about the fish in Tank A and Tank B.

***Responses vary.* The mean tells us that the fish in Tank A are a lot larger than the fish in Tank B. The standard deviation tells us that the fish in Tank A are very similar in size, while the fish in Tank B are more varied in size.**

Tank A

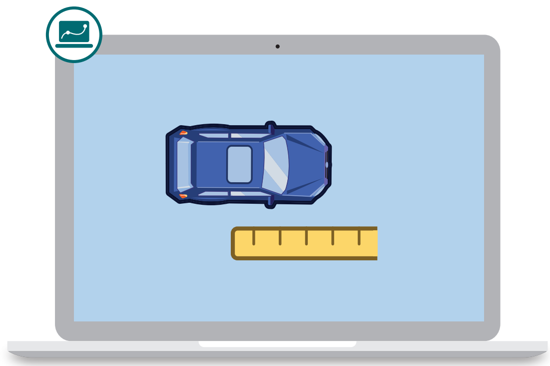
Mean: 20 grams
Standard deviation: 0.5 grams

Tank B

Mean: 5 grams
Standard deviation: 3.7 grams

Things to Remember:

Name: .... Date: .... Period: ....



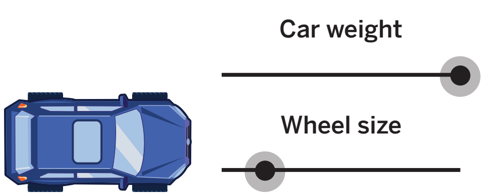
Race Car

Let's compare measures of spread in skewed data sets.

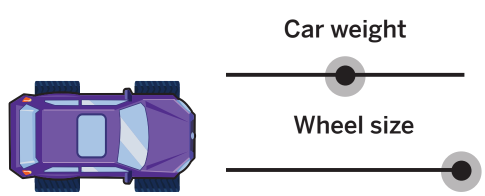
Warm-Up

1 Take a look at two toy race cars with different colors, weights, and wheel sizes.

Car A

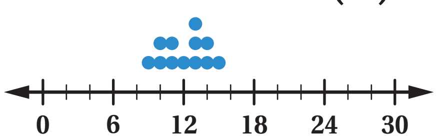


Car B

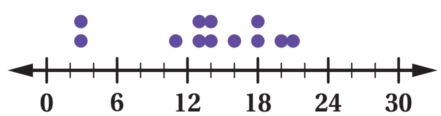


2 **a** Each car was launched 12 times. Compare their results.

Car A's Distances (in.)



Car B's Distances (in.)



b **Discuss:** Which car generally travels farther? How do you know?

Responses and explanations vary.

- **Car A generally travels farther because all of its distances are within 9 and 15, and Car B has several distances that are less than that.**
- **Car B generally goes farther because more of its distances were farther than Car A's.**

Activity 1

Name: .... Date: .... Period: ....

Box Plots and Races

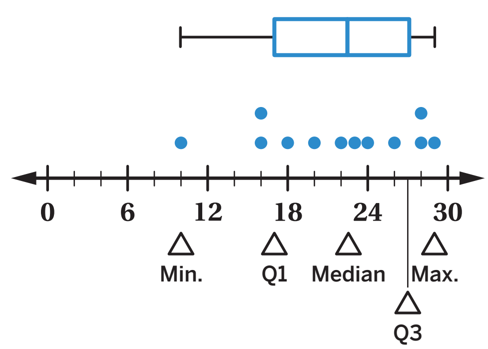
3 Abena launched her car 12 times.

a Let's watch how the dot plot of Abena's data is made into a box plot.

b What do you think are the advantages of each representation? ***Responses vary.***

Dot plot: **You can see each individual distance that Abena's race car traveled. You can also see the number of times her car raced.**

Box plot: **You can see a lot of statistics! For example, you can just look at the different parts of the box plot and know the median, minimum, and maximum.**



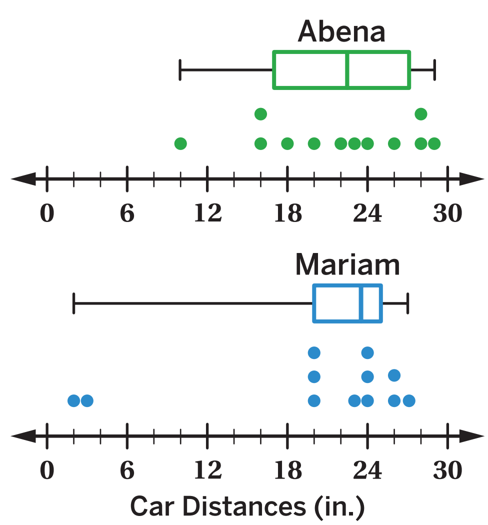
4 Professor Cho is giving out some racing awards.

Discuss:

- What awards do you think Professor Cho should give out?
- For each award you come up with, who would win between Abena and Mariam?

Responses vary.

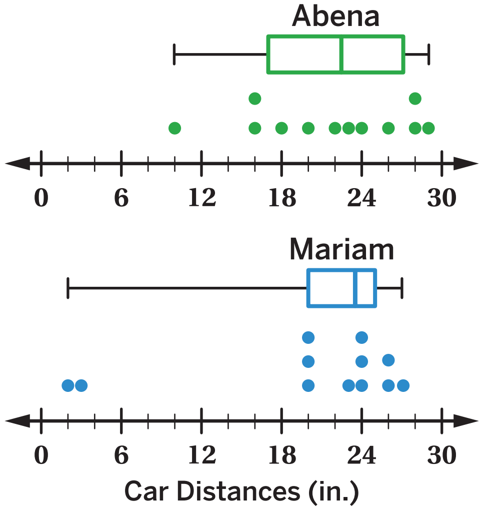
- **I think Professor Cho should give out an award for the farthest distance and an award for the farthest typical distance.**
- **Abena would win the farthest distance award because her maximum distance is greater. Mariam would win the farthest typical distance award because her median is greater.**



Name: Date: Period:

Box Plots and Races (continued)

5 Professor Cho wants to give an award for Most Consistent Race Car using the middle half of each car's data.



Select one question to answer:

A. Do you think Abena's or Mariam's race car is more consistent? Why?

B. Why might Professor Cho look at only the middle half of the data?

C. How might Professor Cho measure the middle half of the data?

Responses vary.

- **Question A: I think Mariam would win the award because the middle half of her data looks less spread out. This means it's more consistent.**
- **Question B: I think Professor Cho might only look at the middle half of the data because it ignores values that might have been mistakes. Overall, Mariam seems more consistent, but if you include all her data, it's more spread out.**
- **Question C: I think Professor Cho could measure the middle half of the data by figuring out how wide the box in the box plot is.**

Name: .... Date: .... Period: ....

Interquartile Range

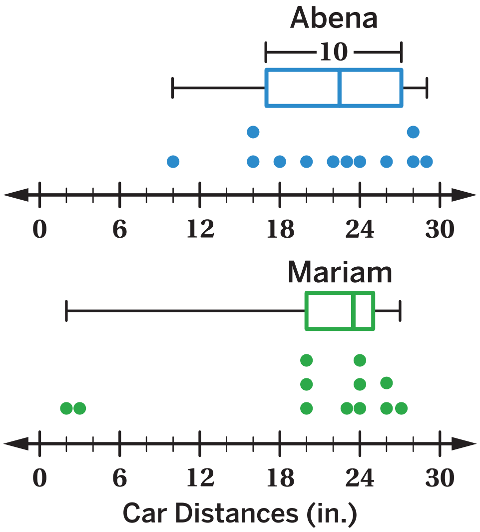
6 One way to measure the middle half of a data set is to use the *interquartile range* (or *IQR*).

IQR is a statistic that measures the spread of a data set.

The middle half of Abena's distances are between 17 and 27 inches, so the IQR of her data is 10 inches.

What is the interquartile range (IQR) of Mariam's data?

5 inches



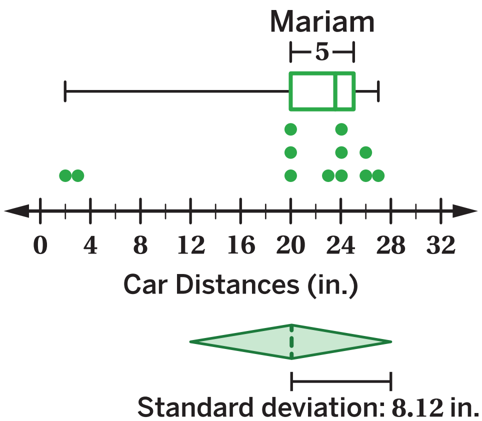
7 Standard deviation and IQR are statistics that measure the spread of a data set.

Which statistic is a more appropriate measure of the spread of Mariam's data? Circle one.

IQR   Standard deviation

Explain your thinking.

***Explanations vary.* Since the standard deviation is based on the mean, and the mean isn't a reliable statistic because the 2 and 3 are included in the data set, the IQR is a more appropriate measure of spread for Mariam's data set.**

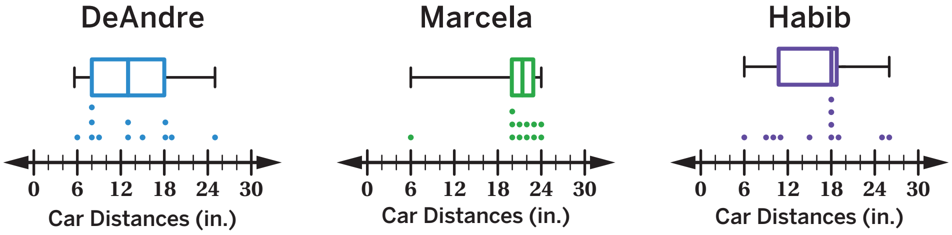


Activity 2

Name: ________ Date: ________ Period: ________

Interquartile Range (continued)

8 Help Professor Cho give out the award for Most Consistent Race Car.



a Determine the IQR for each student's data.

Student	IQR (in.)
Abena	10
Mariam	5
DeAndre	**10**
Marcela	**3**
Habib	**8**

b Who do you think Professor Cho will give the award to? Circle one.

Abena Mariam DeAndre Marcela Habib

Explain your thinking.

Responses and explanations vary.

- **I think Professor Cho will give the award for the most consistent car to Marcela because her IQR is the smallest.**
- **I think Abena should win the award because all of her points are close together. There are no big gaps.**

Activity 3

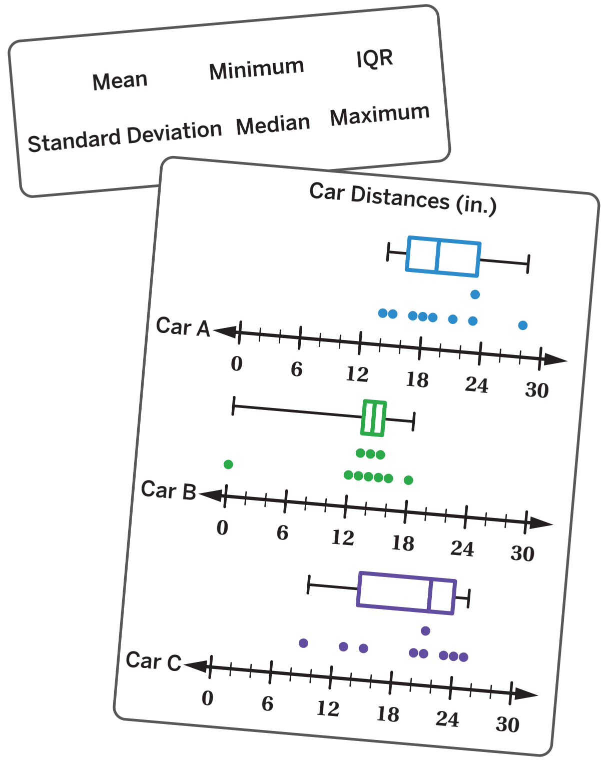
Name: .... Date: .... Period: ....

Which Statistic?

9 Here is the data for three new cars.

Which statistic would you use to answer each of the following questions?

Question	Statistic
Which car had the best individual launch?	**Maximum**
Which car was the most consistent?	**IQR**
Which car typically traveled the farthest distance?	**Median**



10 Which car would you give the Best Overall Car award to? Circle one. ***Responses vary.***

Car A Car B Car C

Use vocabulary from this unit to justify your thinking.

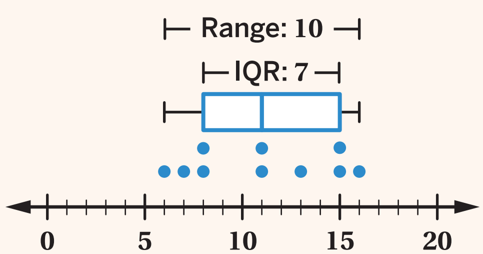
Explanations vary.

- **Car A. The maximum distance is 28 inches. That means Car A went really far on one of the launches! None of the other race cars went that far.**
- **Car B. The IQR is 2, which is much smaller than the IQR of the other data sets. This means Car B is very consistent.**
- **Car C. The median is 21 inches, which is quite a bit higher than the medians of the other data sets. That means the typical distance Car C travels is farther than the other cars.**

Explore More

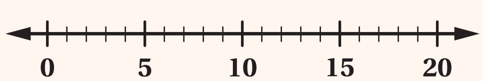
11 Range and IQR are both measures of spread.

Here is a box plot with an IQR of 7 and a range of 10.



Can you make a second box plot with the following features? Select *all* the box plots that are possible to create.

- [x] **A.** A smaller IQR and larger range
- [x] **B.** A larger IQR and smaller range
- [ ] **C.** An IQR of 10 and a range of 7
- [x] **D.** An IQR of 0 and a large range

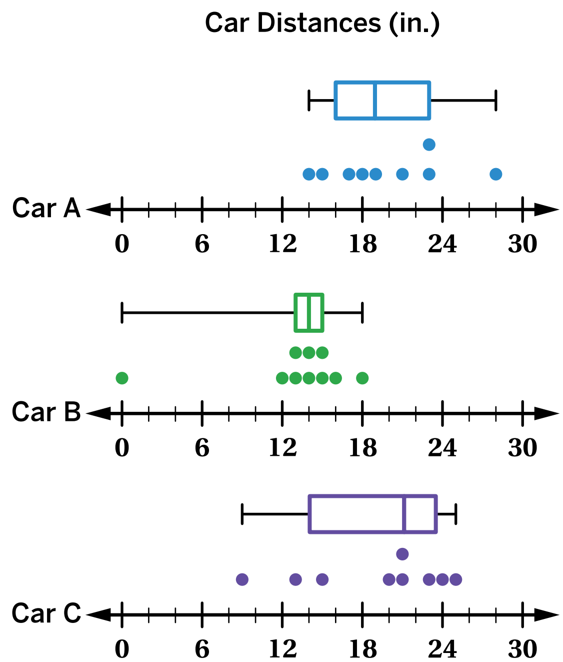


12 Synthesis

How can the interquartile range help you compare data sets?

Use the example data if it helps with your thinking.

***Responses vary.* The interquartile range, or IQR, helps us determine which data set is more consistent. By measuring the middle half of the data between quartile 1 and quartile 3, we can compare the spread of each data set without the effect of any data that is far away from the middle.**



Things to Remember:

Unit 3
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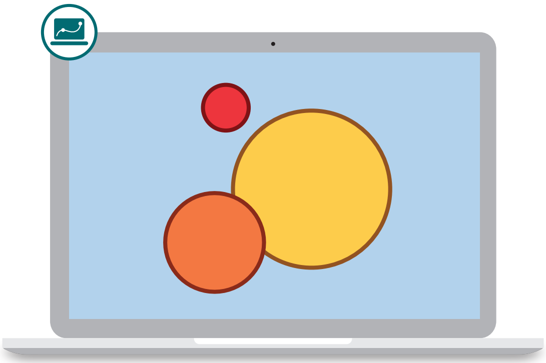
11

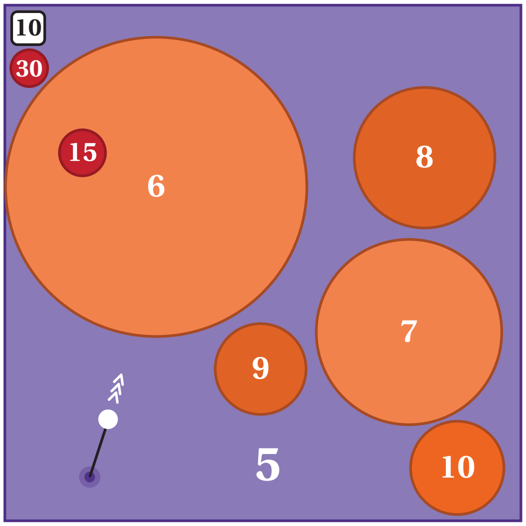
Name: Date: Period:



Far Out

Let's determine whether a data point is an outlier and consider its effect on the mean and median.

Warm-Up



1 Use the digital activity to play a game.

Play the game as many times as you want.
Scores vary.

Activity 1

Name: Date: Period:

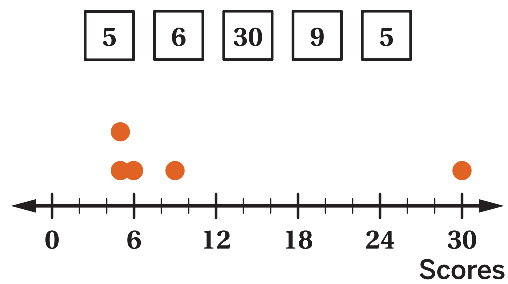
Outliers and Their Effects

2 Koharu played the game 5 times.

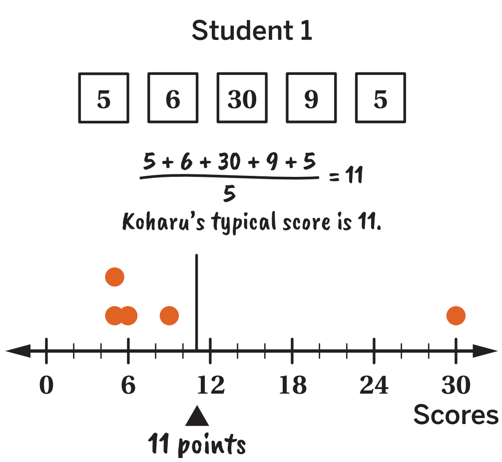
Here are her scores.

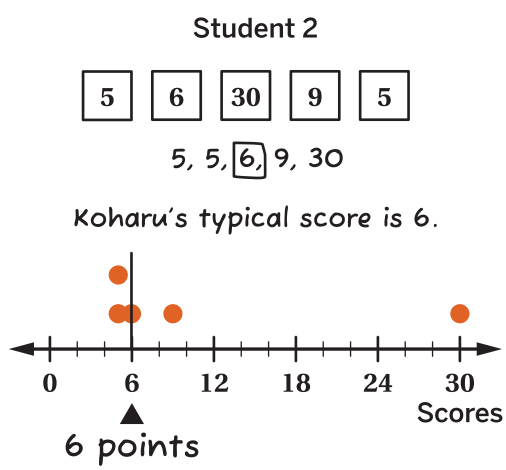
What do you think is her typical score?

Responses vary.



3 Here are two different strategies for determining Koharu's typical score.





Discuss: How are Student 1's and Student 2's strategies alike? How are they different?

Responses vary.

Alike
- They each rewrote the scores to help them think about the scores.
- They each graphed their calculated typical score on the dot plot.

Different
- Student 1 added up all the scores and divided by the number of scores. Student 2 put the scores in order and circled the middle one.
- Student 1 calculated the mean, and Student 2 determined the median.

Activity 1

Name: .... Date: .... Period: ....

Outliers and Their Effects (continued)

4 Koharu says: *You shouldn't use the mean for the typical score because the* 30 *messes it up*.

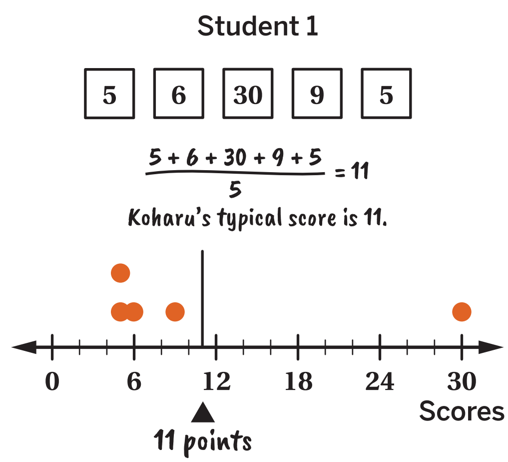
Do you agree? Circle one.

Yes No I'm not sure

Explain your thinking.

Responses and explanations vary.

- **Yes. The mean is higher than 4 out of 5 of Koharu's scores, so it isn't a good typical score.**
- **I'm not sure. The mean does seem really high, but Korahu got a 30 once and it probably wasn't a mistake.**

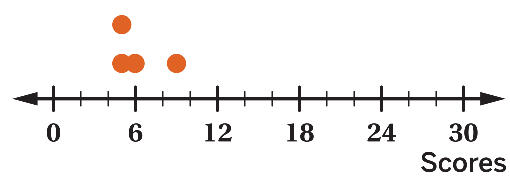


5 In Koharu's data, 30 is an *outlier* because it's far from other values in the data set.

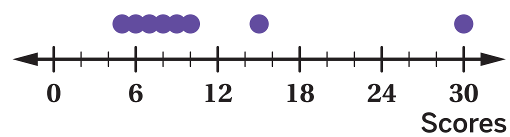
Outliers can have a big impact on the mean of a data set.

Circle the point(s) in Yona's data that you think are outliers.

***Responses vary.* Some students may select 30, while others may select both 15 and 30.**



Koharu



Yona

6 Let's look at a dot plot and a box plot for Yona's data.

a Watch what happens when we click and unclick the "Exclude outliers" checkbox.

b **Discuss:** How does the checkbox change the box plot?

***Responses vary.* The checkbox shows each outlier as its own empty dot and shortens the right section of the box plot. The rest of the box plot didn't change.**

Activity 2

Name: ______ Date: ______ Period: ______

Outliers and Box Plots

7 You can make a box plot in the Desmos Graphing Calculator to determine whether there are any outliers in a data set.

a Look at the rows in the folder "Yona's Data" in the digital activity to see how to make a box plot.

b Turn on and open the folder labeled "Brandon's Data".

c Use the next available row to make a box plot for Brandon's data.

d **Discuss:** Are there any outliers?

15 and 30 are both outliers in Brandon's data.

8 Making a box plot can help us determine the outliers in any data set.

a Let's watch an animation.

b Use the Desmos Graphing Calculator to answer:

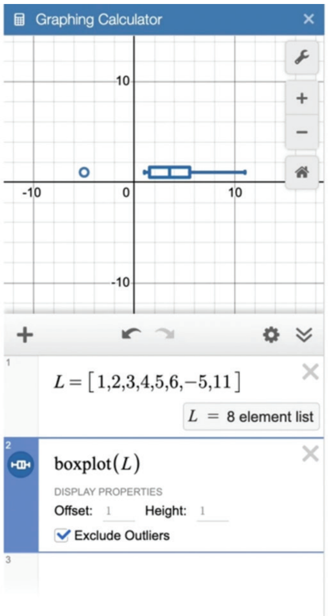
Which value is an outlier in this data set?
[1, -1, 0, 0, 1, -7, 2, 3, 7]

A. -7

B. -1

C. 3

D. 7



Activity 2

Name: ________ Date: ________ Period: ________

Outliers and Box Plots (continued)

9 **a** In the digital activity, drag the point to see when a value becomes an outlier.

b How do you think the calculator determines if a value in a data set is an outlier?

***Responses vary.* I think it has something to do with the IQR. When the IQR is smaller, there seems to be more outliers.**

10 One way the Desmos Graphing Calculator decides if a value is an outlier is by looking at its distance from *quartiles* 1 or 3.

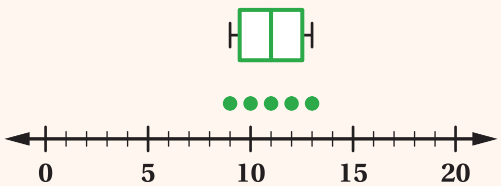
a Let's watch an animation to see what we mean.

b **Discuss:** How would you describe this strategy for determining whether a point is an outlier?

***Responses vary.* First, figure out the IQR. Then, move 1.5 times the IQR to the left of quartile 1 and to the right of quartile 3. Any points that are beyond those lines are outliers.**

Explore More

11 Use the digital activity to add points to the dot plot.



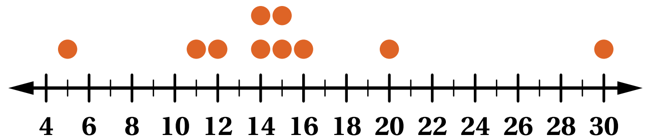
Can you make a dot plot that has:

- One outlier?
- Two outliers?
- Three outliers?
- Four outliers?
- More than four outliers?

***Dot plots vary.* All situations are possible.**

12 Synthesis

Describe how to use the Desmos Graphing Calculator to determine which values in a data set are outliers.



Use the Desmos Graphing Calculator if it helps with your thinking.

***Responses vary.* First, make a list of all the data points in the calculator. Then, create a box plot and make sure "Exclude outliers" is checked. Use the magnifying glass so that you can see the entire box plot. Any points that are outside of the box plot are outliers.**

Things to Remember:

Unit 3
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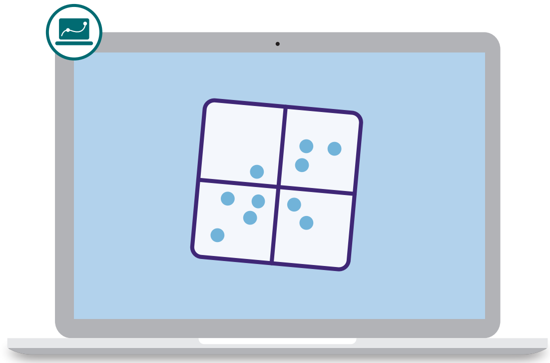
Name: .... Date: .... Period: ....

Correlation Coefficient



Let's learn about the correlation coefficient (r-value) as a way to measure the strength and direction of a linear relationship.

Warm-Up

1 Play a few rounds of Polygraph with your classmates!

You will use a Warm-Up Sheet. For each round:

- You and your partner will take turns being the Picker and the Guesser.
- Picker: Select a scatter plot from the Warm-Up Sheet. Keep it a secret!
- Guesser: Ask the Picker yes-or-no questions, eliminating scatter plots until you're ready to guess which scatter plot the Picker chose.

Record helpful questions from each round in the space below.

Activity 1

Name: .... Date: .... Period: ....

Linear Associations and Scatter Plots

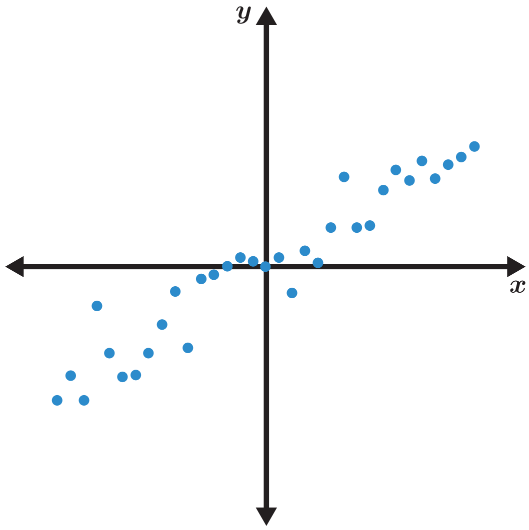
2 In Polygraph, you looked at different *scatter plots*.

Now it's your turn to make your own!

a Drag the sliders in the digital activity to make a scatter plot you like.

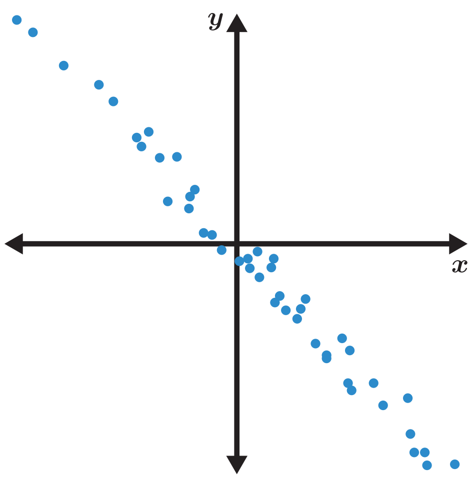
b Describe your scatter plot.

***Scatter plots and responses vary.* All the points are bunched together along an imaginary line. It looks like the slope of this line is positive.**

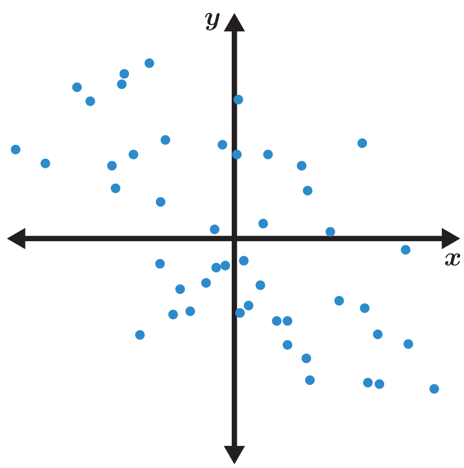


3 Here are the scatter plots Mio and Ayaan made.

Mio



Ayaan



Discuss: How are they alike? How are they different? ***Responses vary.***

Alike:
- **Both scatter plots follow an imaginary line.**
- **Both scatter plots show a negative association.**

Different:
- **The points in the scatter plot on the left are clustered close to an imaginary line. On the other scatter plot, the points are more scattered.**

Activity 1

Name: ____________ Date: ________ Period: ________

Linear Associations and Scatter Plots(continued)

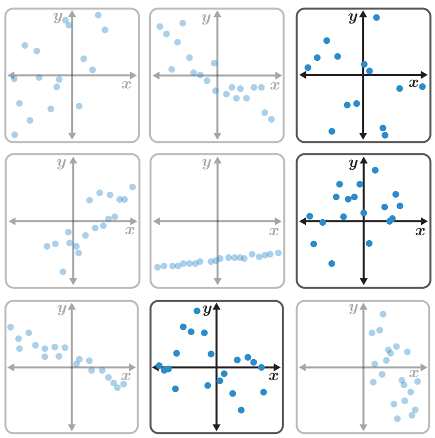
4 When the points on a scatter plot follow a line, we say there is a *linear association* between x and y.

a Here are some terms that describe linear associations.

b **Discuss:** What does each term mean? ***Responses vary.***

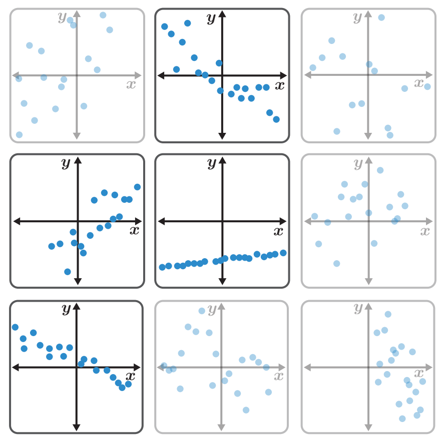
Strength

Weak



The points are scattered and don't follow a clear line.

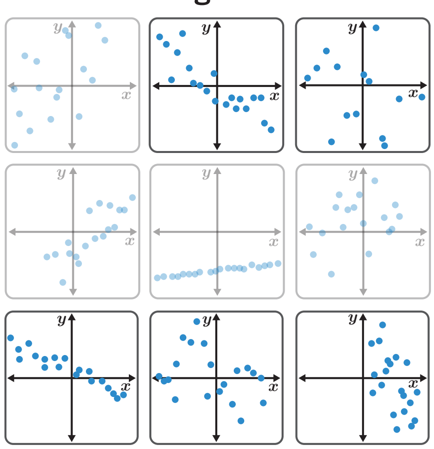
Strong



The points closely follow an imaginary line. There is a clear trend in the points.

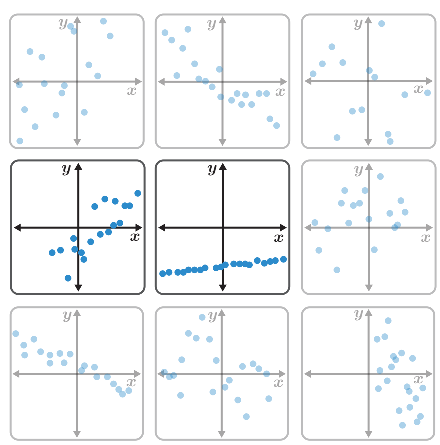
Direction

Negative



The points follow a line with a negative slope.

Positive



The points follow a line with a positive slope.

Activity 2

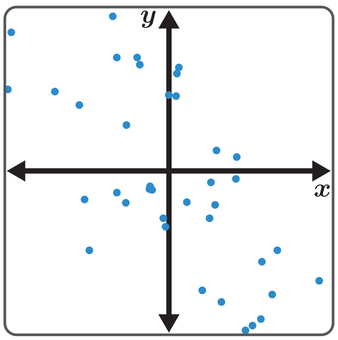
Name: ______ Date: ______ Period: ______

The r-value

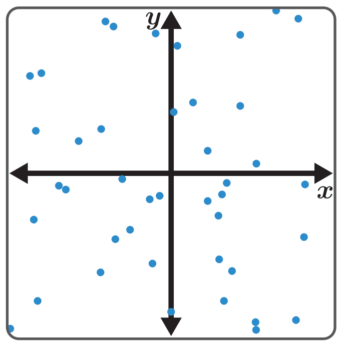
5 The **r-value** is a number that measures the strength and direction of a linear association.

a Take a look at three scatter plots with different r-values:

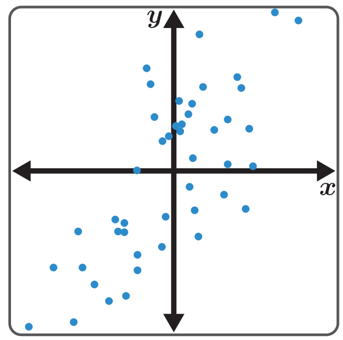
$r = -0.7$



$r = 0$



$r = 0.8$



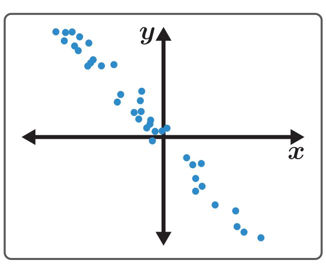
b What do you notice and wonder about the r-value? ***Responses vary.***

- **I notice that there is a positive association when r is positive.**
- **I notice that the association gets stronger when r approaches 1 or -1.**
- **I wonder if non-linear associations can have an r-value.**
- **I wonder if the r-value is the same for linear associations that don't go through (0, 0).**

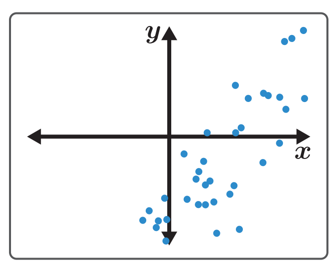
6 Match each scatter plot to its r-value.

-0.99	-0.65	0.86	0.99

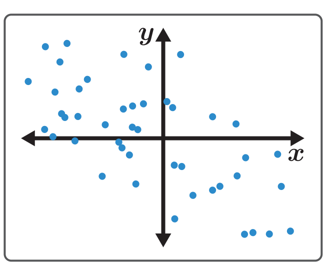
-0.99



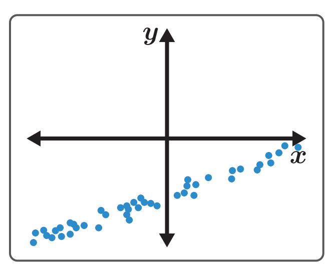
0.86



-0.65



0.99



Activity 3

Name: .... Date: .... Period: ....

Correlation Coefficient

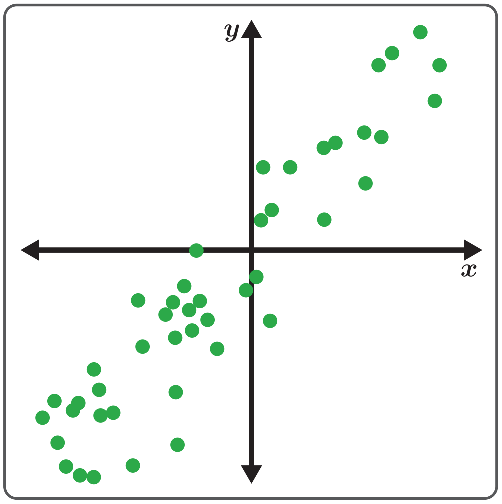
7 The r-value is also known as the **correlation coefficient**.

a **Discuss:** What do you think *correlation* means?

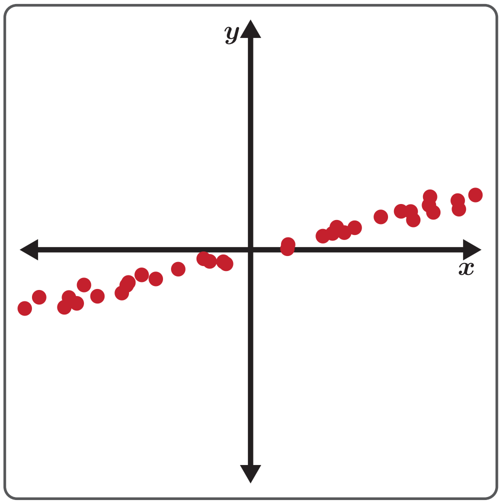
Responses vary. **I think correlation means that two things are related to each other.**

b Circle the scatter plot that has a greater correlation coefficient. Revisit Problem 5 if it helps with your thinking.

A



B



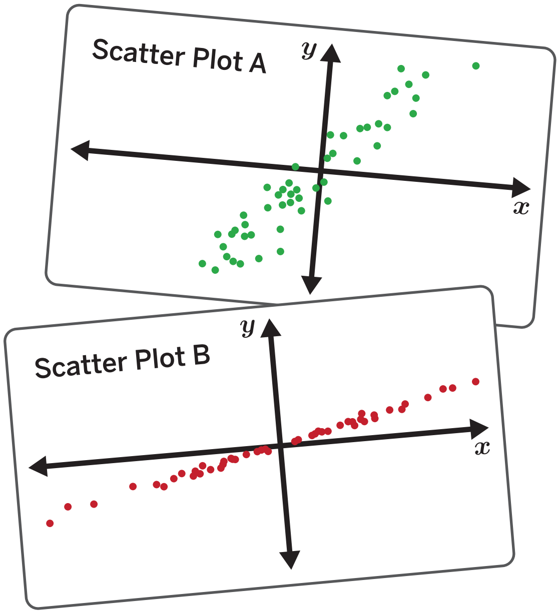
Explain your thinking.

Explanations vary. **Scatter Plot B has a greater correlation coefficient because there is a strong linear association.**

8 Mio says Scatter Plot A has a greater correlation coefficient because the slope of its line is larger.

What would you say to Mio to help her understand her mistake?

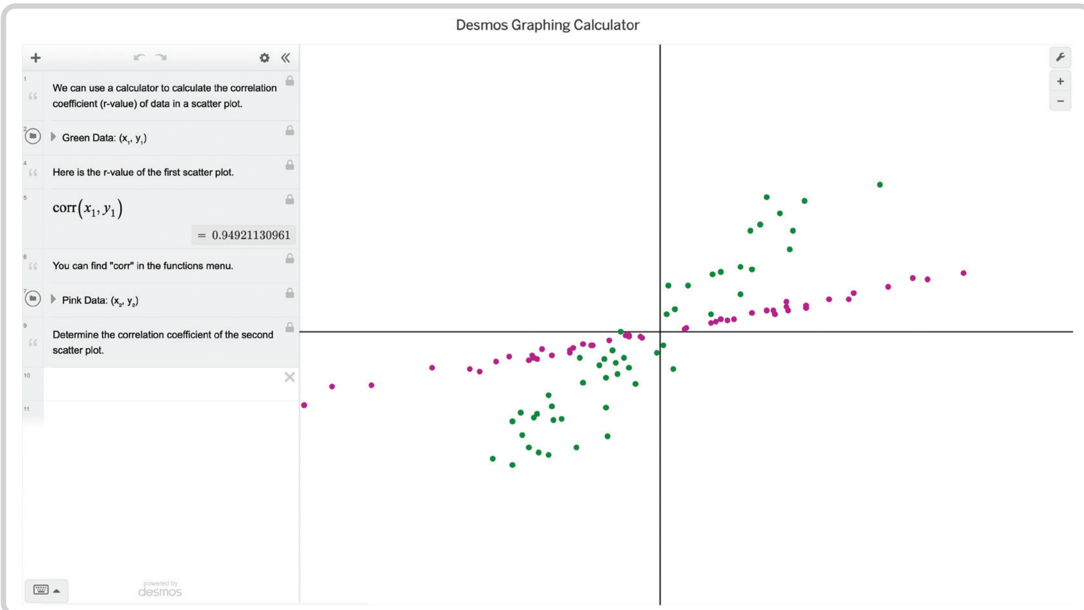
Responses vary. **I would tell Mio that the slope of the line doesn't matter. The r-value tells us how close the points are to a line, not the slope of the line.**



Name: .... Date: .... Period: ....

Correlation Coefficient (continued)

9 We can use the Desmos Graphing Calculator to calculate the correlation coefficient (r-value) of data in a scatter plot.



Use the digital activity to determine the correlation coefficient of the second scatter plot.

0.9948

Explore More

10 Use the digital activity to drag the sliders and see how the scatter plot changes.

a What does the top slider control?

***Responses vary.* The top slider controls the slope of the line the data follows. It also controls whether the r-value is negative or positive.**

b What does the bottom slider control?

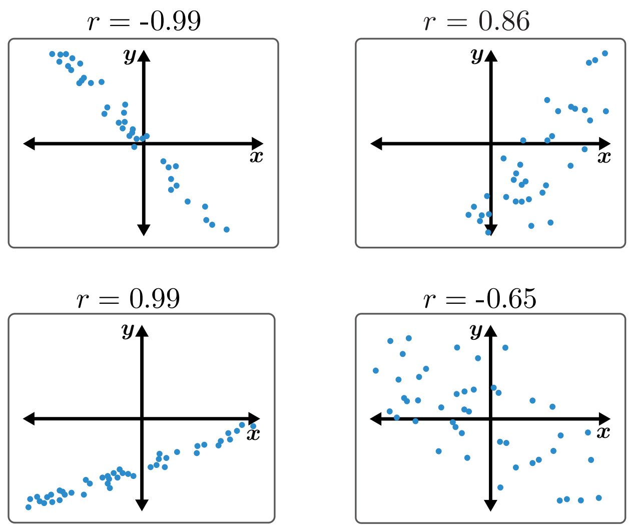
***Responses vary.* The bottom slider controls the strength of the linear association. When you drag the slider to the left, the r-value is close to 0. When you drag it to the right, the r-value is close to -1 or 1.**

11 Synthesis

What does the correlation coefficient tell us about the data in a scatter plot?

Use these scatter plots if they help with your thinking.

Responses vary. **The correlation coefficient tells us whether there is a positive or negative association between the two variables. It also tells us if the association is strong or weak.**

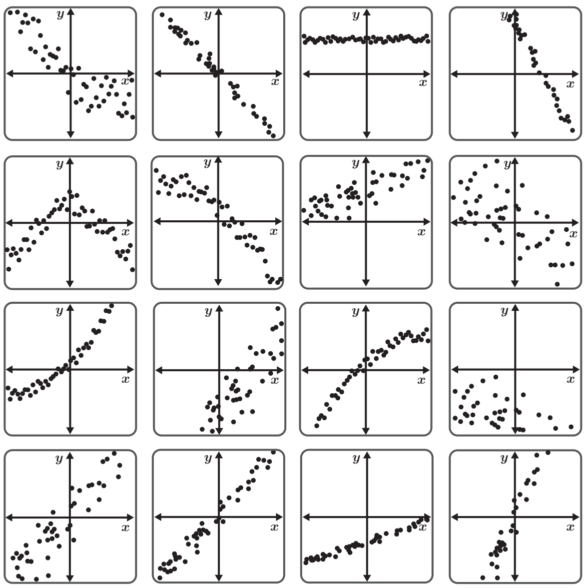


Things to Remember:

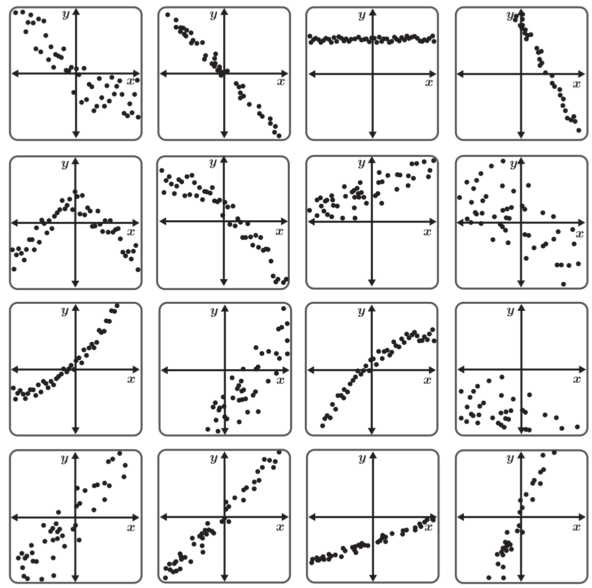
Name: ______________________ Date: ____________ Period: ____________

Polygraph Set A

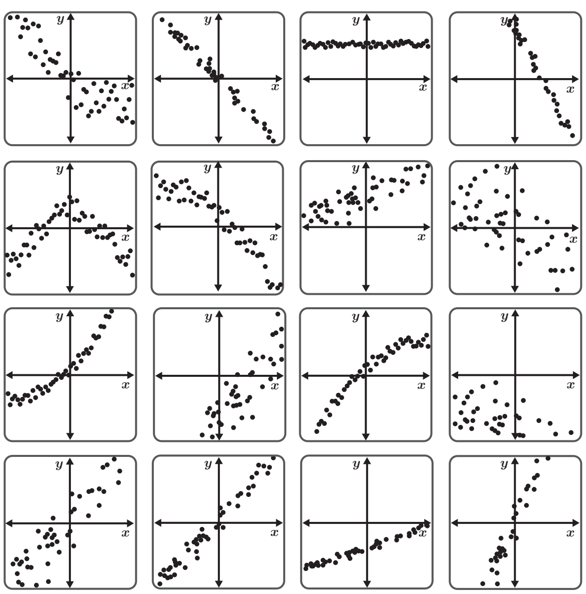
Round 1



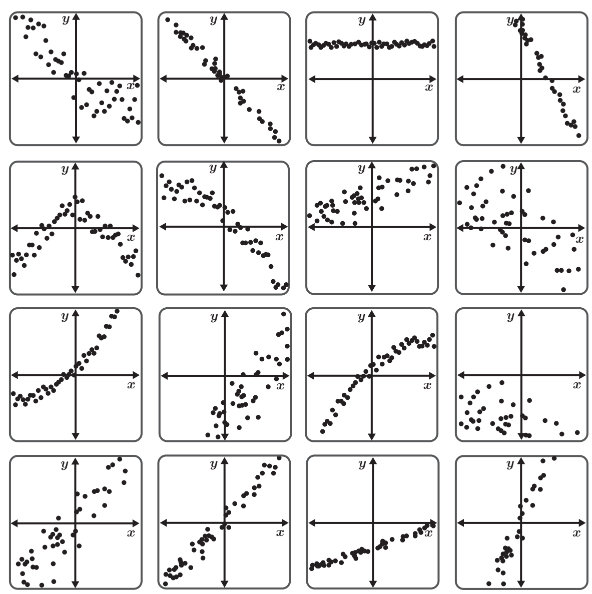
Round 2



Round 3



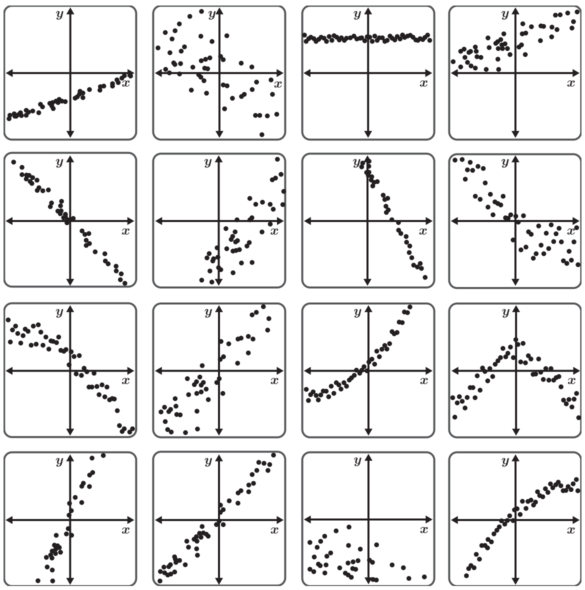
Round 4



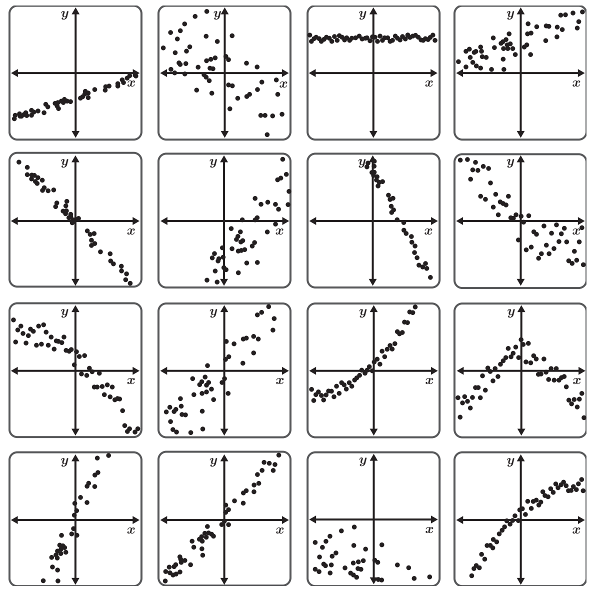
Name: .... Date: .... Period: ....

Polygraph Set B

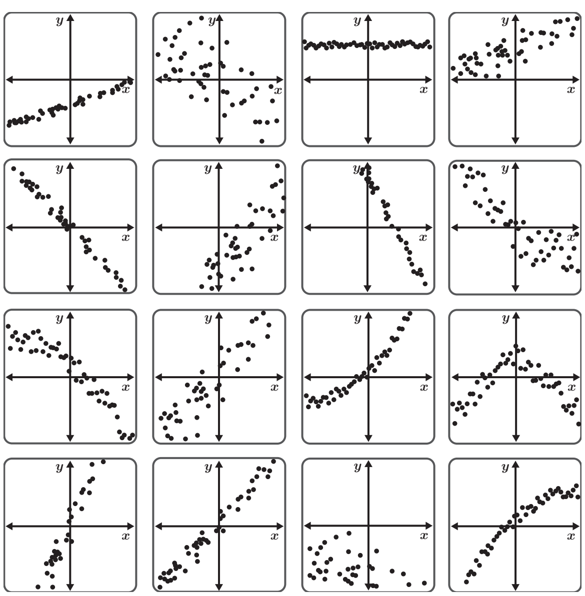
Round 1



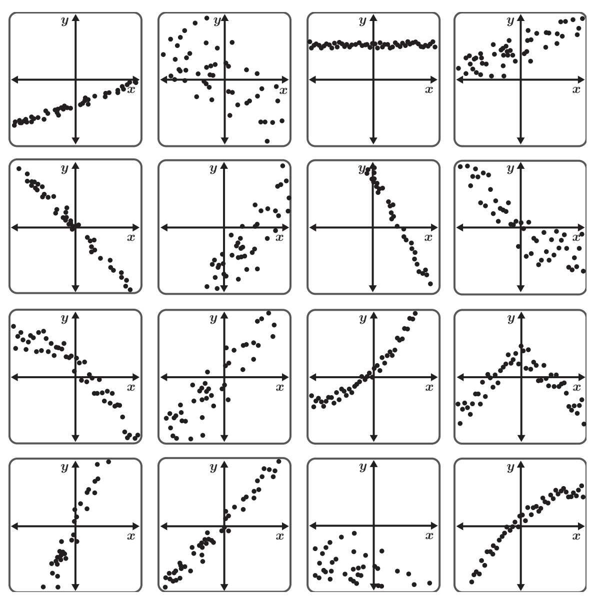
Round 2



Round 3

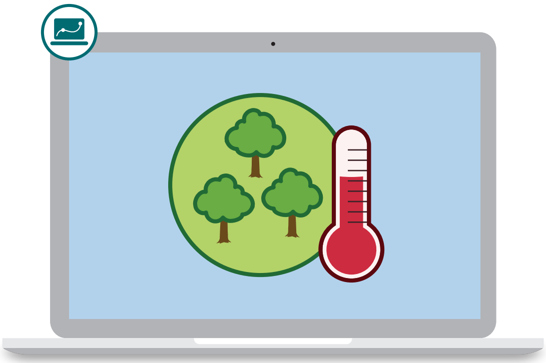


Round 4



Unit 3
Lesson
14

Name: ______ Date: ______ Period: ______



How Hot Is It?

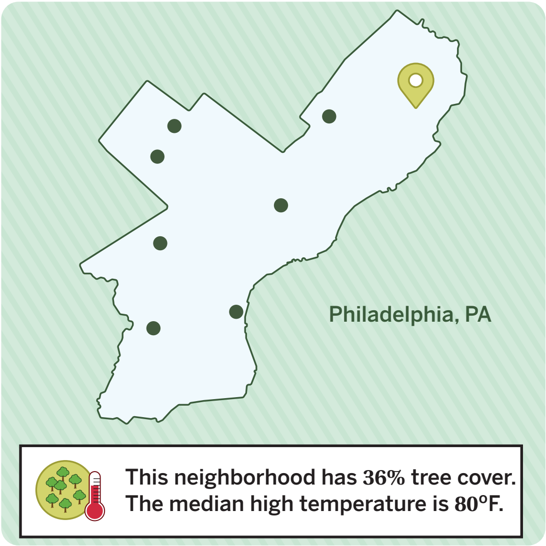
Let's use correlation coefficients to analyze relationships between income, tree cover, and average temperature.

Warm-Up

1 Here is a map of Philadelphia, PA.

Let's look at a few neighborhoods.

What do you notice? What do you wonder?
Responses vary.



Source: *PLoS One*

I notice:

- **I notice that some places have a high percentage of tree cover and other places have very low tree cover.**
- **I notice that some places with a high percentage of tree cover also have a lower temperature.**

I wonder:

- **I wonder why some places are hotter than others.**
- **I wonder if tree cover and temperature are related.**

Activity 1

Name: .... Date: .... Period: ....

Tree Cover vs. Temperature

2 Laila talked to people in different parts of Philadelphia to learn more about tree cover and temperature.

a Let's watch the data get collected.

b **Discuss:** Based on this data, do you think there is a relationship between tree cover and temperature?

Responses vary.

- **I think there is a relationship. The temperature seems cooler where there are more trees. When there is 4% tree cover, the temperature is 90°, and when there is 88% tree cover, the temperature is 74°.**
- **I don't think so. We only have data for eight neighborhoods, which isn't really enough data to know for sure.**

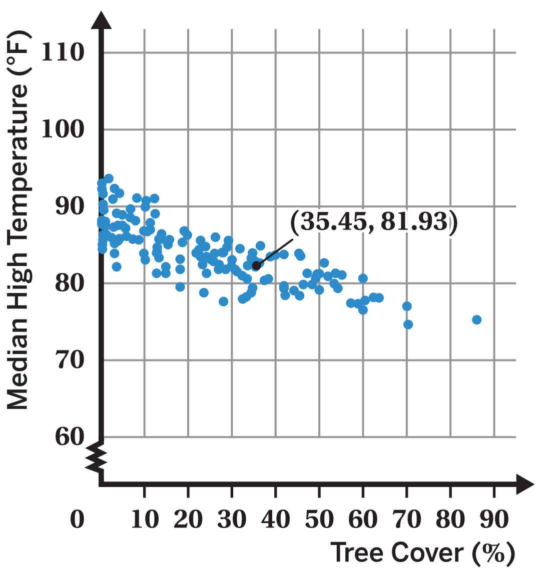
Tree Cover (%)	Median High Temperature (°F)
43	78
88	74
20	88
27	81
36	80
78	75
4	90
8	85

3 Here is some data from 150 blocks in Philadelphia.

One of the coordinates is shown.

Describe what the coordinates tell you about that block.

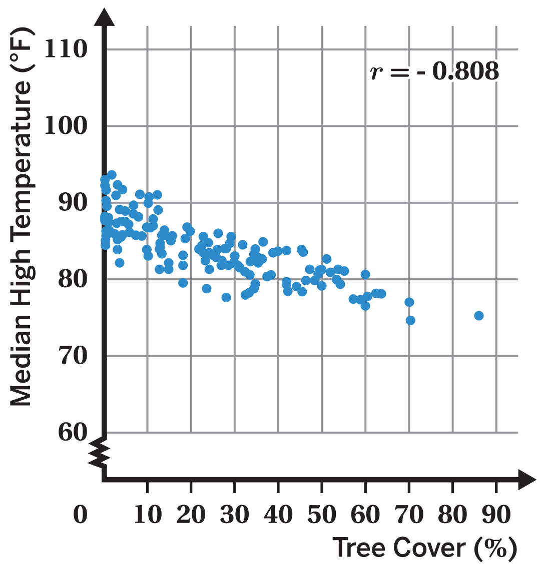
Responses vary. **On this block, about 35% of the ground is covered with trees, and the median high temperature is about 82°F.**



Name: Date: Period:

Tree Cover vs. Temperature (continued)

4 Here is the correlation coefficient (the r-value) of the data.



a Based on the r-value, what kind of association is there between tree cover and temperature? Circle one.

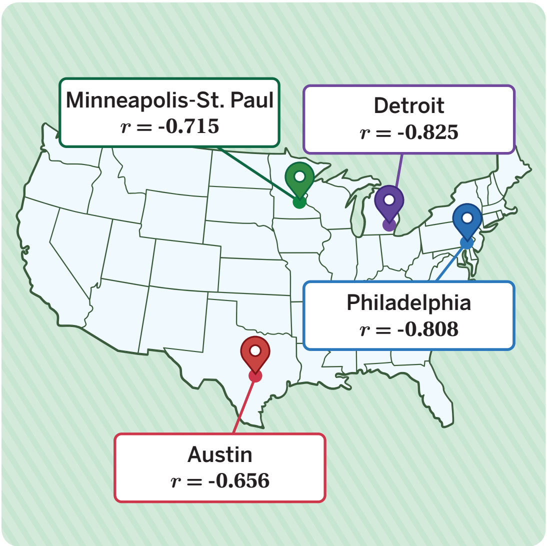
Positive   Negative   No association

b What is the strength of the association? Circle one.

Weak   Strong

5 Laila wonders how other cities compare to Philadelphia.

Let's look at other city data on the map.



Discuss: What do you notice? What do you wonder?

Responses vary.

- I notice that the strongest correlation is in Detroit. I wonder what that means about Detroit.
- I notice that Austin's data is higher on the graph than the other cities, and that Austin is the only one of the cities that's in the South. I wonder if those are related.
- I notice that the slope of Philadelphia's graph is the most steep. I wonder what that would actually look like if you lived there.

Name: .... Date: .... Period: ....

Including Income

6 Temperature is one of many variables associated with tree cover.

What other variables do you think could be associated with tree cover?

Responses vary.

- **Number of homes (vs. number of businesses)**
- **Population**
- **Income**
- **Age of buildings/when most buildings were built**
- **Distance to the city center**

7 Laila wonders: *Is there an association between income and tree cover?*

a Make a prediction: What kind of association do you expect between these variables? (E.g., weak positive or strong negative.)

Responses vary.

- **I think there will be no association because the number of trees isn't related to how much money people make. Usually the city plants trees, so how many trees they plant shouldn't change no matter what the neighborhood's income is.**
- **I think there will be more trees in richer neighborhoods, which means there will be a relationship between the two variables.**

b Let's look at the data.

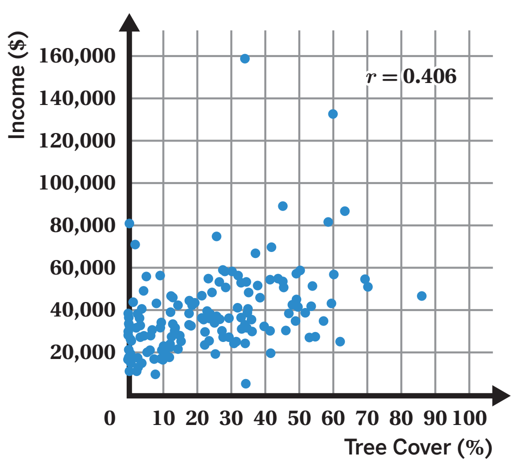
Discuss: Was your prediction correct?

Responses vary.

8 This graph shows the average income and the percentage of tree cover for 150 blocks in Philadelphia.

What does the r-value say about the association between income and tree cover?

***Responses vary.* It looks like there is a positive association between income and tree cover.**



Activity 2

Name: ______ Date: ______ Period: ______

Including Income (continued)

9 The Philadelphia data has a correlation coefficient of 0.406.

Here is some data about income and tree cover in Detroit.

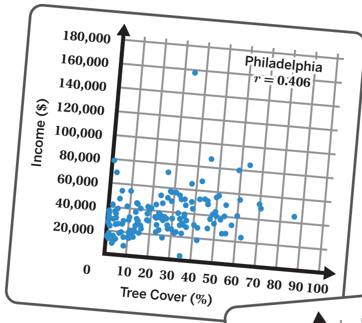
Which could the r-value for Detroit be?

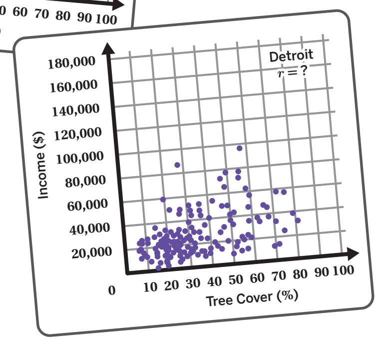
A. $r = 0.35$

B. $r = 0.85$

C. $r = -0.35$

D. $r = -0.85$

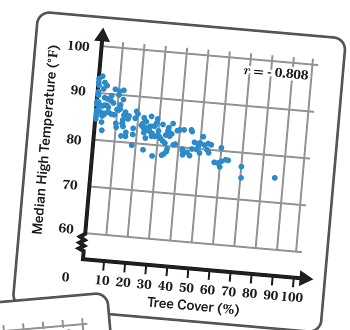


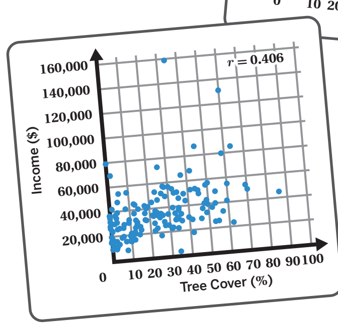


10 At a neighborhood meeting, someone said:

It is unfair that lower income neighborhoods are hotter in Philadelphia.

Philadelphia, PA





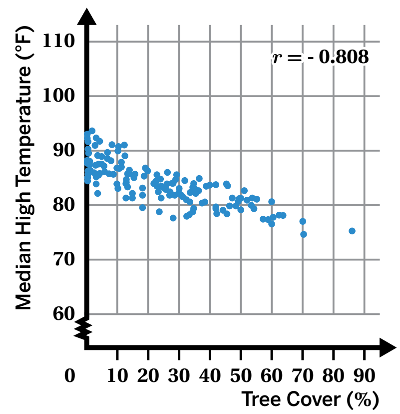
a How does the data support this statement? ***Responses vary.* The data shows that lower income neighborhoods have fewer trees and are therefore hotter. This is unfair because higher temperatures could mean plants don't grow as well or that people's health is worse.**

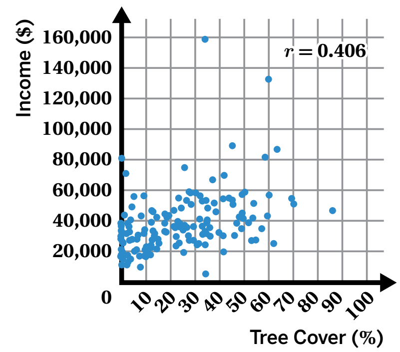
b What changes do you think should be made? ***Responses vary.* I think more trees should be planted in lower income neighborhoods. This could mean turning parking lots into parks or community gardens, or planting more trees on sidewalks or between buildings.**

11 Synthesis

How can correlation coefficients help us describe the relationship between two variables in the real world?

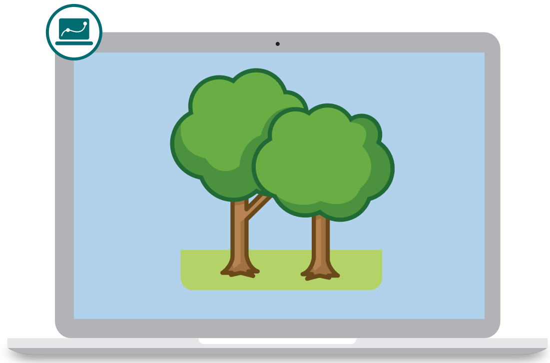
Responses vary. Correlation coefficients can help us know whether there really might be something going on with those two variables or not. For example, the correlation coefficient between temperature and tree cover is high, which means that there is some association between the two. We don't know if trees are causing the temperature change, but we could investigate more.





Things to Remember:

Name: ________ Date: ________ Period: ________



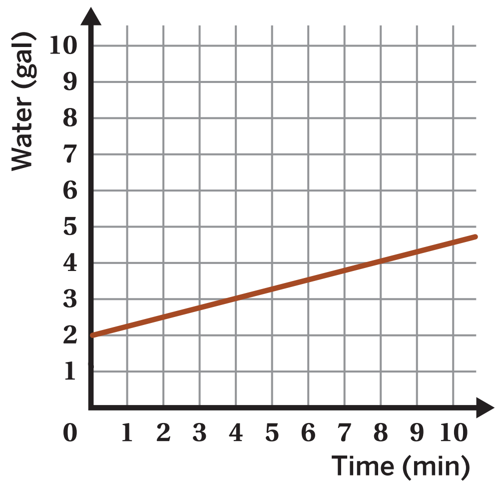
City Slopes

Let's use a line of fit to describe the relationship between two variables and make predictions.

Warm-Up

1 An equation for this line is $y = \frac{1}{4}x + 2$.

Show or explain where you see $\frac{1}{4}$ and 2 in the graph.



Source: *PLoS One*

$\frac{1}{4}$: **You can see the $\frac{1}{4}$ in the slope of the line because the line goes up 1 and right 4.**

2: **You can see the 2 at the y-intercept, $(0, 2)$.**

Name: ______ Date: ______ Period: ______

Lines of Fit

2 Here's the graph of temperature and tree cover for Austin, Texas.

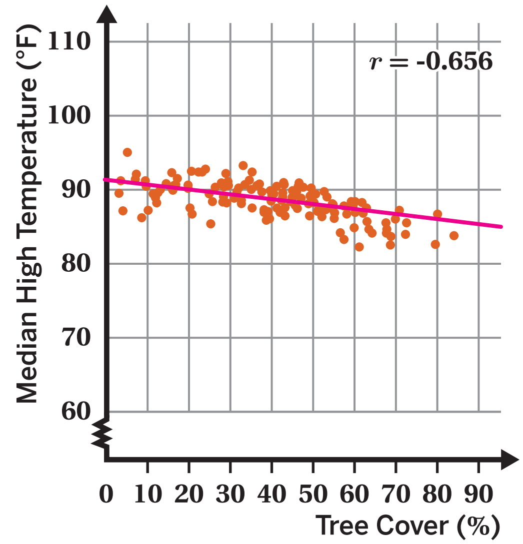
Mathematicians use a *line of fit* to describe relationships and make predictions.

a. **Discuss:** Why is a line a good fit for this data?

***Responses vary*. The correlation coefficient is -0.656, which says there is a medium strength correlation.**

b. Draw a line of fit for the data.

Sample shown on graph.

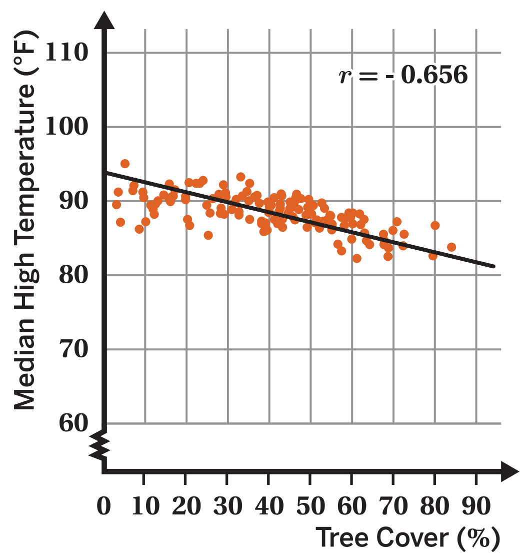


3 Here is the line of fit a student drew.

Jamal lives in Austin, on a block that has 75% tree cover.

What might the median high temperature be on his block?

Responses between 82°F and 88°F are considered correct.

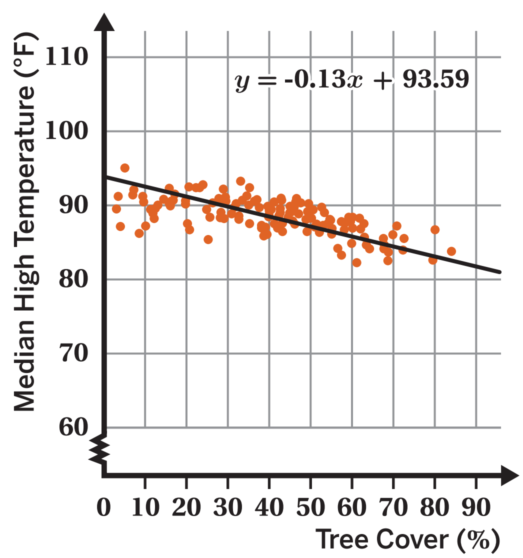


4 Here is an equation for this line of fit.

What does the *slope* tell us the relationship between temperature and tree cover? Circle one.

When the tree cover increases by 1%, the temperature decreases by 0.13°F.

When the tree cover increases by 1%, the temperature decreases by 93.59°F.



Activity 2

Name: ______ Date: ______ Period: ______

Interpreting in Context

5 Let's compare some cities.

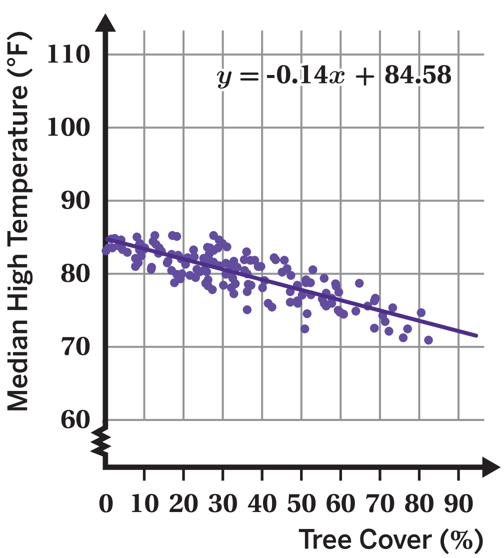
Discuss: What do you notice? What do you wonder?

Responses vary.

- **I notice that for all the cities listed, the more tree cover there is, the lower the temperature gets.**
- **I notice that Austin has the hottest median temperatures.**
- **I notice that the slope of the line of fit is steepest for Philadelphia and least steep for Austin.**
- **I wonder how cities decide where to plant trees.**
- **I wonder why tree cover seems to have a greater effect on the temperature in Philadelphia.**

6 Here is an equation for Detroit's line.

What do the -0.14 and 84.58 tell us about the relationship between temperature and tree cover?



Responses vary.

-0.14: **As the tree cover increases by 1%, the median temperature tends to decrease by about 0.14°F.**

84.58: **The median temperature in Detroit if there was no tree cover.**

Activity 2

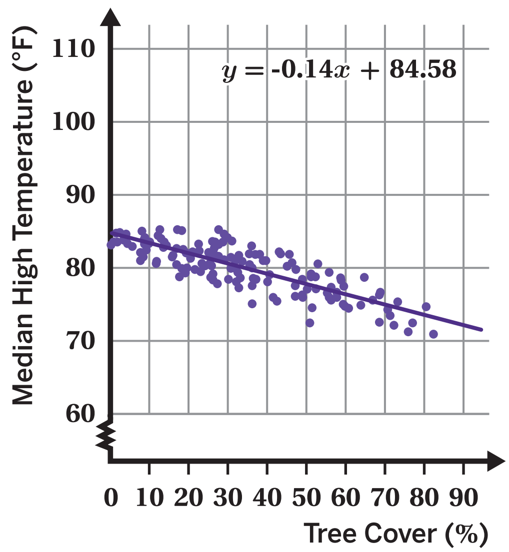
Name: ________ Date: ________ Period: ________

Interpreting in Context (continued)

7 A community in Detroit wants to build a park.

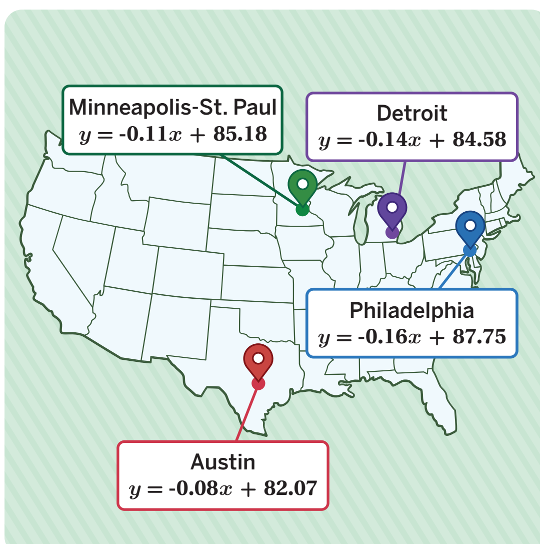
If the park has 80% tree cover, what might its median high temperature be?

Responses between 72°F and 75°F are considered correct.



8 **a** Order the cities according to where tree cover has the greatest impact on temperature.

Philadelphia	Greatest
Detroit	
Minneapolis-St. Paul	
Austin	Least



b **Discuss:** How did you order each city?

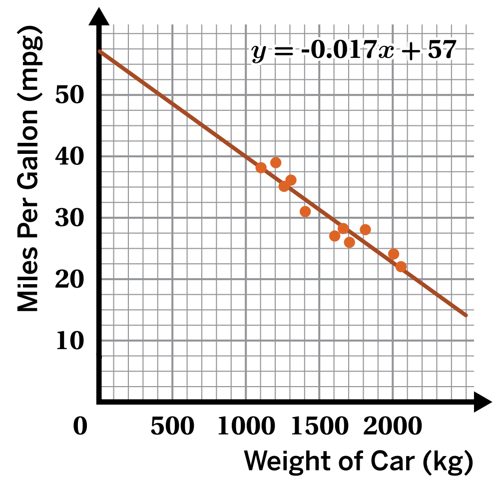
Explanations vary.

- **I ordered each city by the value of the slope. The steeper the line is, the more impact tree cover has on temperature.**

9 Synthesis

How can a line help us make predictions about data?

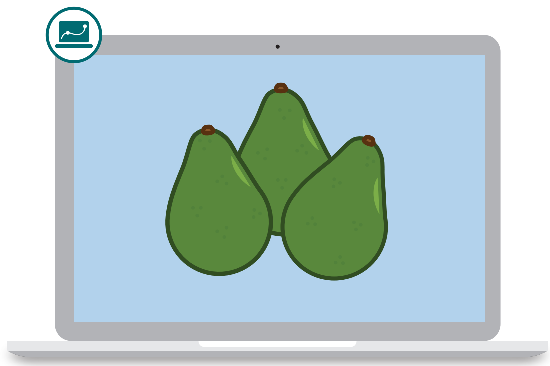
Responses vary. A line can help us make predictions about data because it can predict values that are not seen on the scatter plot.



Things to Remember:

Unit 3
Lesson
16

Name: ________ Date: ________ Period: ________

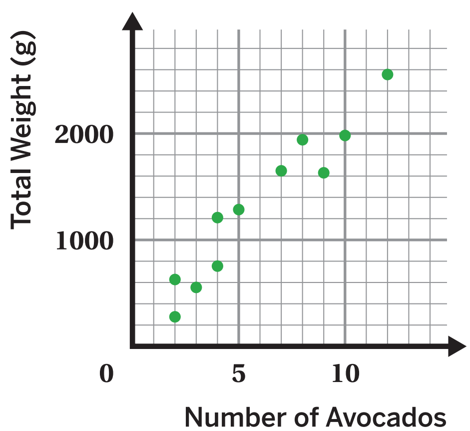


Residual Fruit

Let's use residual plots to determine how well a line fits data.

Warm-Up

1 Brianna has a business that ships different kinds of fruit.



a Let's watch orders being weighed.

b **Discuss:** How much do you think 11 avocados will weigh?

Responses between 2,000 and 2,400 grams are considered correct.

Activity 1

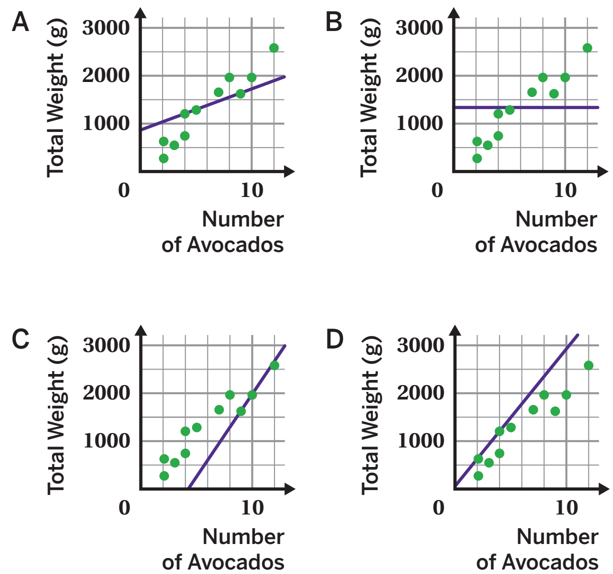
Name: .... Date: .... Period: ....

Predicting With Lines

2 Lines that fit the data well can help us make predictions.

None of these lines fit the data well.

Circle a scatter plot. Explain why the line does not fit the data well.



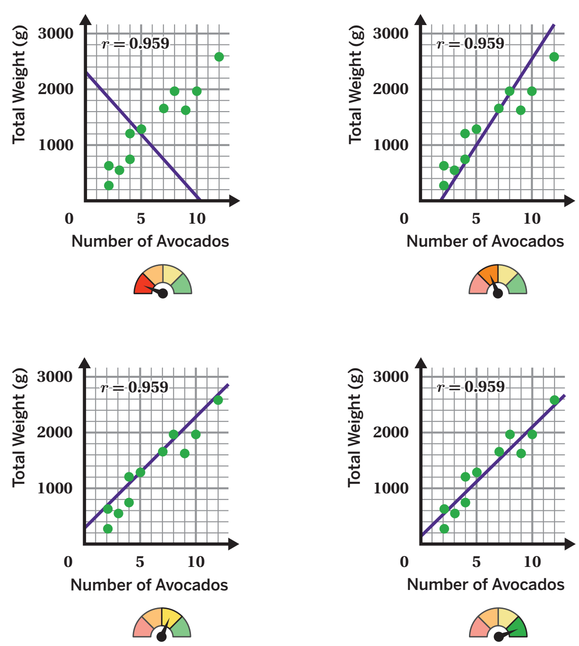
Selections and explanations vary.

- **Line A has a y-intercept near 1,000, which does not make sense in this situation because the y-intercept represents the weight of 0 avocados.**
- **Line B doesn't follow the general direction of the points.**
- **Line C is too low and doesn't go through the middle of the points.**
- **Line D is above most of the points. This will cause the weight predictions to be too high.**

3 **a** Take a look at these scatter plots. The meters show how well each line fits the data.

b Explain to a classmate how to get a high score on the meter.

Explanations vary. **The line should follow the general pattern of the data, increasing or decreasing at the same rate as the data. There should be some points above and below the line.**

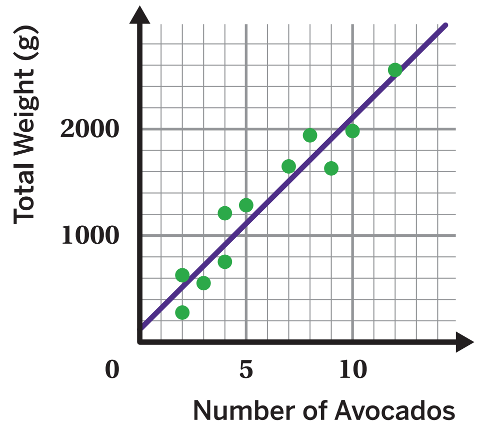


Name: .... Date: .... Period: ....

Predicting With Lines (continued)

4 Brianna has an order to ship 6 avocados.

How could you use this line to predict the weight of the order?



***Responses vary.* I could find 6 on the x-axis and trace up until I hit the line. The y-value when x is 6 is $1{,}316$, which means the line predicts that 6 avocados will weigh $1{,}316$ g.**

5 The line predicts that 6 avocados will weigh $1{,}316$ grams, but 6 avocados actually weigh $1{,}740$ grams.

What is the difference, in grams, between the actual weight and the predicted weight?

424 grams

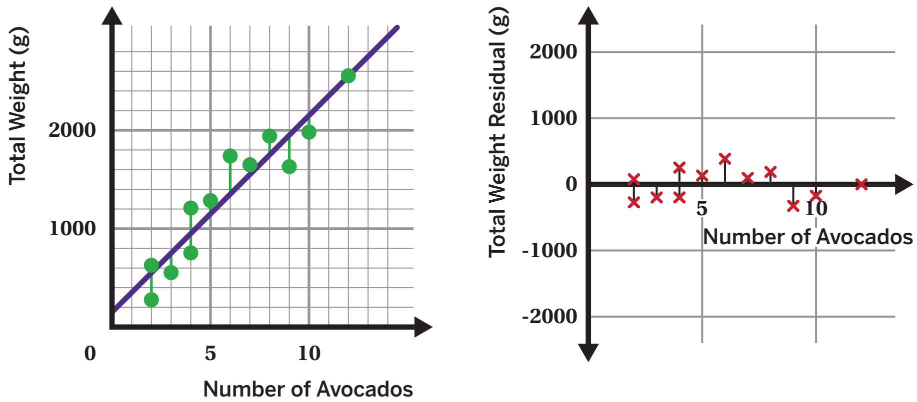
Activity
2

Name: .... Date: .... Period: ....

Residual Plots

6 A **residual** is the difference between the predicted and measured weight.

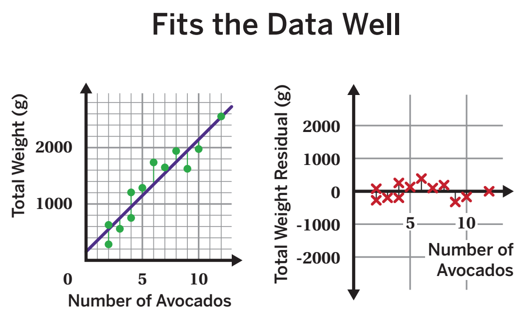
a Let's see how to plot residuals.

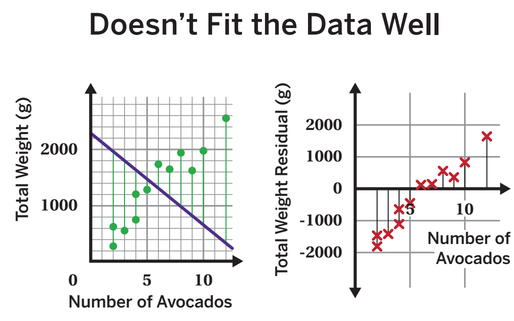


b **Discuss:** How could you make a residual plot?

Responses vary. I could make a residual plot by measuring the distance between each point and the line. Then I would make a new graph with points that represent the distance between each number of avocados and the line.

7 A **residual plot** shows how far each point is from the line of fit.





What does the residual plot look like when the line fits the data well? What about when it doesn't fit the data well?

Responses vary.

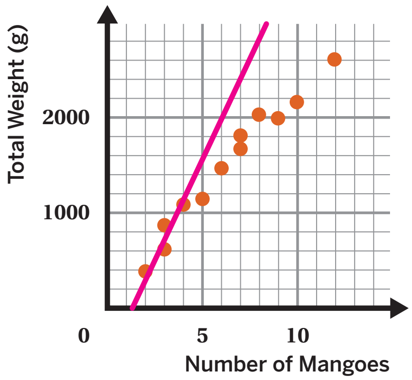
- When the line fits the data well, the points on the residual plot are close to the x-axis, and they're randomly distributed both above and below the x-axis.
- When the line doesn't fit the data well, the points on the residual plot are far from the x-axis, or there are more points either above or below the x-axis.

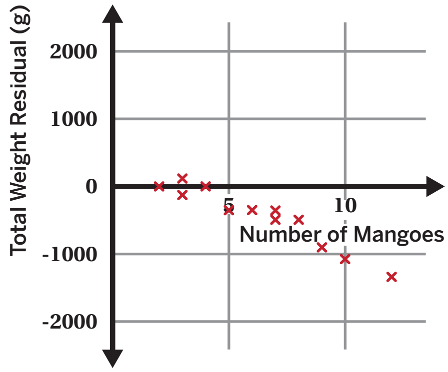
Name: .... Date: .... Period: ....

Residual Plots (continued)

8 Here is the residual plot for a line of fit Brianna created.

Sketch the line of fit you think Brianna made. *Lines vary.*

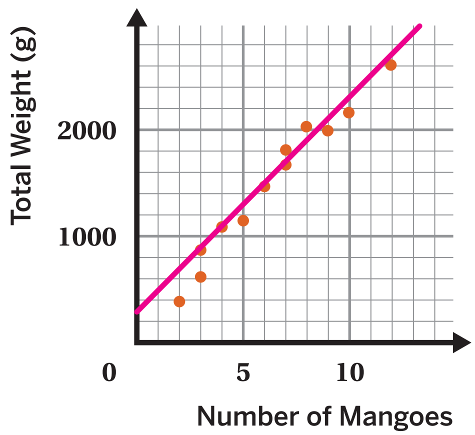




9 Let's look at Brianna's line.

a Draw another line that is a better fit for the data.
Lines vary.

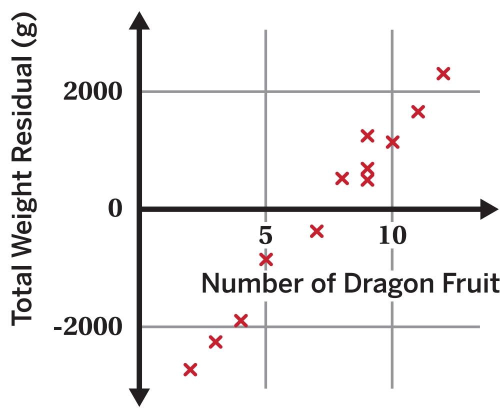
b **Discuss:** How will the residual plot change once the line is a better fit for the data?
Responses vary. The points on the residual plot will move closer to the x-axis and become randomly dispersed both above and below the x-axis.



10 Aditi made a line of fit for this data showing the total weight of different numbers of dragon fruit.

Here is the graph of the residuals from Aditi's line. How well do you think Aditi's line fits the data?

Explanations vary. Aditi's line does not model the data very well because the points on the residual plot are very far from the x-axis. Also, the residuals start off all negative and then turn positive, which shows that the line does not follow the pattern of the data.



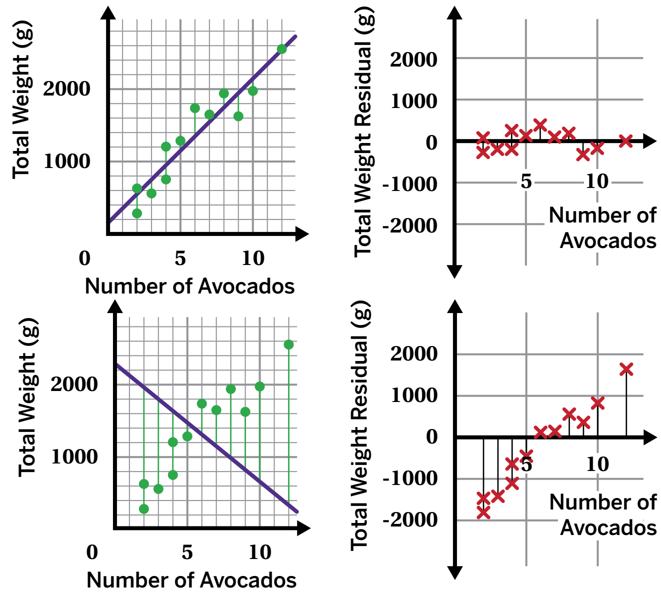
Explore More

11 Use the Explore More Sheet to draw a line of fit.

12 Synthesis

How can you use a residual plot to determine if a line fits the data well?

Responses vary. **I check if the points on the residual plot are close to the x-axis and if they're randomly dispersed both above and below the x-axis. If so, then I know the line fits the data well.**



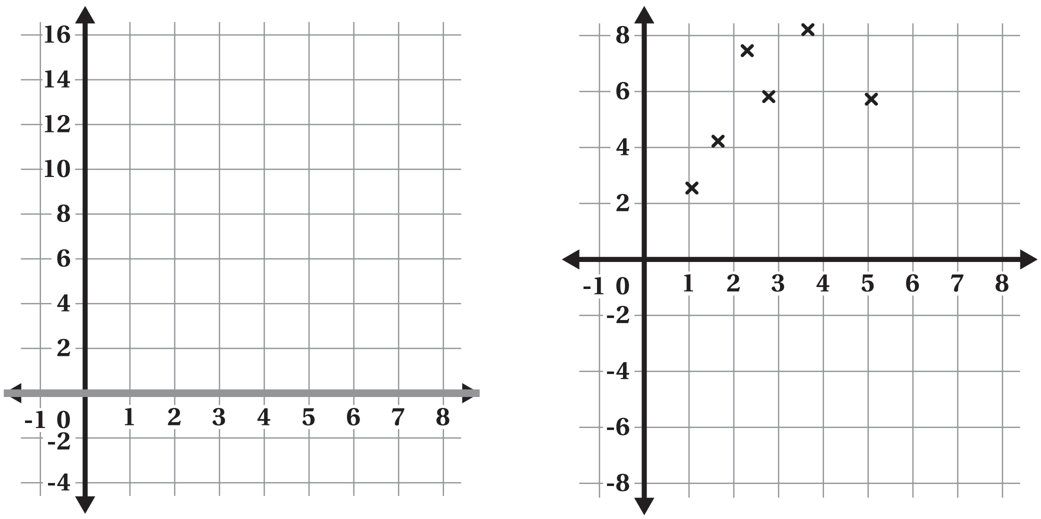
Things to Remember:

Name: ______________________ Date: ____________ Period: ____________

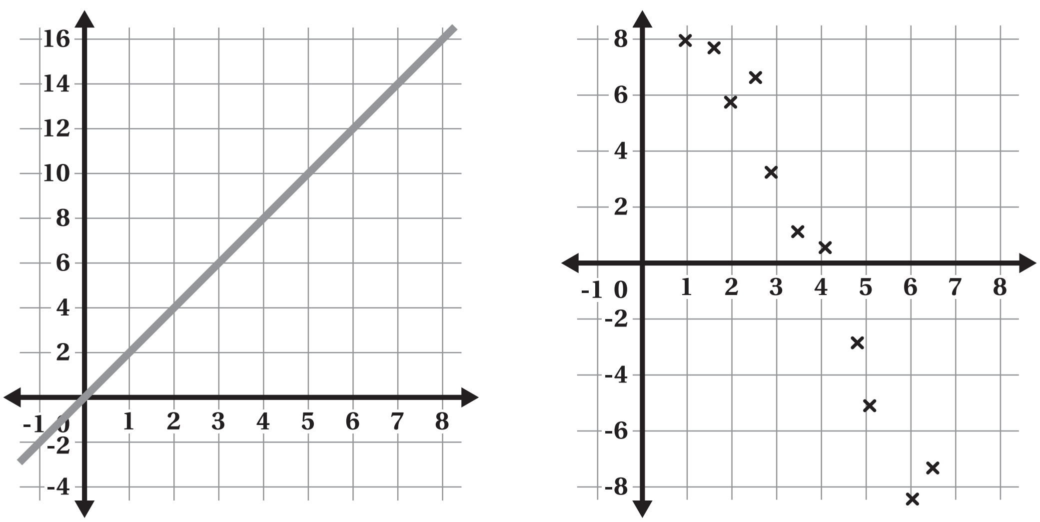
Explore More

11 The data on each scatter plot is hidden. Use the residual plot to draw a line that fits the data better.

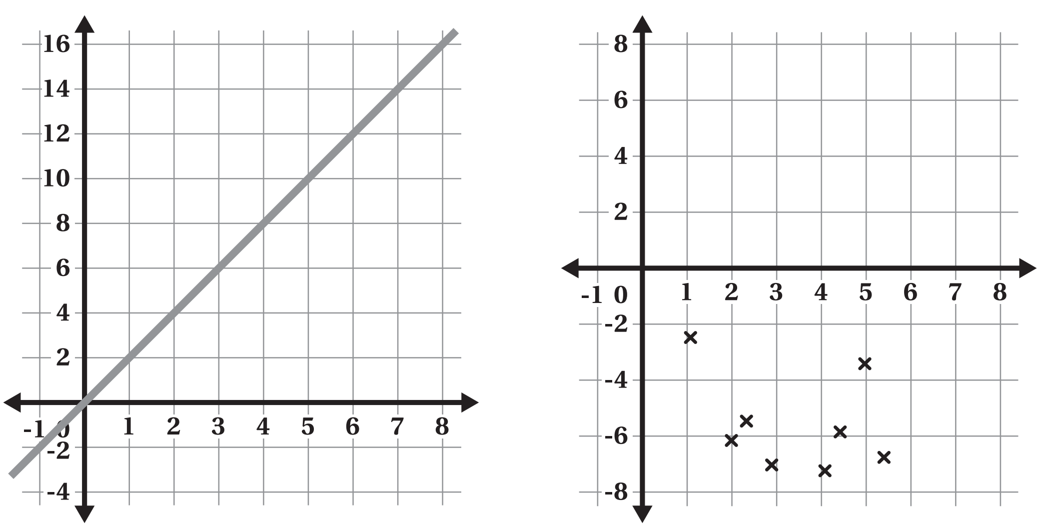
a



b



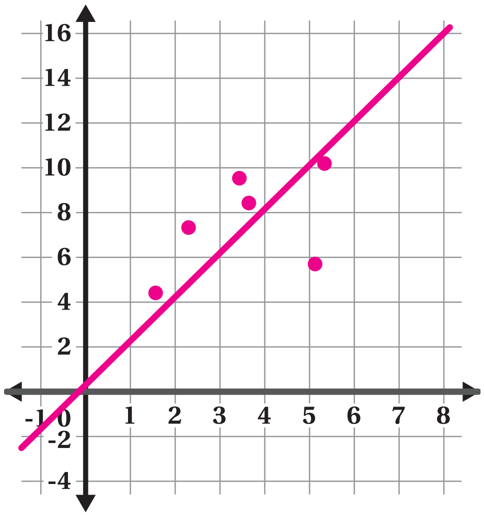
c

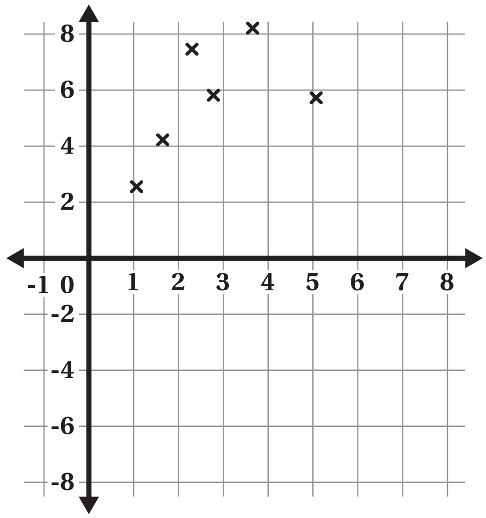


Explore More (answers)

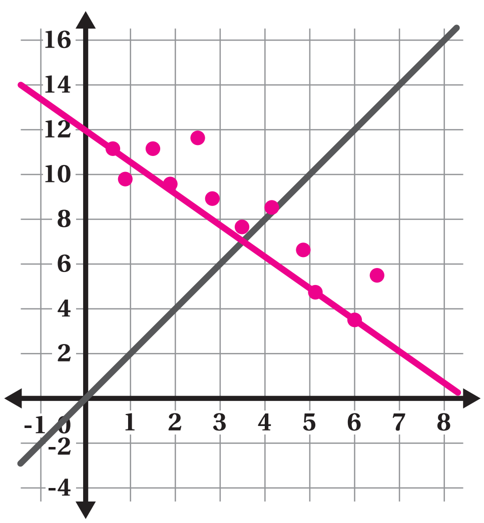
11 The data on each scatter plot is hidden. Use the residual plot to draw a line that fits the data better.

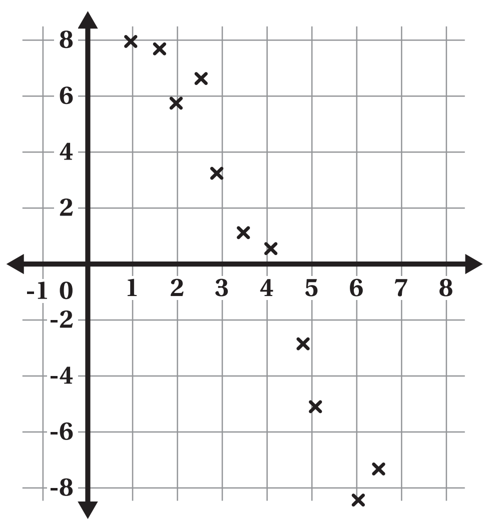
a



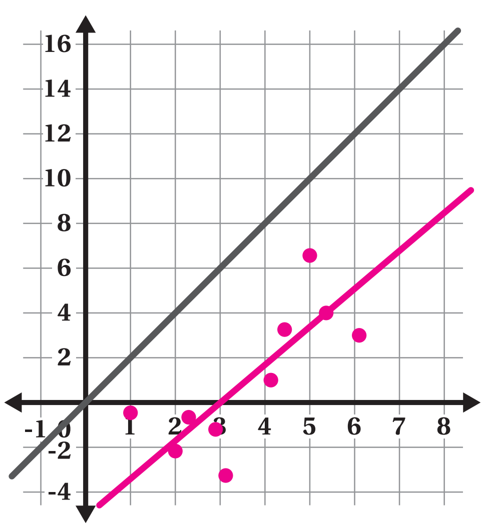


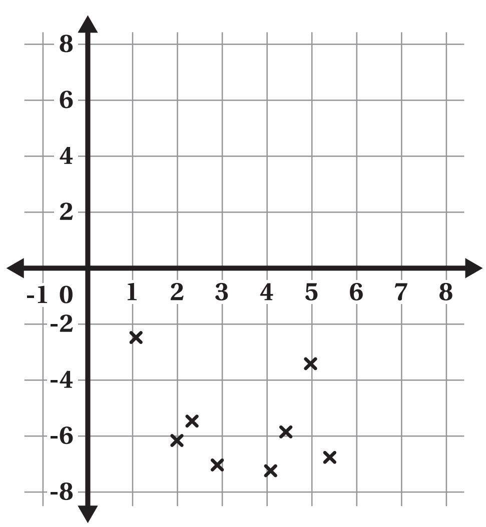
b





c





Name: .... Date: .... Period: ....

Penguin Populations

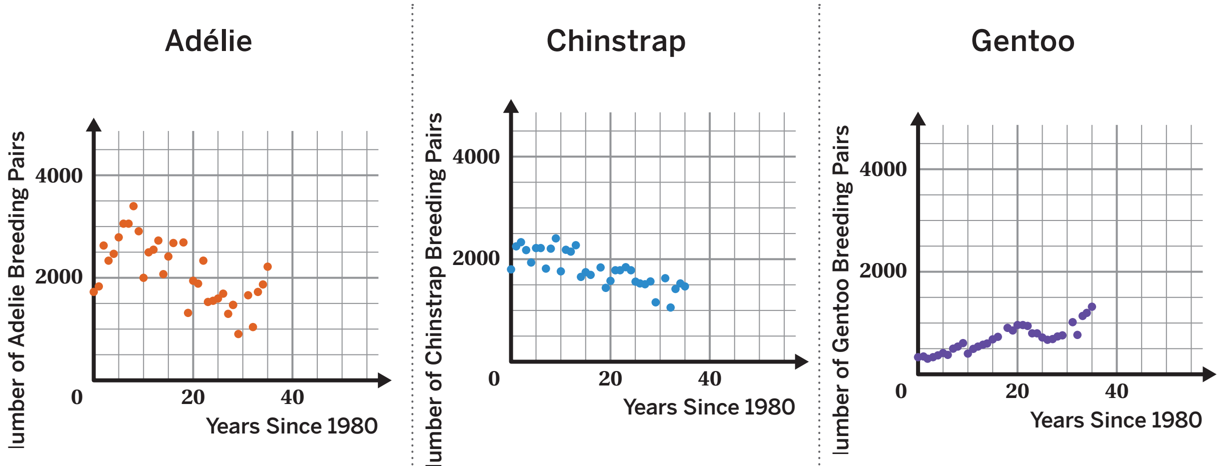
Let's generate and analyze lines of best fit to explore how penguin populations have changed over time.

Warm-Up

1–2 Researchers are conducting a long-term study on the South Orkney Islands in Antarctica.

The goal of this study is to understand how the populations of three species of penguins have changed over the last 40 years.

Here is the study's data for each of the penguin species.



Source: *PLoS One*

How is the population of each species changing over time? What might be affecting these changes?

Responses vary.

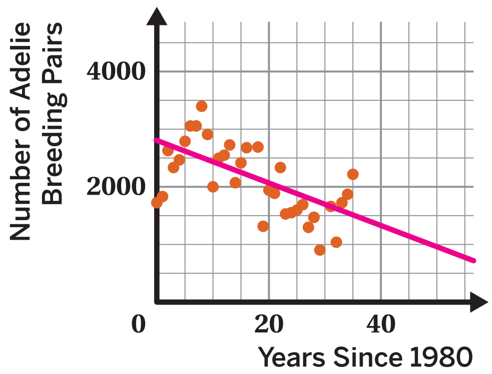
- **The Adélie penguin population seems to be going down. But it was pretty high recently, so I'm not sure. Maybe there are different amounts of fish each year, so the penguin population keeps jumping up and down.**
- **The population of Chinstrap penguins seems like it's decreasing pretty steadily. Maybe there isn't enough food for them to eat, or maybe they're being eaten by predators.**
- **The Gentoo population is increasing! Maybe whatever changes are happening in Antarctica are actually better for them.**

Activity 1

Name: .... Date: .... Period: ....

Predicting With Lines

3 Let's explore how the Adélie penguin population on the South Orkney Islands has changed over time.



a Draw a line that fits the data.

Lines vary. **Sample shown on the graph.**

b What does the line tell you about the penguin population?

Responses vary.

- **The line tells me that there are about 42 fewer breeding pairs of penguins every year.**
- **The slope of the line is negative, so the population is going down over time.**
- **My line has an intercept of 2,873.7, which means there were around 2,873 breeding pairs in 1980.**

4 A calculator can compute the **line of best fit**.

a Let's look at the line of best fit for the Adélie penguin data.

b **Discuss:** What do you notice about the residual plot?

Responses vary. **The points on the residual plot moved closer to the x-axis and became randomly dispersed both above and below the x-axis.**

5 Let's look at the equation of the line of best fit for the Adélie penguin data.

a **Discuss:** What x-value represents the year 2030?

50

b Use the line of best fit to predict how many breeding pairs of penguins there will be in 2030. **$-36.13(50) + 2733.52$ (or equivalent)**

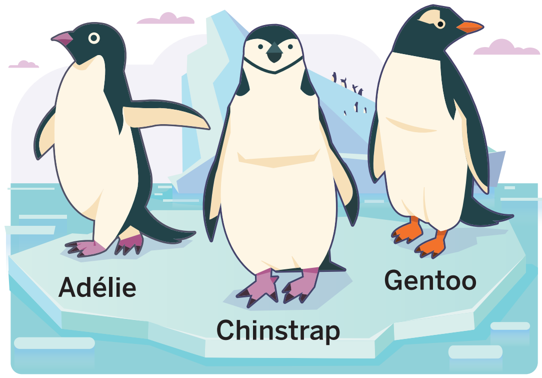
Responses between 926 and 928 are considered correct.

Activity 2

Name: ______ Date: ______ Period: ______

Generating a Line of Best Fit

6–7 In the digital activity, take a look at the table of data showing the Adélie penguin population over time and the equation of the line of best fit.



a **Discuss:** Why do you think this is called a linear regression?

Responses vary. **It might be called a linear regression because it is creating the line of best fit for this data.**

b **Discuss:** What do the parts of the equation represent?

Responses vary.

- **The -36.13 is the slope. It says there are about 36 fewer Adélie penguins each year after 1980.**
- **The 2,733.52 is the number of penguins the line of best fit predicted there would be in the year 1980.**

8 Use the digital activity to generate the line of best fit for the Chinstrap data.

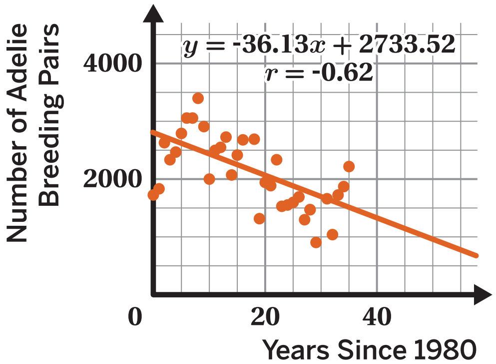
$y = -25.45x + 2239.59$ **(or equivalent)**

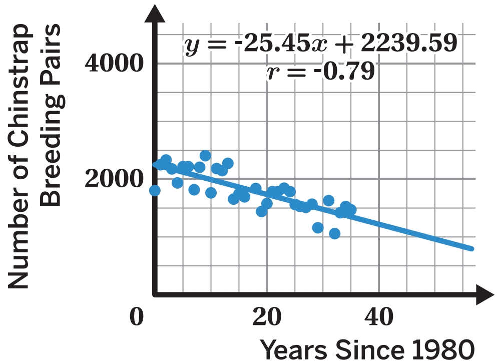
Activity 2

Name: ________ Date: ________ Period: ________

Generating a Line of Best Fit (continued)

9 Compare the data of these two penguin populations.





Discuss: How are they alike? How are they different?

Responses vary.

- Both populations are decreasing over time.
- They both started with more than 2,000 breeding pairs and have had years with around 1,000 breeding pairs.
- Both of the r-values and slopes are negative.
- The Adélie started off with way more breeding pairs on the island (2,733 instead of 2,239).
- The relationship between population and time is stronger for the Chinstrap penguins. This might mean that their relationship has been more predictable so far.
- The Adélie penguin population is declining at a faster rate according to the line of best fit.

Activity 3

Name: .... Date: .... Period: ....

Making Predictions

10 Use the digital activity to analyze the Gentoo data.

Generate the line of best fit and write the equation for the Gentoo data.

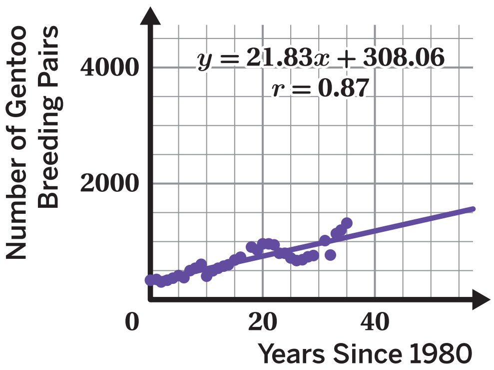
$y = 21.83x + 308.06$ (or equivalent)

11 Here is the line of best fit for the Gentoo data.

How many breeding pairs of penguins does the line of best fit predict there will be in 2030?

$21.83(50) + 308.06$ (or equivalent)

Responses between 1,399 and 1,400 breeding pairs are considered correct.



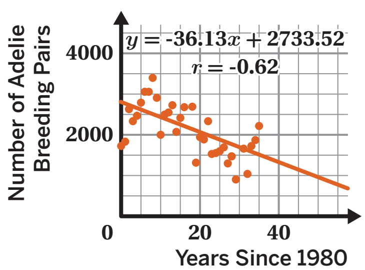
Activity 3

Name: Date: Period:

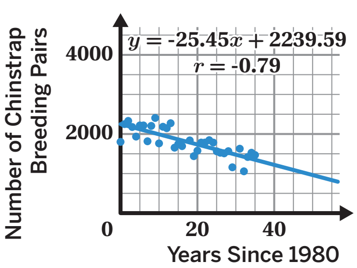
Making Predictions (continued)

12 What questions might researchers have after analyzing the data from the Adélie, Chinstrap, and Gentoo penguin populations?

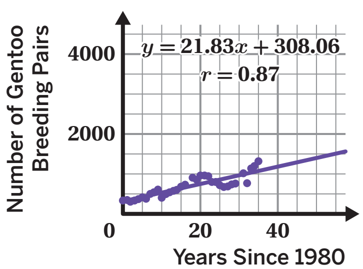
Adélie



Chinstrap



Gentoo



Responses vary.

- **Why might the Adélie and Chinstrap penguin populations be declining while the Gentoo penguin population is growing?**
- **If nothing changes, when would each group of penguins become extinct from the island?**
- **What else might be changing on the island that's associated with these changes in population?**
- **Does the data from 2020 confirm this pattern?**

13 Here's a quote about modeling that you may remember:

All models are wrong, but some are useful.

a Select a model we've explored today. ***Choices vary.***

b Explain how that model is wrong and how it is useful. ***Explanations vary.***

The model is wrong because . . .

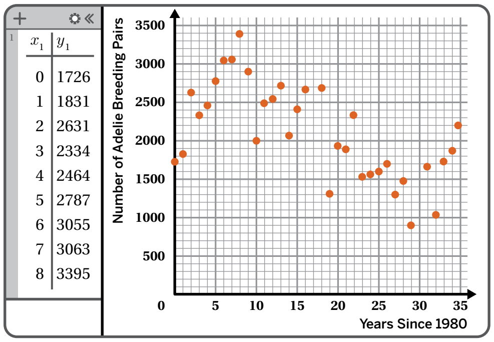
- **The Adélie model is wrong because populations don't always go straight up or down like a line.**
- **The Gentoo model is wrong because the population can't increase forever. At some point, the island would run out of space for more penguins.**

It is useful because . . .

- **The Chinstrap model is useful because the relationship between population and time is pretty strong, which makes me more confident in my predictions.**
- **All three models are useful because they can still help predict the short-term future.**

14 Synthesis

Describe how to use the Desmos Graphing Calculator to generate the line of best fit for data.

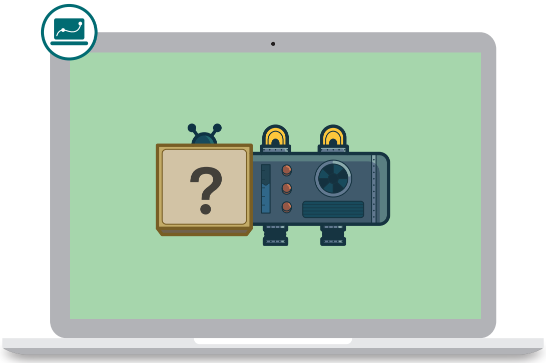


Responses vary. First, make sure that you know which variables the table uses. Use those variables in your expression. If the variables are x_1 and y_1, write $y_1 \sim mx_1 + b$ in a new line in the calculator. The calculator will show you the r-value, the slope m, and the intercept b. You can write an equation for the line of best fit from this information using the m and b values.

Things to Remember:

Unit 4
Lesson
1

Name: ______ Date: ______ Period: ______



Mystery Rule

Let's consider whether or not rules are functions.

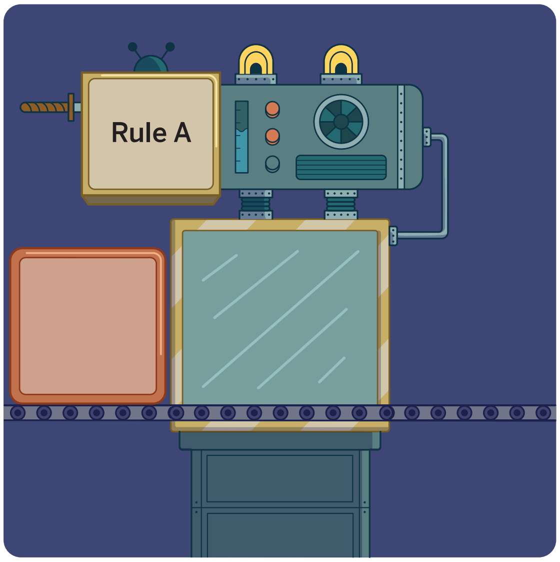
Warm-Up

1 This machine uses Rule A to turn *inputs* into *outputs*.

Let's test several inputs to see how Rule A works. Record the results in the table.

Responses vary.

Input	Output
5	16



2 Predict the output for 101. Explain your reasoning.

304. *Explanations vary.* The pattern is to multiply the input by 3 and then add 1, so $101 \cdot 3 = 303 + 1 = 304$.

Activity 1

Name: ________ Date: ________ Period: ________

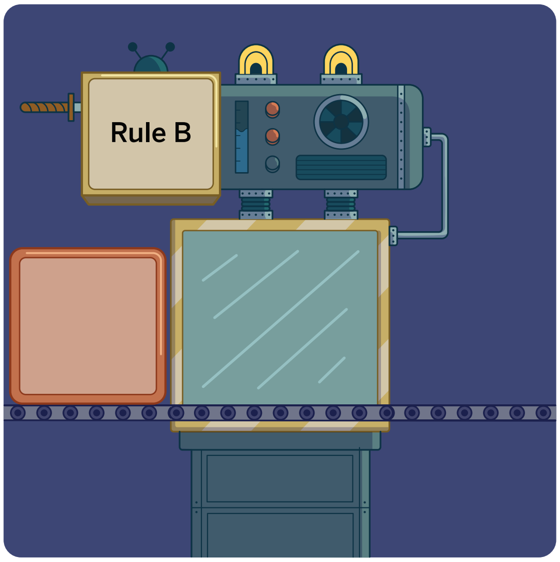
What Is a Function?

3 Rule B's inputs are words.

Let's test several inputs to see how Rule B works. Record the results in the table.

Responses vary.

Input	Output
howdy	8



4 Predict the output for "give". Explain your reasoning.

7. Explanations vary.

- I noticed that "howdy" had an output of 8, and that "h" is the eighth letter of the alphabet, so I think that "give" would have an output of 7.
- Each output is based on the first letter of the word and its position in the English alphabet. So that means words that start with A will have an output of 1, B = 2, C = 3, D = 4, E = 5, F = 6, G = 7, and so on.

Name: .... Date: .... Period: ....

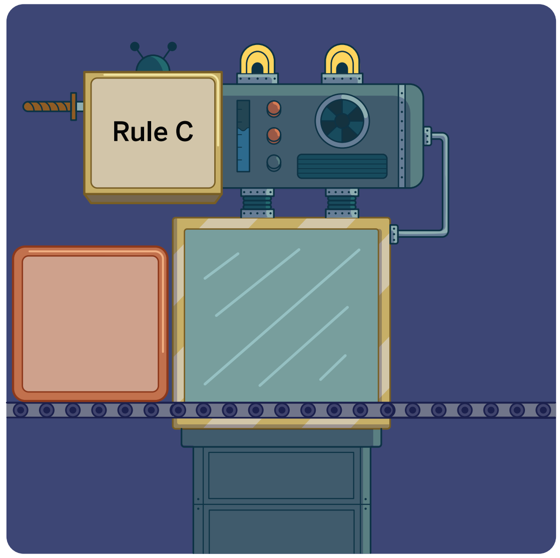
What Is a Function? (continued)

5 Rule C's inputs are whole numbers from 1 to 15.

Let's test several inputs to see how Rule C works. Record the results below.

Responses vary.

Input	Output



6 **a** Predict the output for 6.

***Responses vary.* Any six-letter word.**

b Compare your response with a partner's. How are your responses alike and different?

***Responses vary.* My partner and I both wrote six-letter words, but they were different words.**

7 Rules A and B are examples of a *function*.

Rule C is *not* a function.

What do you think makes Rule C not a function?

Responses vary.

- **Rule C is not a function because the input 9 has two different outputs.**
- **Rule C is not a function because every input does not have only one output. 9 has outputs of both "vegetable" and "classroom."**

Functions

Rule A

Input	Output
5	16
6	19
0	1
5	16

Rule B

Input	Output
howdy	8
face	6
mountain	13
flower	6

Not a function

Rule C

Input	Output
5	watch
9	vegetable
9	classroom
1	a

Rule H

Input	Output
2	5
1	10
0	15
1	20

Name: Date: Period:

Which Rules Are Functions?

8 Here are four rules.

Discuss: Is each rule a function? Why or why not?

Rule D takes temperatures in Fahrenheit and outputs temperatures in Celsius.

Input	Output
50	10
77	25
100	37.8
212	100

Rule D is a function. *Explanations vary.* Every temperature in Fahrenheit has exactly one matching temperature in Celsius.

Rule E takes any integer and outputs one of its factors.

Input	Output
24	6
13	1
13	13
9	3

Rule E is not a function. *Explanations vary.* If 13 is the input, there are two possible outputs: 1 or 13.

Rule F takes any number and rounds it to a whole number.

Input	Output
32.5	33
$\frac{4}{3}$	1
0.1	0
23	23

Rule F is a function. *Explanations vary.* Rounding any number always gets you one whole number as an output.

Rule G takes any word and shifts each letter one place in the alphabet.

Input	Output
bird	cjse
house	ipvtf
hello	ifmmp
world	xpsme

Rule G is a function. *Explanations vary.* You can follow a rule to make this shift the same way every time.

Activity 2

Name: .... Date: .... Period: ....

Which Rules Are Functions? (continued)

9 Rule E is *not* a function.

How can you tell by looking at the table?

Responses vary.

- **I notice that when you have the input 1,200, you get two different outputs for the rule.**
- **Rule E is not a function because the input 1,200 repeats without getting the same output both times.**
- **Rule E is not a function because any integer, including prime numbers, has more than one factor, so you could get more than one output for every input.**

Rule E takes any integer and outputs one of its factors.

Rule E	
Input	**Output**
1200	30
1200	10
20	10

10 Synthesis

How can you decide whether a rule is a function?

Responses vary.

- **In order to determine whether a rule is a function, you need to test different inputs to see if the outputs repeat. If the outputs are different for the same input, then the rule is not a function.**
- **If you have an input value that has two different output values in a rule, then the rule is not a function.**

Rule F

Input	Output
6.8	7
6.6	7
6.4	6
6.4	6

Rule C

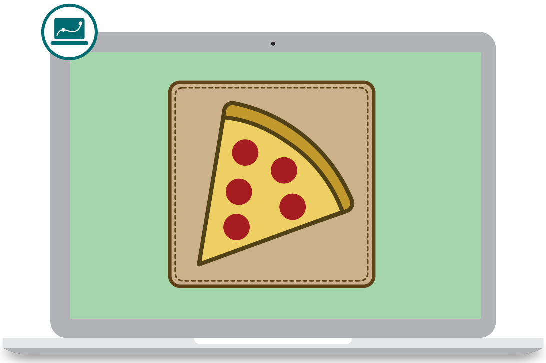
Input	Output
5	watch
9	vegetable
9	classroom
1	a

Rule G takes any word and shifts each letter one place in the alphabet.

Things to Remember:

Unit 4
Lesson
2

Name: ____________ Date: ________ Period: ________



Pricing Pizzas

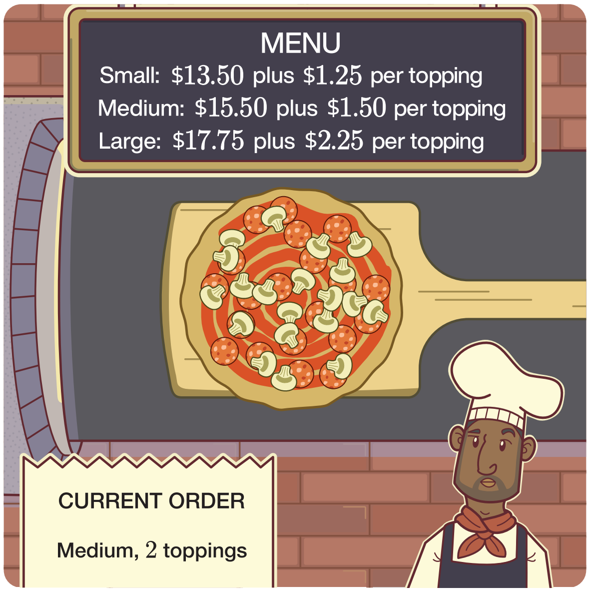
Let's learn what function notation is and interpret function notation statements in context

Warm-Up

1 Desmos Pizza offers small, medium, and large pizzas.

Use the menu to determine the price of each pizza order.

Pizza Order	Price ($)
Medium, 2 toppings	18.50
Large, 4 toppings	26.75
Small, 3 toppings	17.25
Large, 6 toppings	31.25



Activity
1

Name: .... Date: .... Period: ....

Pricing Pizzas

2 A worker at Desmos Pizza made a list of all the large pizza orders one night.

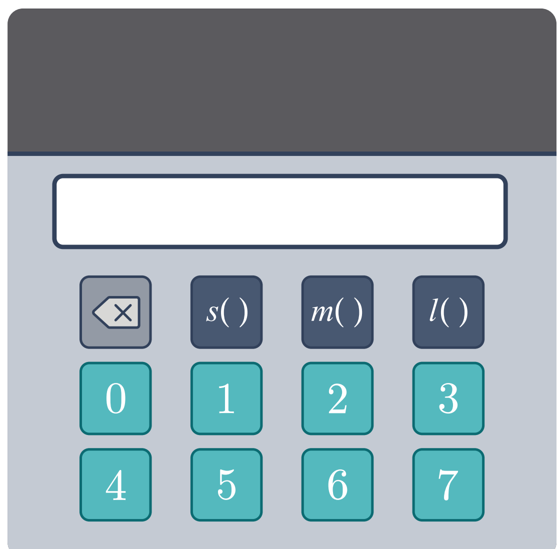
Discuss: How is this list like a function?

***Explanations vary.* This list is like a function because each number of toppings only matches up with one price. But, if this list had prices for medium and small pizzas too, it would not be a function.**

3 Desmos Pizza uses a cash register with three functions: one for each size pizza.

In the digital activity, use the functions to determine the price of each pizza order.

Pizza Order	Price ($)
Small, 3 toppings	$s(3) = 17.25$
Large, 2 toppings	$l(2) = 22.25$
Medium, 1 topping	$m(1) = 17.00$



Activity 1

Name: ________ Date: ________ Period: ________

Pricing Pizzas (continued)

4 $s(3)$ is an example of a statement in **function notation**.

We read $s(3)$ as "s of three."

a Say $s(3) = 17.25$ aloud to a classmate.

b Select *all* the ideas that $s(3)$ represents.

- ☑ **A.** The price of a small pizza with 3 toppings.
- ☐ **B.** The price of a small pizza multiplied by 3.
- ☑ **C.** The function s with an input of 3.
- ☐ **D.** The function s with an output of 3.
- ☐ **E.** The price of 3 small pizzas.

5 In the previous problem, Luca said:

s times 3 is 17.25, so a small pizza with 3 toppings will cost $\$5.75$.

What would you say to help him understand his mistake?

***Responses vary.* Luca thinks that $s(3)$ means to multiply s by 3. When we are reading a statement in function notation, a 3 in parentheses doesn't mean multiplication.**

Luca

$$\frac{s(3)}{3} = \frac{17.25}{3}$$

$$s = 5.75$$

Activity 2

Name: ____________ Date: ________ Period: ________

Interpreting Function Notation

6 Match each function notation statement with its correct interpretation(s). Three cards will have no match.

Card A	Card B	Card C	Card D
The function s with the input 0.	A small pizza that costs $\$0$.	The price of a medium pizza with x toppings.	5 large pizzas
Card E	**Card F**	**Card G**	
The output of l when the input is 5.	The price of a small pizza with 0 toppings.	The price of a medium pizza multiplied by x.	

$l(5)$	$s(0)$	$m(x)$
Card E	Card A Card F	Card C

Cards with no match: Card B, Card D, and Card G

7 Emma and her friends are texting about their pizza order.

Emma writes: $m(7) < l(5)$.

What do you think this means?

***Responses vary.* A medium pizza with 7 toppings costs less than a large pizza with 5 toppings.**

8 Select *all* the true statements.

☑ **A.** $s(2) > s(1)$

☐ **B.** $m(4) < s(4)$

☑ **C.** $l(4) > s(3)$

☑ **D.** $l(1) = m(3)$

☐ **E.** $l(0) > m(2)$

Activity 3

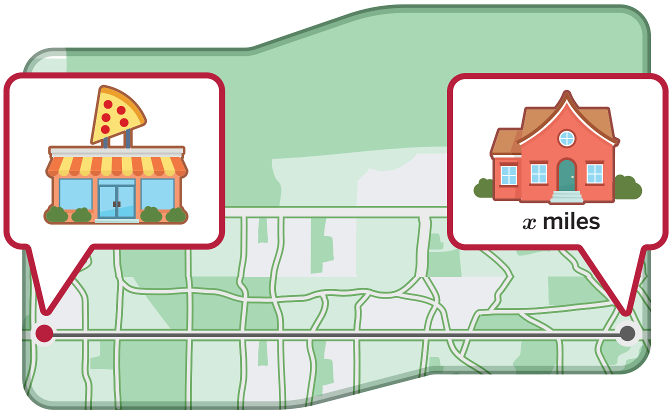
Name: .... Date: .... Period: ....

Delivering Pizzas

9 Desmos Pizza uses the function $d(x)$ to estimate the number of minutes it takes to deliver pizza x miles.

What would $d(2) = 30$ mean in this situation?

***Responses vary.* $d(2) = 30$ means that it takes 30 minutes to deliver a pizza 2 miles.**



10 $d(x)$ estimates the number of minutes it takes to deliver pizza x miles. Match each function notation card to a description. One card will have no match.

Card A
It takes longer to deliver 5 miles away than 1 mile away.

Card B
The number of minutes to make a delivery 5 miles away.

Card C
A delivery 1 mile away will take more than 5 minutes.

Card D
A delivery 5 miles away will take more than 1 minute.

Card E
Delivering 5 pizzas takes longer than delivering 1 pizza.

$d(5) > 1$	$d(5)$	$d(5) > d(1)$	$d(1) > 5$
Card D	Card B	Card A	Card C

Card with no match: Card E

11 Synthesis

This lesson introduced function notation.

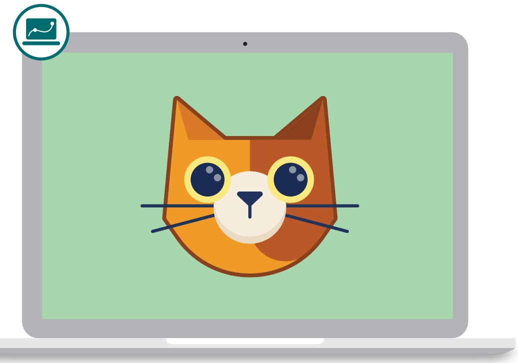
a. Say the equation $m(5) = 23$ aloud to a classmate.

b. Describe what each part of the equation means.

Responses vary. **m is the name of the function that models the number of toppings and pizza price for medium pizzas. 5 is the number of toppings and 23 means $\$23.00$, the price of a medium pizza with 5 toppings.**

Things to Remember:

Name: .... Date: .... Period: ....



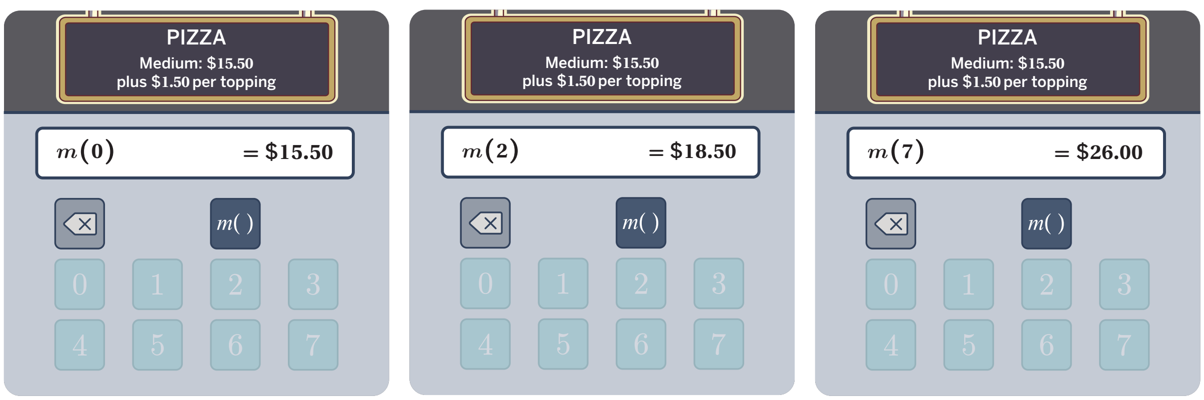
Toy Factory

Let's explore functions represented as equations written in function notation.

Warm-Up

1 The cash register uses $m(x)$ to determine the price of a medium pizza with x toppings.

a Here are three pizzas and their prices.



b Describe how the cash register calculates prices.

***Responses vary.* The cash register takes 15.50 and adds on 1.50 for every topping you have. For example, $m(3) = 20$ because that is 15.50 plus 1.50 times 3.**

2 Which equation represents $m(x)$?

A. $m(x) = 15.5 + 1.5$ **B.** $m(x) = 15.5x + 1.5$ **C.** $m(x) = 15.5 + 1.5x$

Explain your thinking.

***Explanations vary.* x represents the number of toppings and it's $1.50, or 1.5, per topping, so you have to multiply 1.5 by x.**

Activity 1

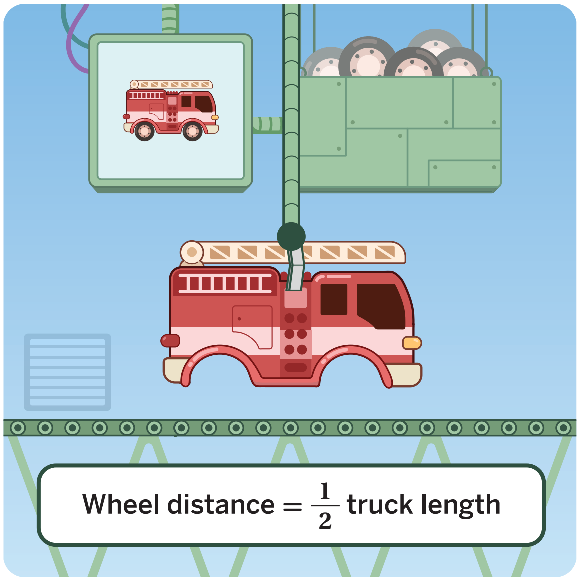
Name: ______ Date: ______ Period: ______

Exploring Equations of Functions

3 A toy factory makes fire trucks in a variety of sizes.

The distance between the truck's wheels is always half the length of the truck.

Complete the table to put wheels on the toy truck.



Truck Length (in.)	Wheel Distance (in.)
10	5
6	3
3	1.5

4 Kanna wrote the function $d(x) = \frac{1}{2}x$ to determine the wheel distance for a truck length of x.

a. **Discuss:** What does $d(x) = \frac{1}{2}x$ mean?

Responses vary. It means that the wheel distance is half of whatever the length of the truck is.

b. What is the value of $d(7)$?

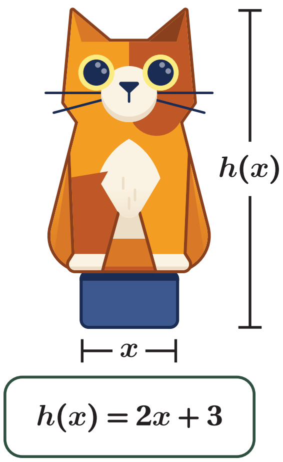
3.5

Name: ________ Date: ________ Period: ________

Exploring Equations of Functions (continued)

5 The factory also makes toy cats.

They use this diagram and function to determine where to place the cat's eyes. All units are in inches.

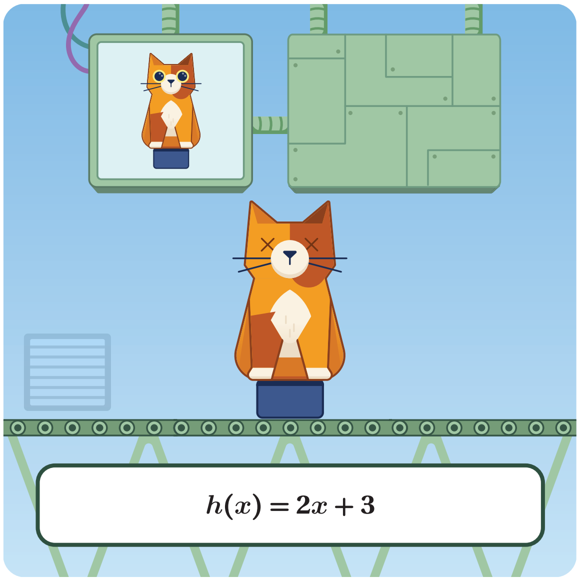


What does $h(x) = 2x + 3$ mean in this situation?

Responses vary. **It says that the cat's eyes should be at a height that is twice the width of the base plus 3 inches.**

6 Calculate the value of each function notation expression.

Expression	Eye Height (in.)
$h(5)$	**13**
$h(3)$	**9**
$h(7.5)$	**18**



Discuss: Why is it useful to write a function as an equation?

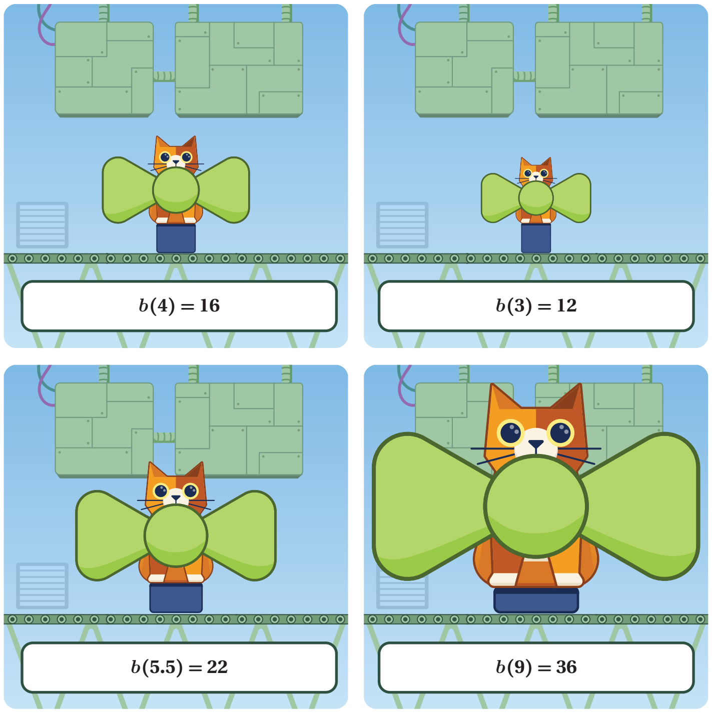
Responses vary. **An equation lets us apply a relationship for lots of different input-output pairs.**

Activity 2

Name: ______ Date: ______ Period: ______

Writing Equations of Functions

7 Each toy cat needs a bow tie. The function $b(x)$ determines the width of the bow tie, where x is the width of the base. All units are in inches.



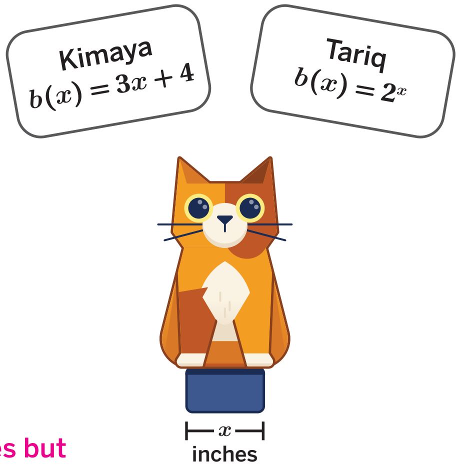
a Let's see how the function $b(x) = 4x$ works.

b Change the equation $b(x) = 4x$ to make bow ties that fit better.

Equations vary.

- $b(x) = 3x$
- $b(x) = 6$
- $b(x) = \frac{10}{x} + 2$

8 $b(x)$ determines the width of the bow tie, where x is the width of the base.



Kimaya says that her function will produce a wider bow tie than Tariq's function for any base width.

Is she correct? Circle one.

Yes No

Explain your thinking.

Explanations vary. Kimaya's statement is true sometimes but not for every base width. For example, if the base width is 5 inches, then Tariq's function makes a wider bow tie.

Activity 2

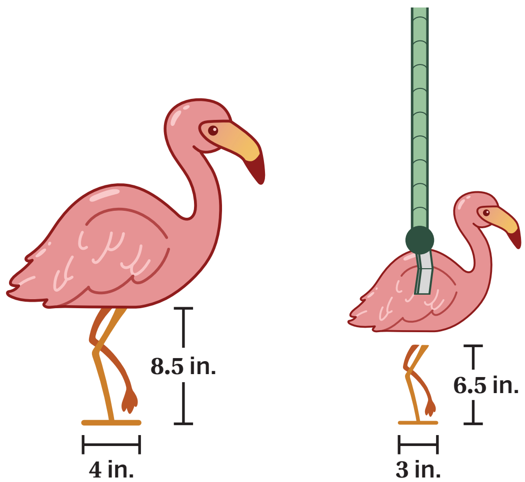
Name: ______ Date: ______ Period: ______

Writing Equations of Functions (continued)

9 Flamingo Frank is made by attaching a flamingo to legs at a specific height.

This height is determined by the width of the base.

Base Width (in.)	Height (in.)
4	8.5
3	6.5
1	2.5
5.5	11.5



How can you determine the height for any base width?

Responses vary. **The height is twice as tall as the base width plus 0.5 inches for the little foot stand.**

10 This machine assembles Flamingo Frank by attaching it to legs at a specific height. That height, $f(x)$, depends on the width of the base, x. Write an equation for $f(x)$.

$f(x) = 2x + 0.5$ **(or equivalent)**

Explore More

11 Here are four figures in a visual pattern. The number of tiles is a function of the figure number.

Figure, n	Number of Tiles, $t(n)$
1	3
2	6
4	18
6	38

Figure 1

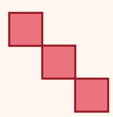


Figure 2

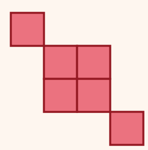


Figure 4

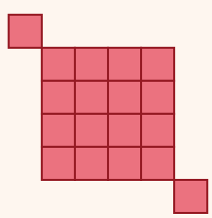
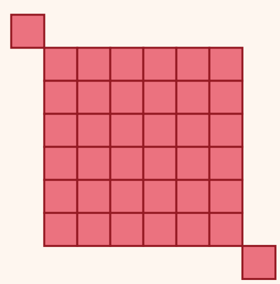


Figure 6

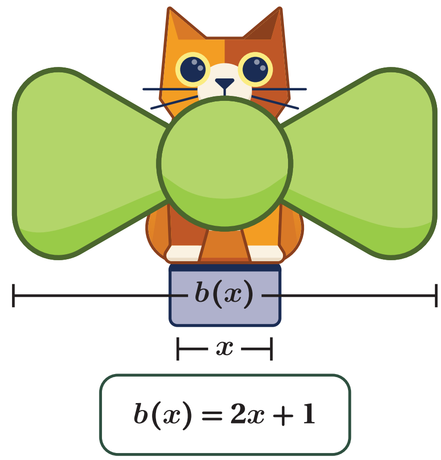


Write an equation for $t(n)$.

$t(n) = n^2 + 2$

12 Synthesis

A toy factory uses this diagram and function to determine the width of the bow tie. All units are in inches.



What does $b(x) = 2x + 1$ mean in this situation?

Responses vary. **$b(x) = 2x + 1$ means that for any base width, the bow tie is twice as wide plus 1 inch.**

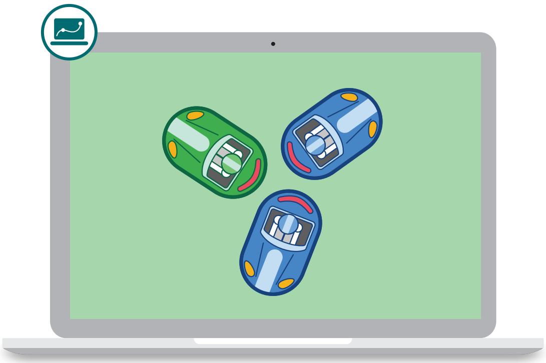
What does $b(4) = 9$ mean?

Responses vary. **$b(4) = 9$ means that when the base is 4 inches wide, the bow tie is 9 inches wide.**

Things to Remember:

Unit 4
Lesson
4

Name: ______ Date: ______ Period: ______



Function Carnival

Let's create and analyze graphs that represent stories.

Warm-Up

1 Let's watch a video.

Write a story about what you see.

Responses vary. What is the cannon doing there? Oh! There is a person in it. The person is launched straight up. When they get close to the top of the screen, they begin to fall back down to the ground. About halfway through their journey down, they pop open their parachute. This slows the Cannon Person down so that they don't hurt themselves. They land successfully and celebrate!

Activity 1

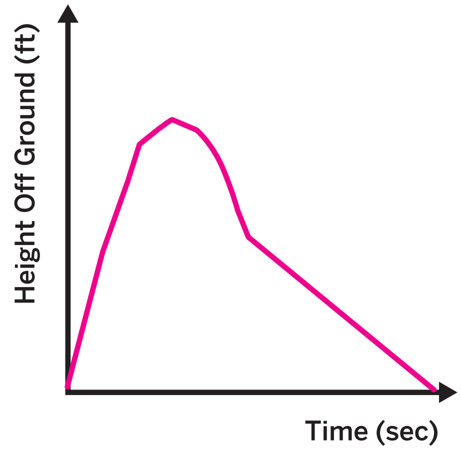
Name: .... Date: .... Period: ....

Cannon Person

2 The function $h(t)$ represents the person's height off the ground at time t.

Sketch a graph of $h(t)$.

Graphs vary.



3 Let's look at a precise graph of $h(t)$.

Select a true statement. ***Responses vary.*** **A and D are true statements.**

A. $h(3) > h(5)$

B. $h(3) = h(5)$

C. $h(0) < h(10)$

D. $h(0) = h(10)$

Explain what the statement says about the Cannon Person.

Explanations vary.

- **$h(3) > h(5)$ says the Cannon Person was higher off the ground at 3 seconds than at 5 seconds.**
- **$h(0) = h(10)$ says that the height of the Cannon Person is the same at 0 seconds as at 10 seconds.**

Activity 2

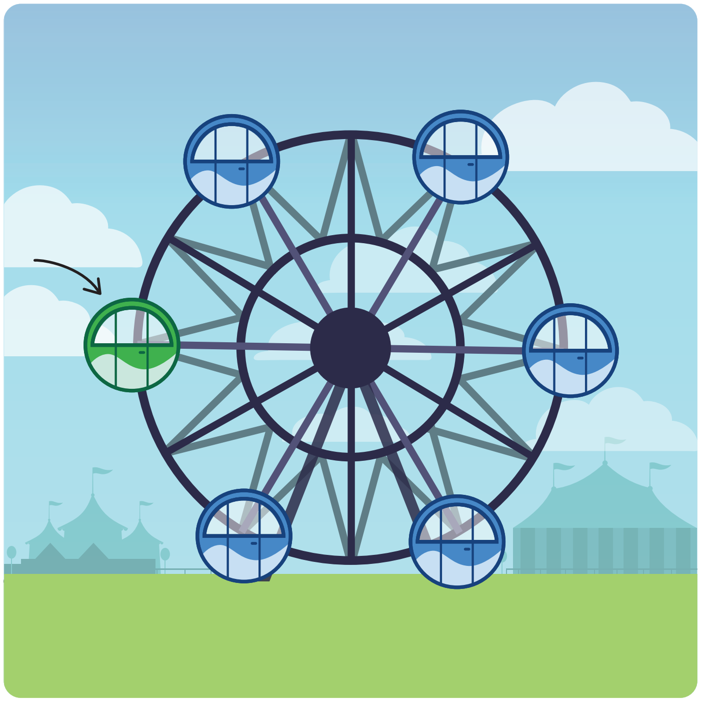
Name: .... Date: .... Period: ....

Ferris Wheel

4 Let's watch a video of a Ferris wheel.

Describe what happens to the height of the green cart.

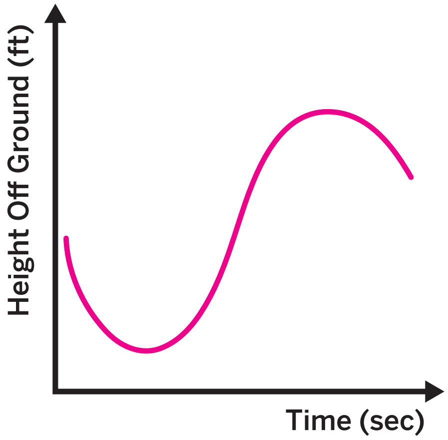
Responses vary. **At first the cart went down, and then the cart went up, and then the cart came back down.**



5 The function $h(t)$ represents the height of the Ferris wheel at time t.

Sketch a graph of $h(t)$.

Graphs vary.



6 Let's look at the graph that Liam drew.

What do you think happened to the green cart when Liam pressed play?

Responses vary. **For about a second, there won't be any green cart. Then, the cart will appear and almost immediately split into two, with one going up and one going down. Then, they'll come back toward each other until they very briefly come together and then immediately disappear.**

Activity 3

Name: ______ Date: ______ Period: ______

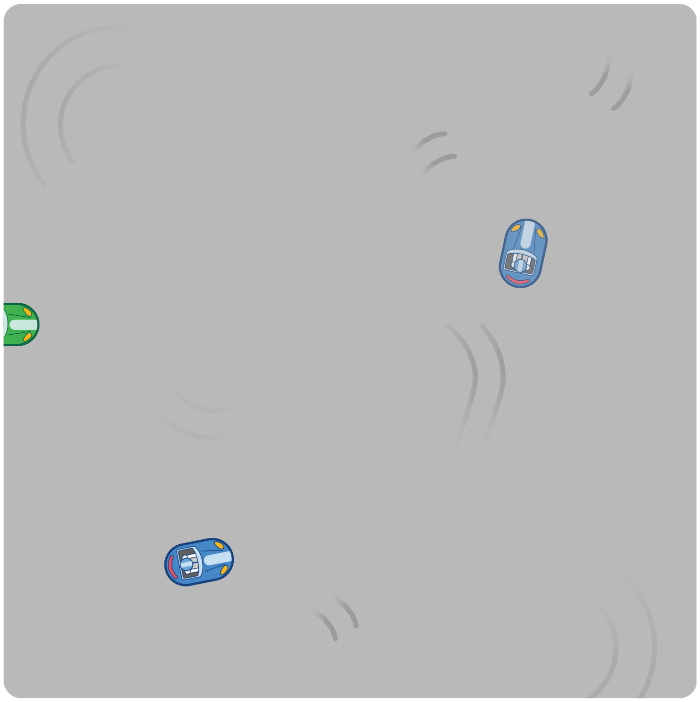
Bumper Cars

7 Let's watch a video of some bumper cars.

What are some different things you could measure and display in a graph?

Responses vary.

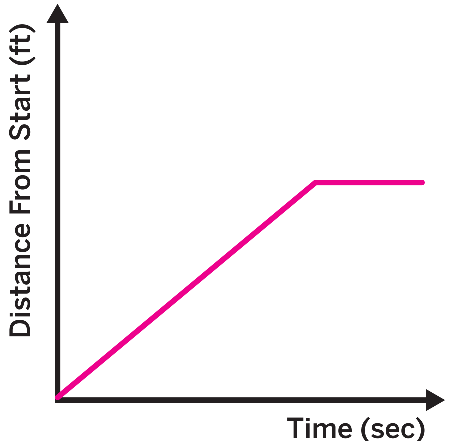
- **Number of cars that are visible**
- **Distance traveled by the green car**
- **Direction the green car is facing**
- **Distance between the green car and the closest car to it**
- **Length of the trail behind the green car**



8 The function $d(t)$ represents the distance traveled by the green car at time t.

Sketch a graph of $d(t)$.

Graphs vary.



Name: ______ Date: ______ Period: ______

Bumper Cars (continued)

9 Match each description to its graph. Two graphs will have no match.

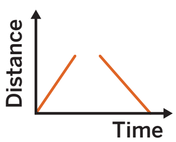
Note: Distance measures distance from the start.

a The roller coaster moves slowly, stops for a moment, then goes very fast.

b The roller coaster moves forward, stops, and then backs up.

c The roller coaster starts in the middle, backs up, and then splits into multiple roller coasters.

______



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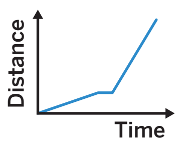

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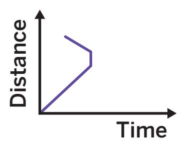
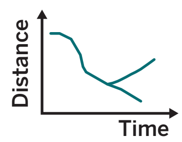


c



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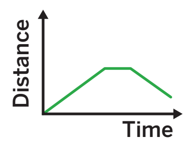
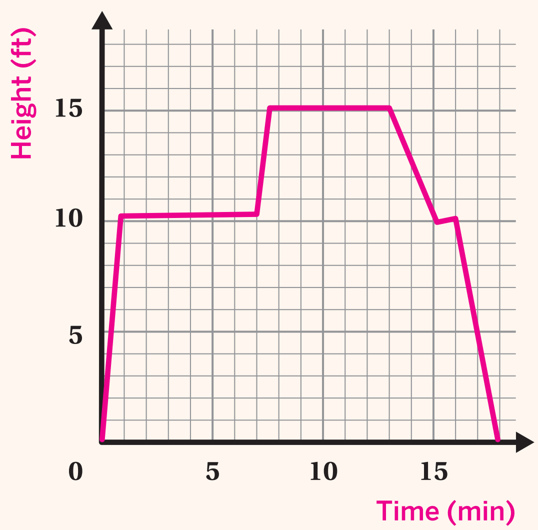


Explore More

10 Draw your own graph and label the x-axis and y-axis. Then write a story about what it describes.

Graphs and stories vary. **Lucia is cleaning up her house after a winter storm and wants to sweep branches off her roof. She climbs up the ladder to the first section of roof, which is 10 feet off the ground, and sweeps off debris. She then climbs up the ladder 5 more feet to reach the top of her house where she finishes sweeping before going back down slower than she went up to be safe. She pauses for a minute to gather her tools before climbing all the way down the ladder.**

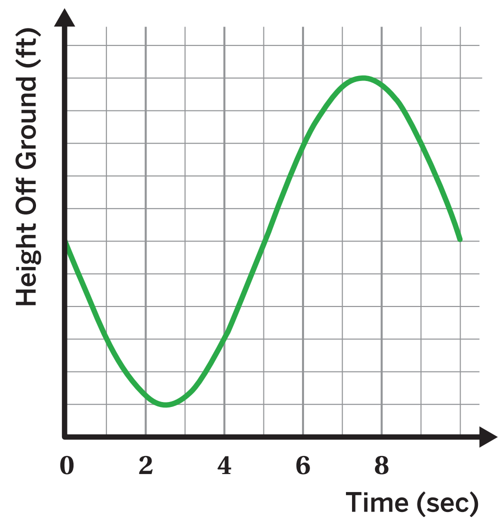


12 Synthesis

How can a graph help tell a story about a situation?

Use this example of a Ferris wheel graph if it supports your explanation.

***Responses vary.* Graphs allow us to see what is happening over time all at once, and provide us with details so we can tell a more precise story. For example, the graph of the Ferris wheel cart says that the height started halfway up, then decreased and then increased past where it started, and then decreased back to the starting height.**



Things to Remember:

Unit 4
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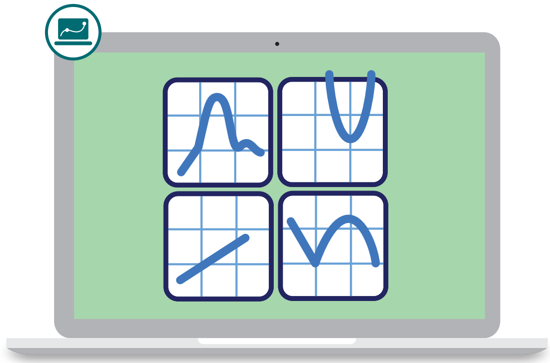
5

Name: ______ Date: ______ Period: ______



Craft-a-Graph

Let's describe and create graphs of functions using key features.

Warm-Up

1 Play a few rounds of Polygraph with your classmates!

You will use a Warm-Up Sheet with functions. In each round:

- You and your partner will take turns being the Picker and the Guesser.
- Picker: Select a function from the Warm-Up Sheet. Keep it a secret!
- Guesser: Ask the Picker yes-or-no questions, eliminating functions until you're ready to guess which function the Picker chose.

Record helpful questions from each round in the space below.

Activity 1

Name: .... Date: .... Period: ....

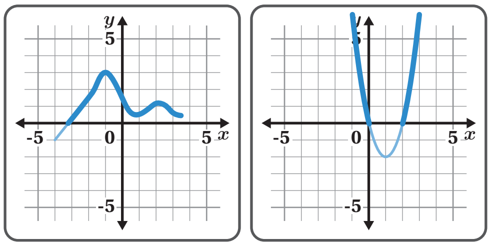
Describe It

2 Here are some functions from Polygraph, along with some terms that describe parts of their graphs.

a Take a look at each term and where it appears on the graph.

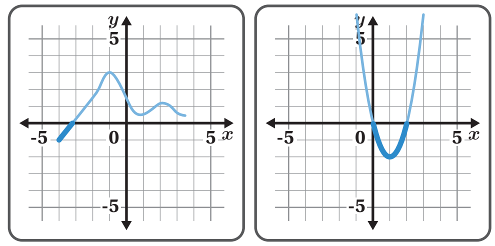
b **Discuss:** What does each term mean? ***Responses vary.***

Positive



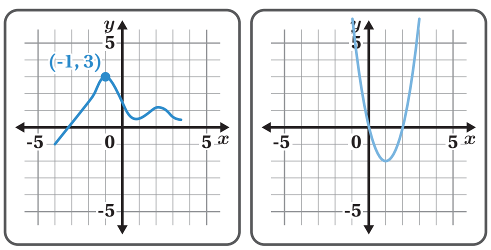
Where the function's graph is above the x-axis.

Negative



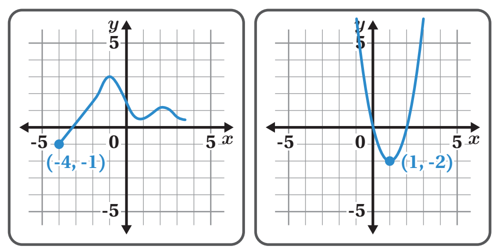
Where the function's graph is below the x-axis.

Maximum



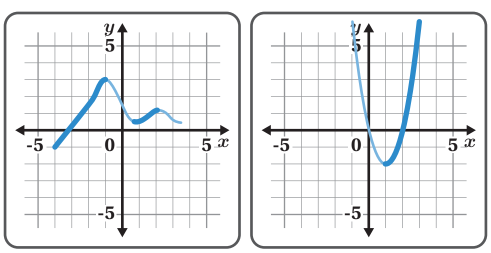
The highest point on a function's graph.

Minimum



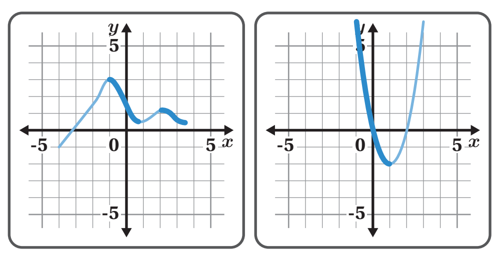
The lowest point on a function's graph.

Increasing



Where a function is going up.

Decreasing



Where a function is going down.

Activity 2

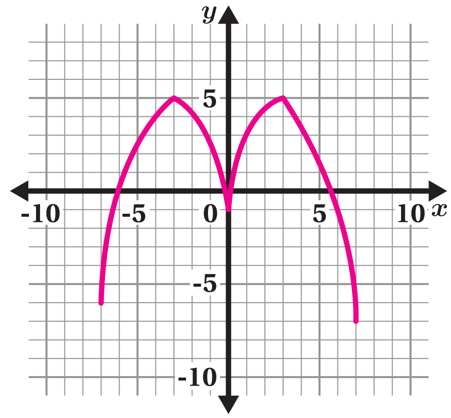
Name: .... Date: .... Period: ....

Build It

3 Now it's your turn to make a function!

Describe your function using some of the terms from the previous problem.

***Functions and descriptions vary.* My function both increases and decreases. It's positive in the middle and has a minimum at (7, -7). It looks like a curvy letter M.**

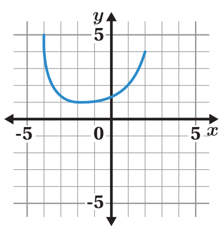


4 Latifah described her function this way:

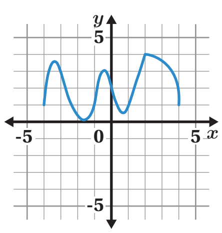
- My function is always positive.
- The maximum is at $(2, 4)$.

Select *all* the functions that could be Latifah's.

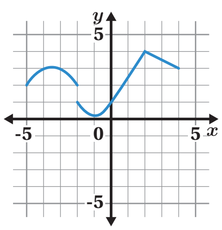
☐ A.



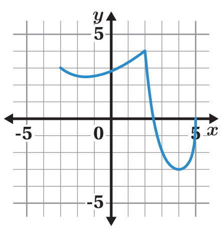
☑ B.



☑ C.



☐ D.



Activity
2

Name: ______ Date: ______ Period: ______

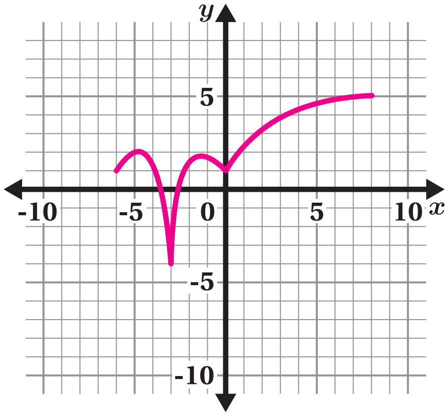
Build It (continued)

5 Haruto described his function this way:

- My function is increasing when $x > 0$.
- It has a minimum at $(-3, -4)$.

Sketch a function that could be Haruto's.

Responses vary.



6 Andrea described her function this way:

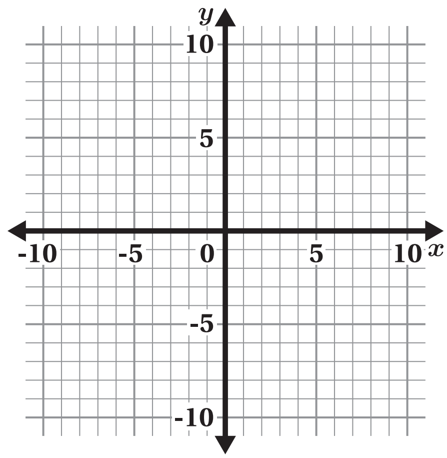
- Positive when $x > -2$.
- Decreasing when $x > 1$.

Is it possible for a function to have both features? Use the graph if it helps with your thinking. Circle one.

Possible Impossible I'm not sure

Explain your thinking.

Possible. *Explanations vary.* A function can be positive and decreasing at the same time because positive numbers can still be decreasing. For example, $5, 4, 3, 2, 1$.



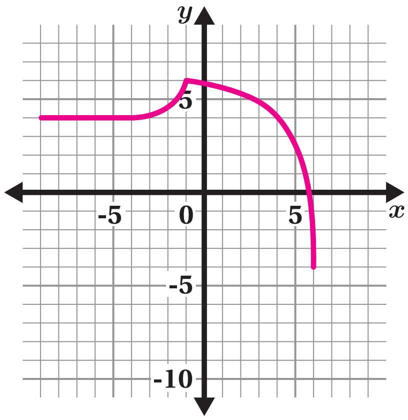
Activity 3

Name: .... Date: .... Period: ....

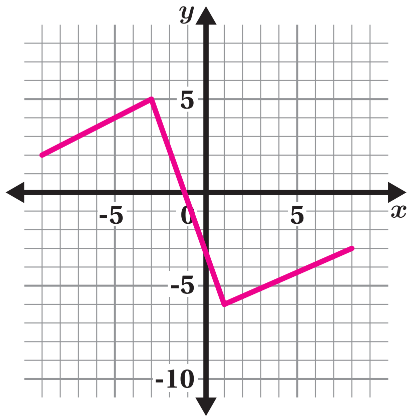
Repeated Challenges

7 Sketch a function that meets these criteria. ***Graphs vary.***

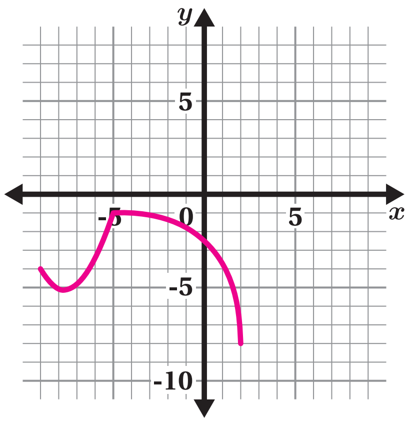
Positive when $x < 4$
Decreasing when $x >$ -1



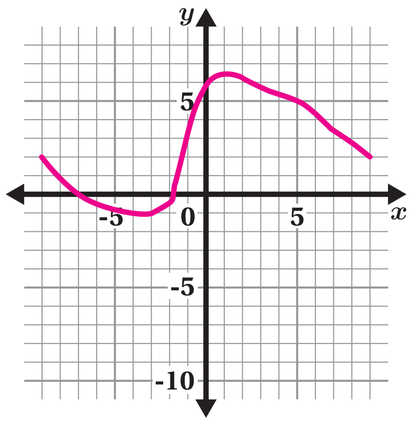
Increasing when $x > 1$
Maximum at (-3, 5)



Negative when $x < 2$
Decreasing when $x >$ -5



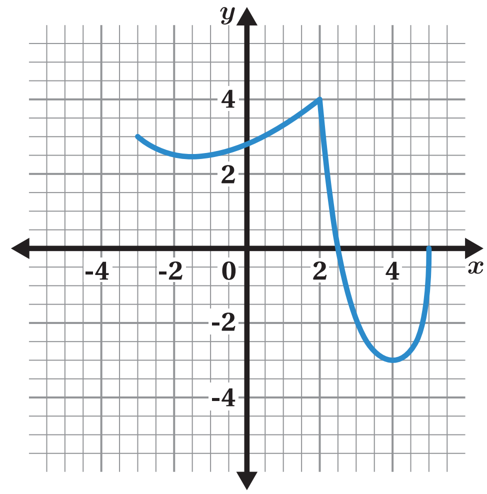
Positive when $x >$ -2
Minimum at (-3, -1)



8 Synthesis

Here are some of the terms to describe functions that we learned about today.

Positive	Negative
Maximum	Minimum
Increasing	Decreasing



Select three terms. Write the meaning of each term you selected.

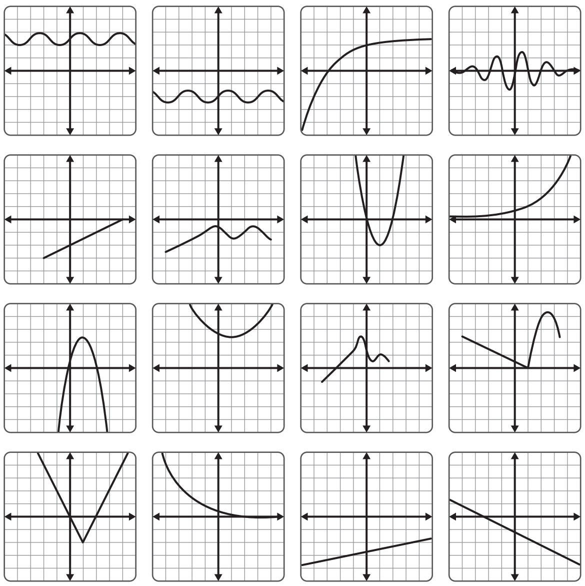
***Responses vary*. See the definitions from the glossary in the Summary below.**

Things to Remember:

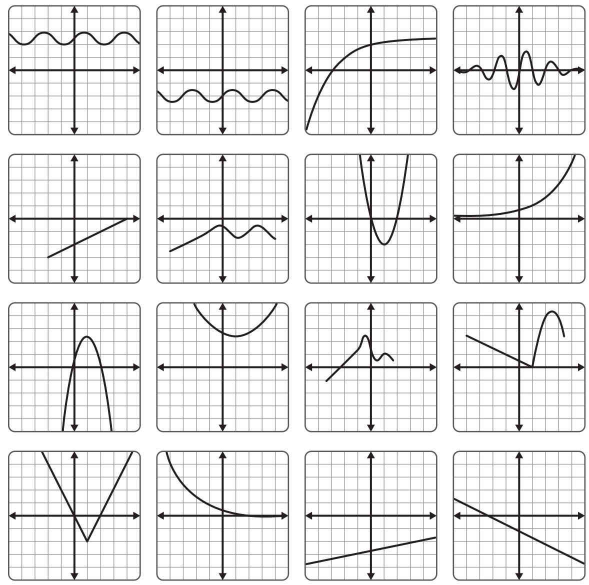
Name: ______________________ Date: __________ Period: __________

Polygraph Set A

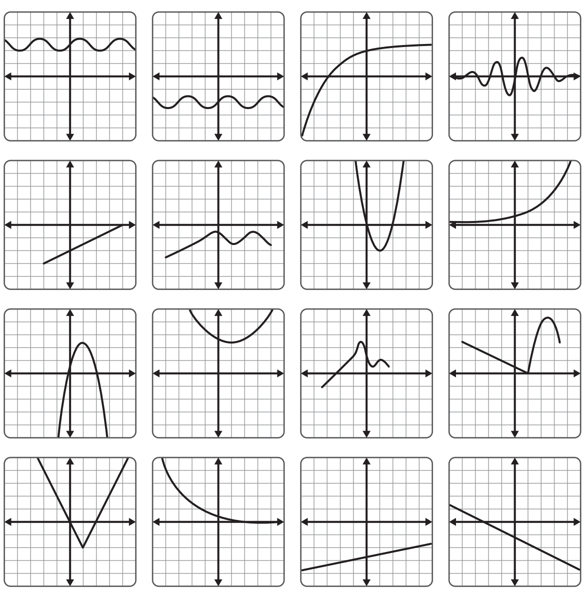
Round 1



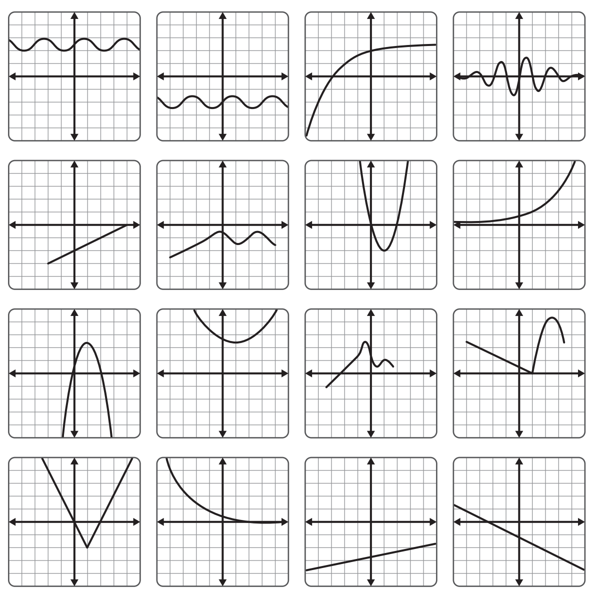
Round 2



Round 3



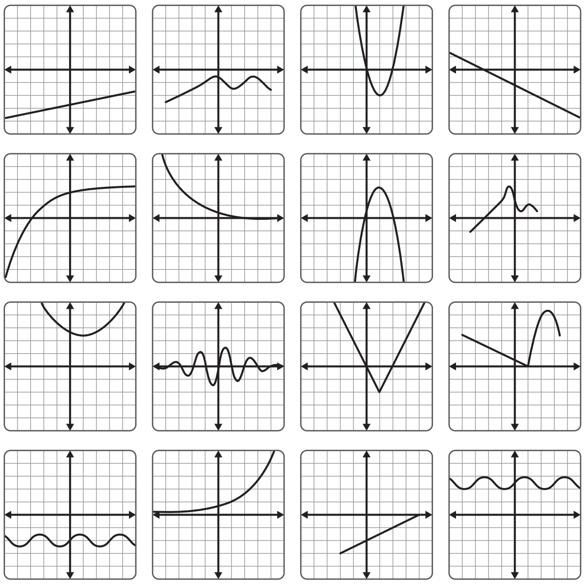
Round 4



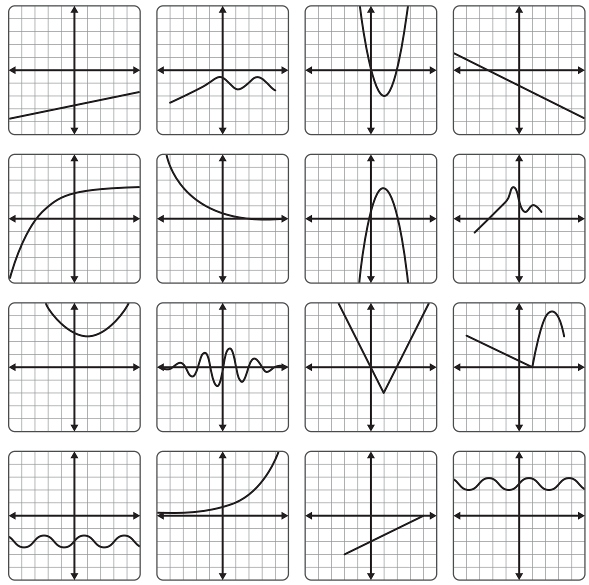
Name: .... Date: .... Period: ....

Polygraph Set B

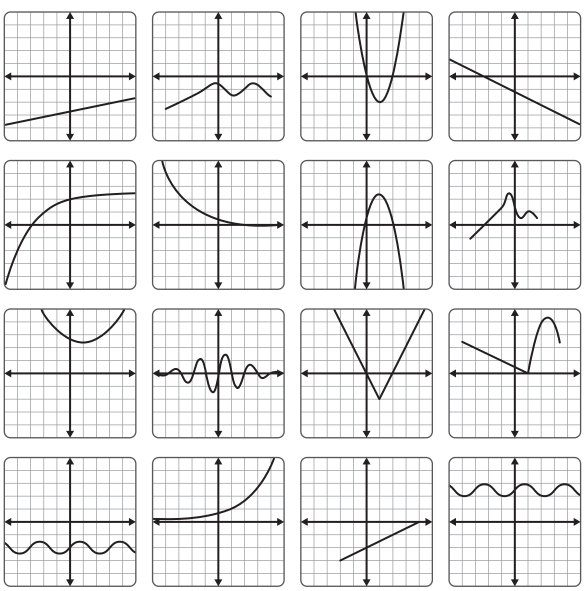
Round 1



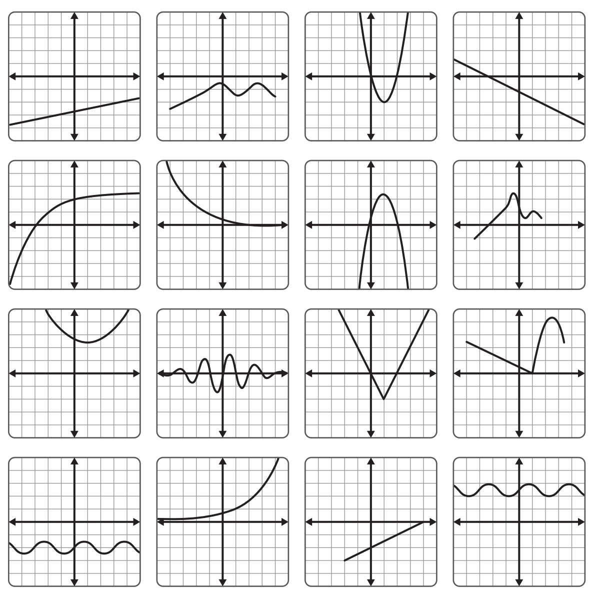
Round 2



Round 3



Round 4



Name: .... Date: .... Period: ....

Plane, Train, and Automobile

Let's calculate the average rate of change over a specified interval.

Warm-Up

1 A wedding is happening in New York City! Many relatives are coming from out of town.

a Take a look at three wedding guests and how they traveled to the wedding.

b What do you notice? What do you wonder? ***Responses vary.***

I notice:

- **I notice Mayra traveled the farthest distance and chose to fly, and Arjun traveled the shortest distance and chose to drive.**
- **I notice Mayra flew east past New York City and then had to travel west.**

I wonder

- **I wonder whose trip was the quickest.**
- **I wonder which trip cost the most money or which trip impacted the environment the most.**

Activity
1

Name: Date: Period:

Arjun’s Automobile Trip

2 Arjun’s car trip was 130 miles and took 3.25 hours.

Imagine Arjun traveled at a constant speed the whole trip.

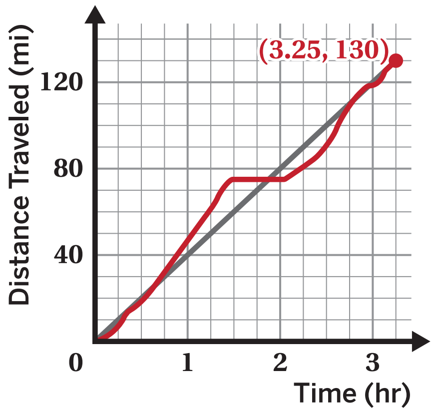
What would be Arjun’s speed in miles per hour?

40 miles per hour

3 **a** Take a look at the map and the graph of Arjun’s actual trip from Hartford, Connecticut to New York City.

b Tell a story about Arjun’s trip.

***Responses vary.* At first, Arjun was traveling at about 40 miles per hour, but he needed to stop and get gas. He stayed there for 30 minutes, maybe to get food and use the bathroom. Then Arjun got back in the car and finished the trip, traveling a bit faster than he did before the stop.**

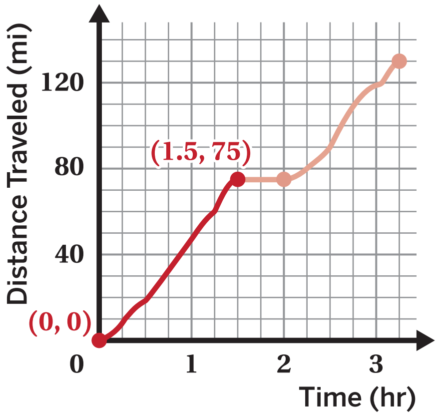


4 The **average rate of change** is the *slope* of the line that connects two points.

For Arjun’s trip, the average rate of change was 40 miles per hour.

We can also look at average rate of change for an **interval**, such as 0 to 1.5 hours (highlighted in the graph).

How would you calculate the average rate of change for that interval?



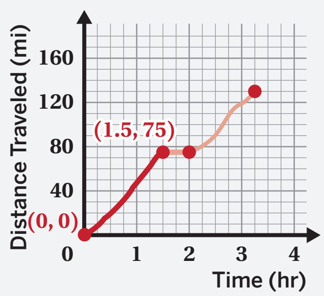
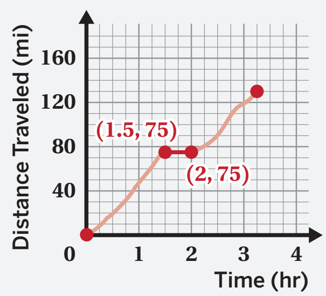
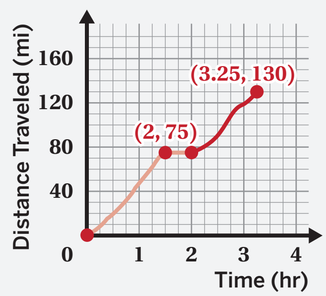
***Responses vary.* I can use the same strategies I used for calculating the slope of a line. I would divide 75 by 1.5 to find the average rate of change for the first interval because the other endpoint is (0, 0).**

Name: .... Date: .... Period: ....

Arjun's Automobile Trip (continued)

5 Let's examine some intervals of Arjun's trip.

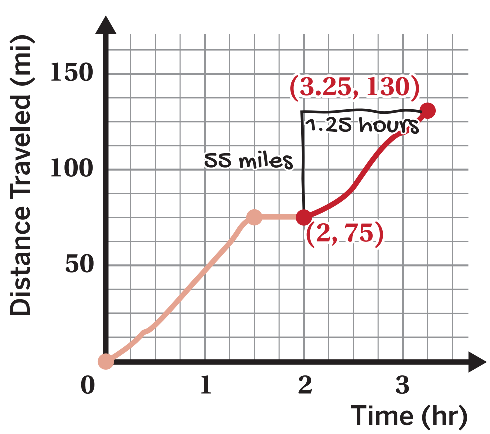
Determine the average rate of change of each interval.

Interval	0 to 1.5 hours	1.5 to 2 hours	2 to 3.25 hours
Graph			
Average Rate of Change (mph)	50	0	44

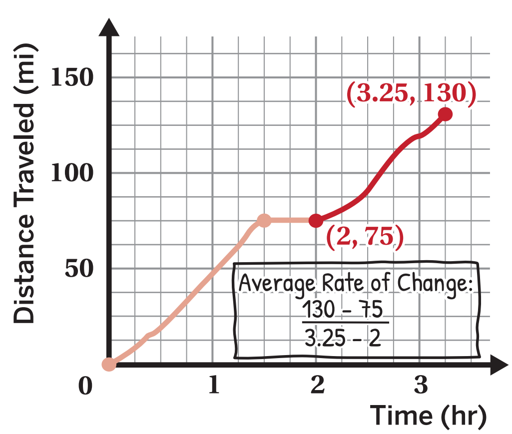
6 Two students calculated the average rate of change between 2 and 3.25 hours.

a Take a look at Naoki's and LaShawn's work.

Naoki



LaShawn



b How do their methods compare?

Responses vary. Naoki's strategy is similar to LaShawn's because he found how many miles Arjun traveled between 130 and 75 miles, and how much time had passed from 2 hours to 3.25 hours, and calculated the average rate of change between those points.

Activity 2

Name: ________ Date: ________ Period: ________

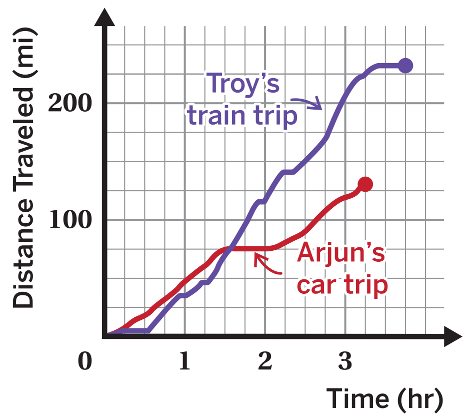
Troy's Train Trip

7 Let's watch an animation of Troy's trip.

Here is the graph for Arjun's and Troy's trips.

What are some ways Troy's trip is different from Arjun's?

Responses vary. **Troy traveled more miles, and it took more time than Arjun's trip. He also wasn't moving at the beginning of his trip because he was waiting for the train but then traveled steadily, while Arjun took a break in the middle.**



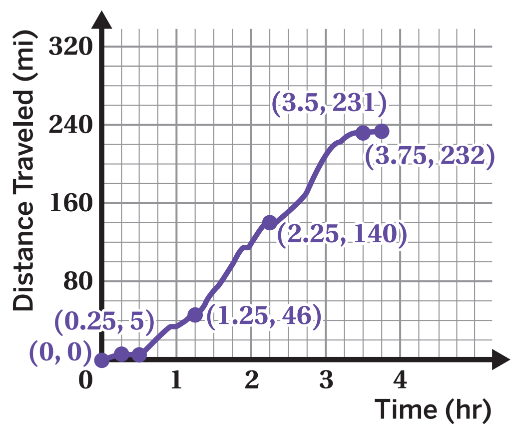
8 Let's examine some intervals of Troy's trip.

Choose two points on the graph. Then calculate the average rate of change for the interval you selected. ***Responses vary based on the points students select.***

Can you find:

- Troy's average rate of change for the full trip?
- An interval where Troy moved fast? Slow?

Interval	Average Rate of Change (mph)
Full trip: 0 to 3.75 hours	**≈61.87**
An interval where Troy moved fast: 1.25 to 2.25 hours	**94**
An interval where Troy moved slow: 0 to 0.25 hours	**20**

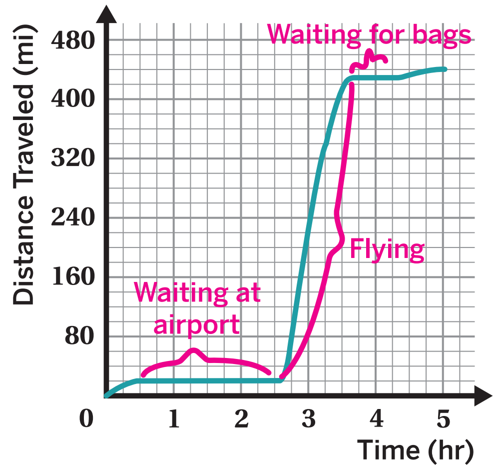


Activity 3

Name: .... Date: .... Period: ....

Mayra's Flight

9 Mayra's plane trip from Pittsburgh was 440 miles and took 5 hours.



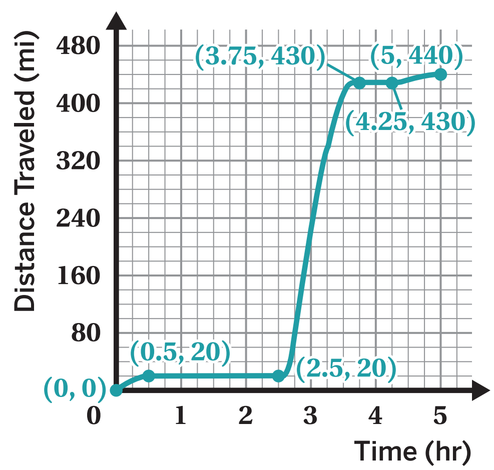
a **Discuss:** What questions do you have about Mayra's trip? ***Responses vary.***

- **How long did it take her to get to the wedding from the airport?**
- **Why was Mayra's average rate of change 0 for such a long interval?**

b Label three or more intervals of the graph with what you think was happening at that time.

Responses vary.

10 Mayra said: *The flight felt fast, but the trip felt slow.*



a Choose two points on the graph. Calculate the average rate of change for the interval you selected. ***Responses vary based on the points students select.***

b Can you find:

- Mayra's average rate of change for the full trip?
- Mayra's average rate of change *during the flight*?

Interval	Average Rate of Change (mph)
Full trip: 0 to 5 hours	**88**
During the flight: 2.5 to 3.75 hours	**328**
Waiting at airport: 0.5 to 2.5 hours	**0**

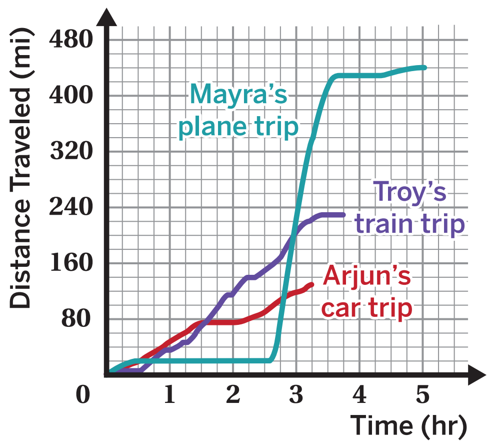
Activity 3

Name: ________ Date: ________ Period: ________

Mayra's Flight (continued)

11 There are many reasons why someone may choose to travel by car, train, or plane.

Here are the graphs for all three people's trips.



Circle the trip you think is best.

Arjun (car) Troy (train) Mayra (plane)

Explain why you chose that trip.

Responses and explanations vary.

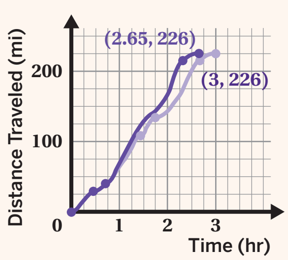
- **I think traveling by car is best, especially if you are carpooling with a group of people because I love to find new and interesting places to stop along the way.**
- **I think traveling by train is best because it's better for the environment due to lower carbon dioxide emissions and the ability to transport more people at a time.**
- **I think traveling by plane is best because it gets you there fast and you don't have to be responsible for driving.**

Explore More

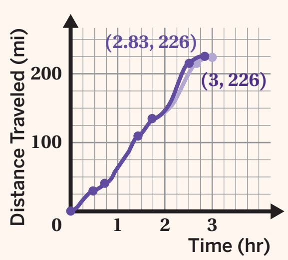
12 Public transit agencies often look at average speed (rate of change) when they look for ways to improve service.

a Explore these three proposals to improve the train from Washington to New York:

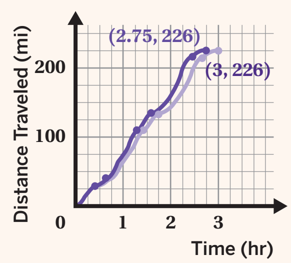
Proposal A: Skip stops along the way.



Proposal B: Upgrade one section of the tracks.



Proposal C: Buy trains with a higher top speed.



b **Discuss:**

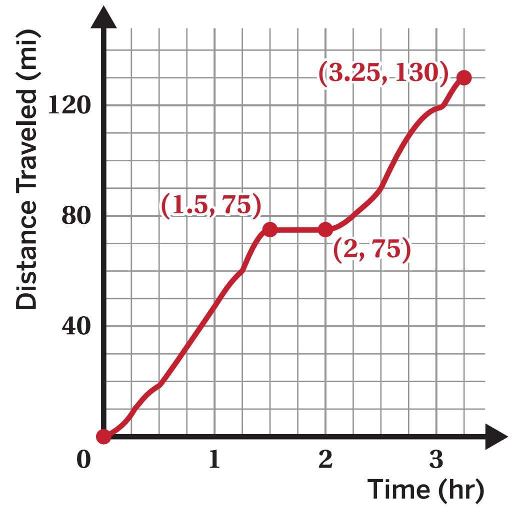
- What is the train's average speed?
- Who might benefit and who might be harmed?

***Responses vary.* See Teacher Edition for sample responses.**

13 Synthesis

How can you calculate the average rate of change for an interval of a function?

Use this graph if it helps to show your thinking.



***Responses vary.* I can calculate the average rate of change by calculating the slope between two endpoints.**

Things to Remember:

Name: ____________ Date: ____________ Period: ____________

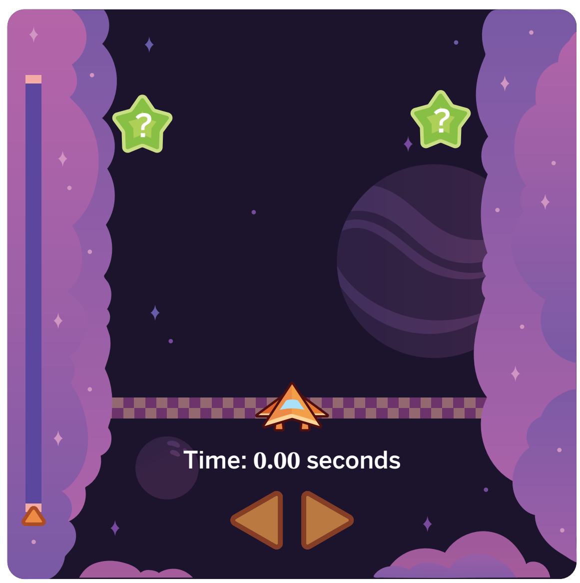
Space Race

Let's make connections between function notation and key features of graphs.

Warm-Up

1 It's time to play a game called Space Race!

Your mission: Get your spaceship to the finish line as quickly as possible.



a. Use the digital activity to play Round 1.

b. **Discuss:** How does this game work?

***Responses vary.* I noticed that different mystery stars cause different things to happen to the spaceships. Some stars speed up the spaceships, some cause them to stall out, and others make them change directions and go backward for a few seconds.**

2 Use the digital activity to race your spaceship several more times. Then compare your graphs with a partner's graphs.

Responses vary.

Activity 1

Name: .... Date: .... Period: ....

Comparing Graphs By Key Features

3 Nekeisha also played a round of Space Race.

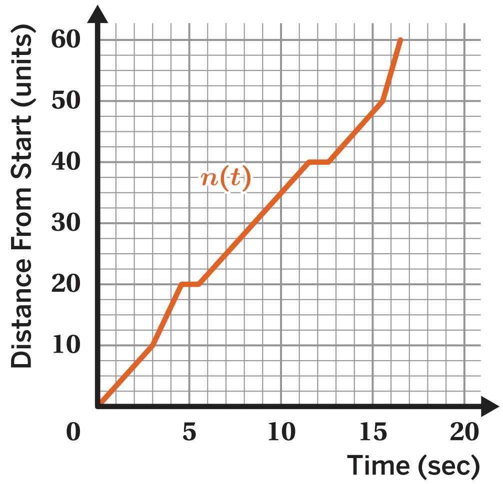
$n(t)$ represents the distance of Nekeisha's spaceship after t seconds.

What is a value of t for which $n(t) = 10$?

$t = 3$

What does that tell you about the situation?

Responses vary. **Nekeisha's spaceship went 10 units in 3 seconds.**



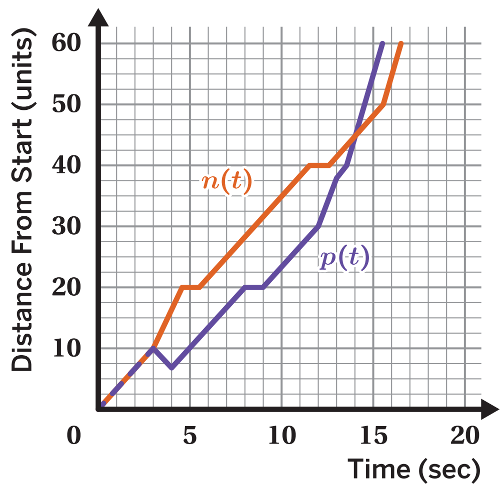
4 Let's watch Nekeisha and Polina race their spaceships.

Discuss:

- What were some interesting moments during this race?
- Where do you see those moments on the graph?

Responses vary.

- **There was a moment when Polina's triangle spun around and then went backwards. This is the part of the graph where Polina's function is decreasing.**
- **At the end of the race, Polina started zooming forward. This is the part of the graph where Polina's graph is very steep.**

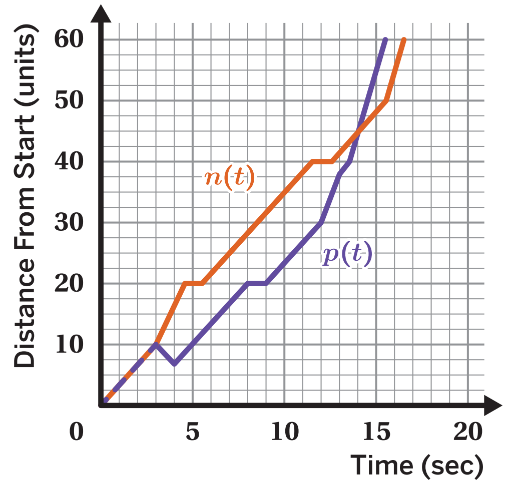


Activity 1

Name: ______ Date: ______ Period: ______

Comparing Graphs By Key Features (continued)

5 $n(t)$ and $p(t)$ represent the distances of Nekeisha's and Polina's spaceships after t seconds.



One student described part of the graph like this: *Polina's function was decreasing, and Nekeisha's function was increasing.*

Write a value of t that is in this interval of time.

***Responses vary.* t-values greater than 3 seconds and less than 4 seconds are considered correct.**

Describe what was happening in the race at that moment.

***Responses vary.* After 3 seconds of racing, Polina hit a star that made her spaceship go backward, while Nekeisha hit a star that zoomed her spaceship forward.**

6 Who had a greater average rate of change from 12 to 14 seconds?

A. Polina **B.** Nekeisha **C.** Their average rates of change were the same

Explain your thinking.

***Explanations vary.* Polina's graph was lower than Nekeisha's at $t = 12$ but at the same point as Nekeisha's at $t = 14$. This means Polina's spaceship needed a faster average rate of change to meet up with Nekeisha's ship at $t = 14$.**

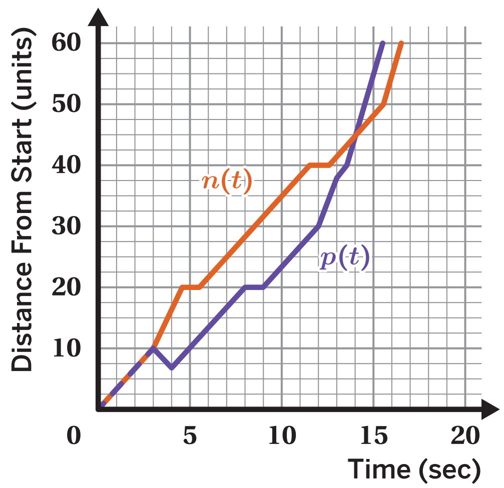
Activity 2

Name: ______ Date: ______ Period: ______

Comparing Using Function Notation

7 Nekeisha traveled farther than Polina after 11 seconds.

You can express that by writing $n(11) > p(11)$.



a **Discuss:** How can you use the graph to tell that $n(11) > p(11)$?

Responses vary.

- At 11 seconds, Nekeisha is around 38 units from the start, while Polina is only 27 units from the start.
- If you draw a vertical line at $t = 11$, the graph of $n(t)$ intersects the line at a higher y-value than the graph of $p(t)$.

b Which statement could you use to show who traveled farther after 15 seconds?

A. $n(15) > p(15)$

B. $n(15) = p(15)$

C. $p(15) \geq n(15)$

D. $p(15) > n(15)$

8 We know that $n(11) > p(11)$.

What is a different value of t for which $n(t) > p(t)$?

Responses vary. Any value of t greater than 3 seconds and less than 14 seconds is considered correct.

Activity 2

Name: ________ Date: ________ Period: ________

Comparing Using Function Notation (continued)

9 What is a value of t for which $n(t) = p(t)$?

***Responses* in the interval $0 \leq x \leq 3$ and 14 are considered correct.**

Describe what was happening in the race at that moment.

***Descriptions vary.* At that moment, Nekeisha and Polina were the same distance from the start.**

10 Valeria and Zion also raced their spaceships.

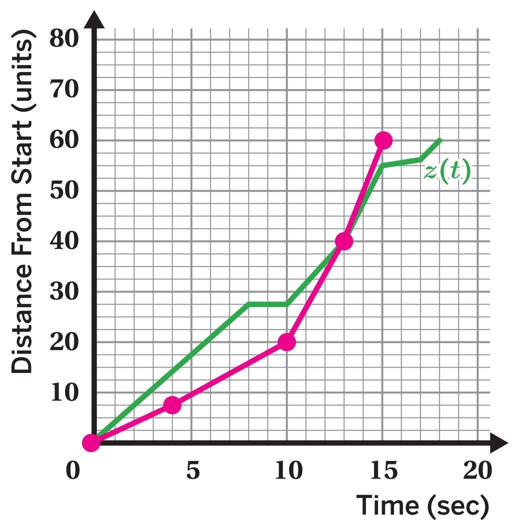
$v(t)$ and $z(t)$ represents the distances of Valeria's and Zion's spaceships after t seconds.

Here is some information about their race:

- $v(0) = z(0)$
- $v(4) < z(4)$
- $v(10) = 20$
- $v(13) = z(13)$
- $v(15) > z(15)$

Make a graph that could represent Valeria's distance traveled after t seconds.

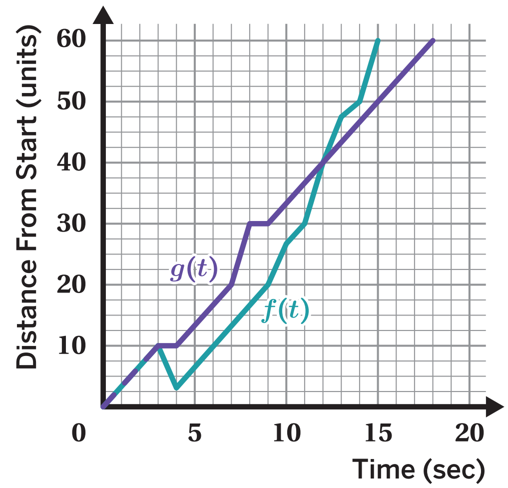
***Graphs vary.* Sample shown.**



8 Synthesis

How can statements in function notation and terms like *maximum*, *increasing*, and *average rate of change* help us compare graphs of functions?

Use the example if it helps to show your thinking.



Responses vary.

- **The terms help me know when one function is moving faster or slower, when it finishes ahead or behind, or when it's moving forward or backward in comparison to another function on the graph.**
- **Function notation helps me compare the functions at a specific moment. $f(t) = g(t)$ tells me that these objects were at the same distance at the same time.**

Things to Remember:

Name: ______ Date: ______ Period: ______

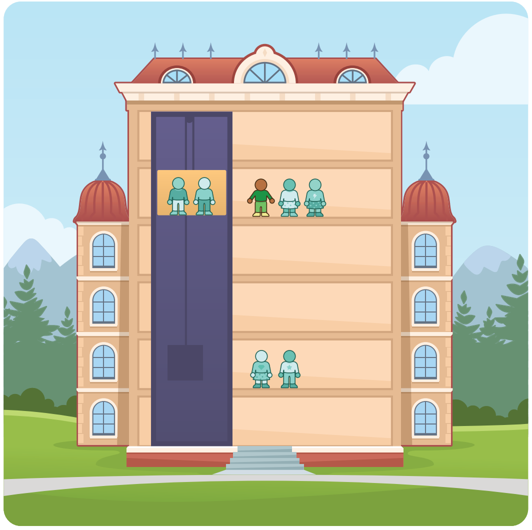
Elevator Stories

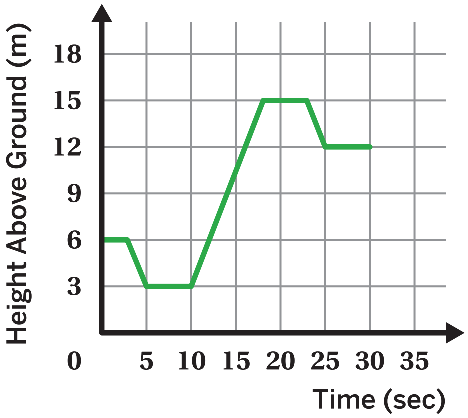
Let's use compound inequalities to describe the domain and range of functions from their graphs.

Warm-Up

1 Let's watch an animation of Amari's elevator ride.

Tell a story about what you see.





Responses vary. Amari is staying at the Four Quadrants Hotel in a room on the third floor. She wants to attend a party on the fifth floor with some friends who are staying on the sixth floor. She gets on the elevator with two people who press the button for Floor 2 while she selects Floor 6. The elevator first goes down to drop them off on the second floor, then travels to the sixth floor, where Amari's friends hop on. Then the elevator travels to the fifth floor where they all get off to go to the party!

Activity
1

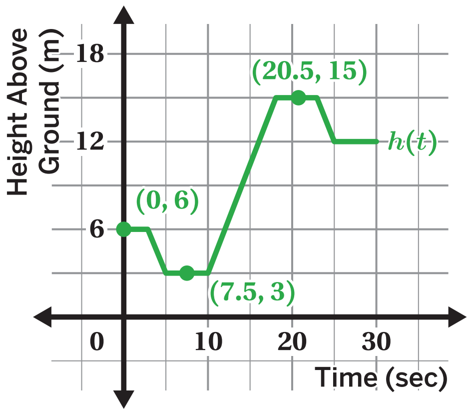
Name: ________ Date: ________ Period: ________

Describing the Domain

2 Here is a graph of $h(t)$, which represents the height of the elevator at a certain time, t, during Amari's ride.

Complete the input and output table for $h(t)$.

t	$h(t)$
5	**3**
20	**15**
30	**12**



3 **a** Select *all* the numbers that are in the domain of $h(t)$.

- ☐ A. -5
- ☑ B. $\frac{1}{2}$
- ☑ C. 2
- ☑ D. 12
- ☑ E. 18
- ☐ F. 23.5
- ☑ G. 32
- ☐ H. 60

b Describe the domain of $h(t)$.

Responses vary. **All the numbers from 0 to 30.**

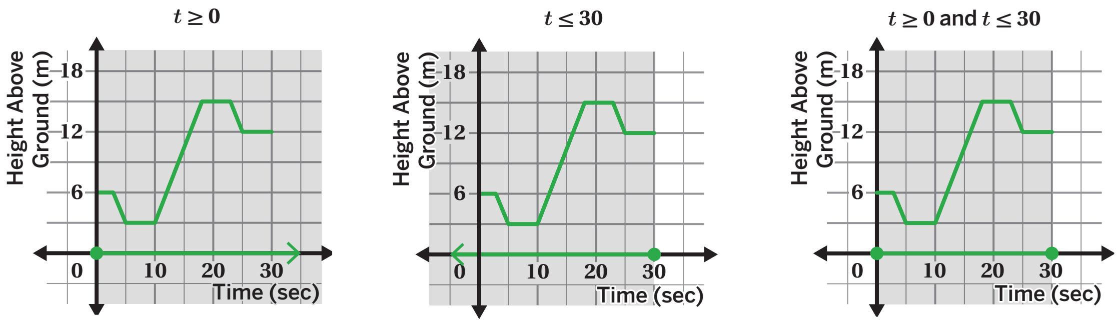
Activity 1

Name: Date: Period:

Describing the Domain (continued)

4 The domain of $h(t)$ is all the numbers from 0 to 30.

a Think about these inequalities and their relationship with the graph of $h(t)$.



b **Discuss:**

- Which inequality describes the domain of $h(t)$?
- Why don't the other inequalities describe the domain?

Responses vary.

- **$t \geq 0$ and $t \leq 30$ describe the domain of $h(t)$ because they represent all the numbers between 0 and 30.**
- **$t \geq 0$ doesn't describe the domain of $h(t)$ because it includes values that are larger than 30. $t \leq 30$ doesn't describe the domain of $h(t)$ because it includes negative values.**

5 Each of these **compound inequalities** accurately describes the domain of $h(t)$.

$t \geq 0$ and $t \leq 30$
$0 \leq t$ and $t \leq 30$
$0 \leq t \leq 30$

a **Discuss:** How are they alike? How are they different?

***Responses vary*. They are alike because each compound inequality has a minimum value that includes 0 and a maximum value that includes 30. They are different because the last compound inequality is written as one expression and the others are two expressions.**

b Explain how a compound inequality can help you describe the domain of $h(t)$.

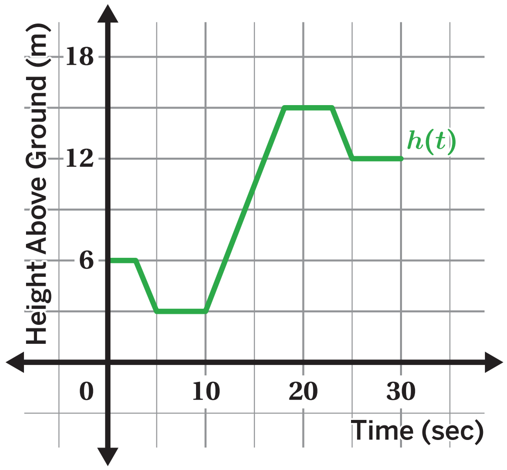
***Explanations vary*. Compound inequalities can help you describe an interval that has two conditions that need to both be true at the same time. In the domain of $h(t)$, we need x to be greater than or equal to 0 while also being less than or equal to 30.**

Activity 2

Name: ______ Date: ______ Period: ______

Distinguishing Domain and Range

6 **a** Write a compound inequality that describes the range of $h(t)$.

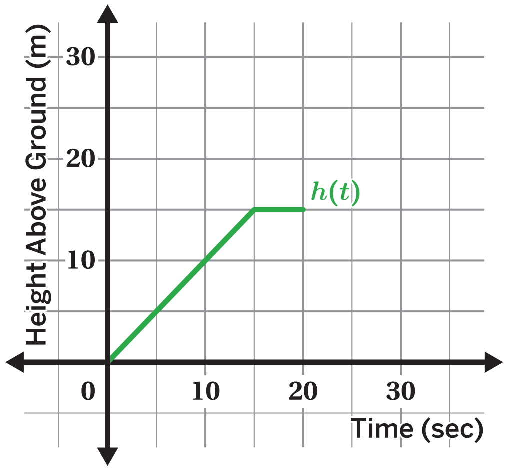


3 $\le h(t) \le$ **15**

b Describe in words what the compound inequality says about the range.

Responses vary. **The compound inequality says that the range of $h(t)$ is all the numbers between 3 and 15.**

7 Here is the graph of another elevator ride.



Two students described the range of this function.

- Ali says the range is $0 \le t \le 15$.
- Shanice says the range is $0 \le h(t) \le 15$.

Whose thinking is correct? Circle one.

Ali's Shanice's Both Neither

Explain your thinking.

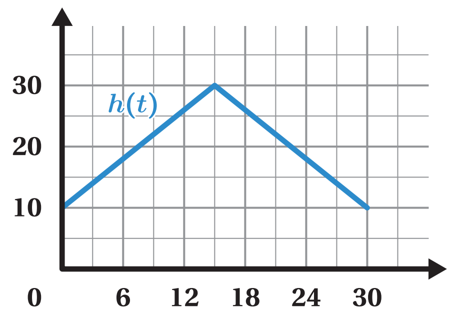
Explanations vary. **$h(t)$ represents the outputs, so Shanice's answer describes the range. Ali's answer uses t, which represents the inputs and describes the domain.**

Name: ______ Date: ______ Period: ______

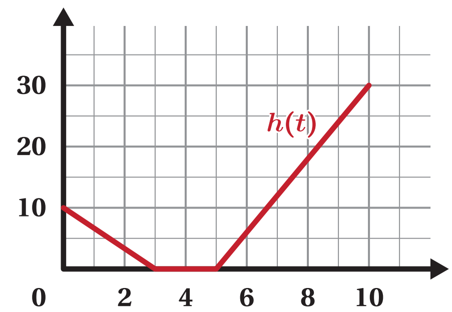
Distinguishing Domain and Range (continued)

8 Match each graph of a function with its domain and range. Two inequalities will have no match.

$0 \le t \le 10$	$0 \le h(t) \le 30$	$0 \le t \le 30$
$10 \le h(t) \le 30$	$0 \le h(t) \le 10$	$10 \le t \le 30$



$0 \le t \le 30$
$10 \le h(t) \le 30$



$0 \le t \le 10$
$0 \le h(t) \le 30$

9 The function $h(t)$ represents the height of the elevator at a certain time, t.

The range of $h(t)$ is $-30 \le h(t) \le 0$.

What could you say about this elevator ride?

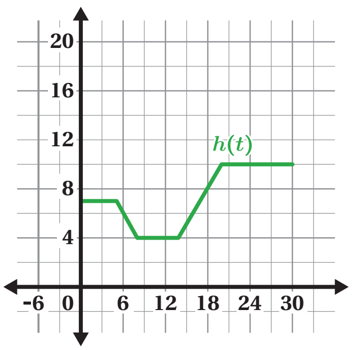
***Responses vary*. This elevator operates in a place that goes below ground, like a parking garage or a building with a basement.**

Name: .... Date: .... Period: ....

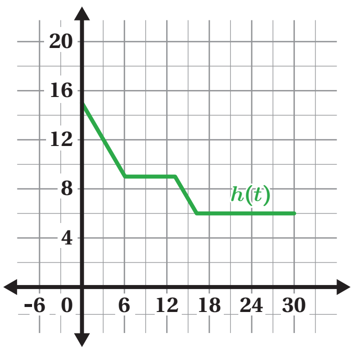
Writing Compound Inequalities

10 Complete the compound inequality to describe the range of $h(t)$.

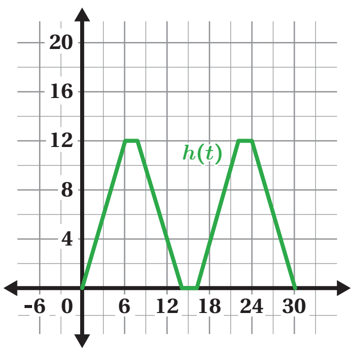
4 $\leq h(t) \leq$ **10**



6 $\leq h(t) \leq$ **15**



0 $\leq h(t) \leq$ **12**



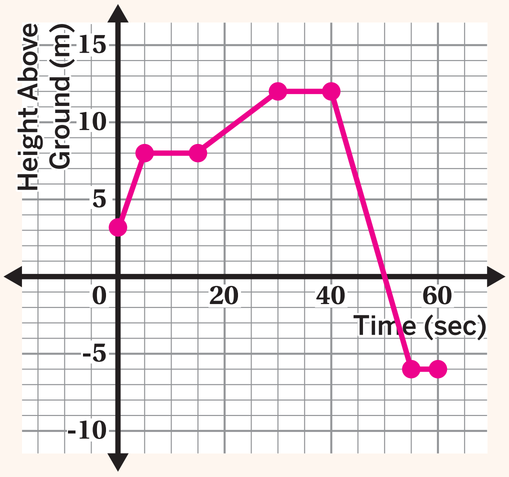
Explore More

11 $h(t)$ represents the height of an elevator at a certain time, t, during an elevator ride.

- The domain is $0 \leq t \leq 60$.
- The range is $-6 \leq h(t) \leq 12$.

a Create a graph that matches the domain and range.

***Graphs vary*. Sample shown.**



b Tell a story about your graph.

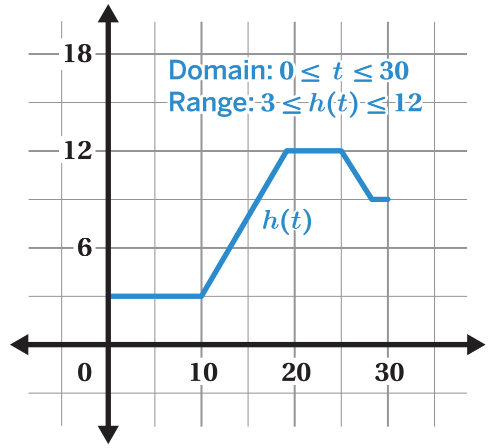
***Responses vary*. I got on the elevator on the 3rd floor of the building. I traveled up to the 8th floor, where several people got on. The elevator then slowed down as it went up to the 12th floor, where it stopped so people could get off. Then we plummeted rather quickly to the basement, which is 6 floors below ground.**

12 Synthesis

How can you determine the domain and range of a function from its graph? Draw on the graph if it helps with your thinking.

Explanations vary.

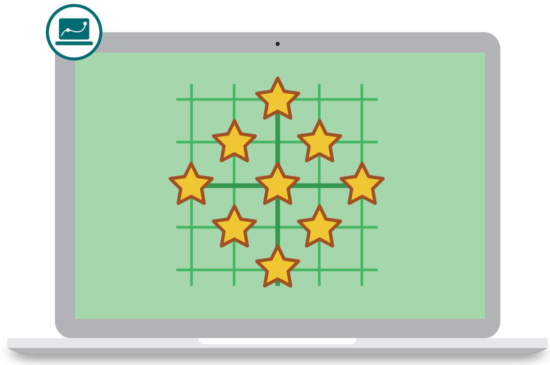
- **Domain: I look for the smallest input and the largest input on the graph to create a compound inequality.**
- **Range: I look for the minimum value and the maximum value of the function to create a compound inequality.**



Things to Remember:

Unit 4
Lesson
10

Name: .... Date: .... Period: ....



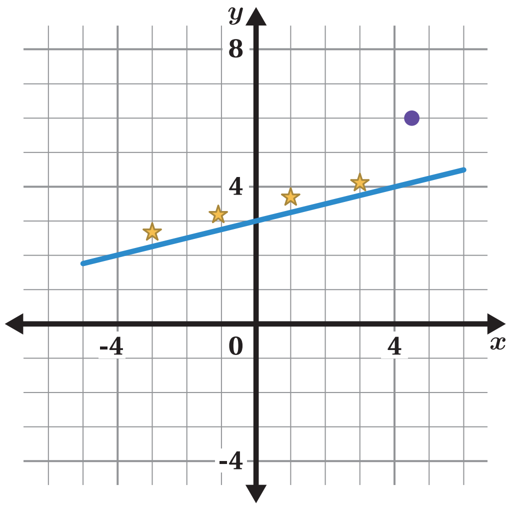
Marbleslides

Let's practice restricting the domain and range of a graph.

Warm-Up

1 Your goal is to capture all the stars.

Use the digital screen to see what happens when we press "Launch."



Activity 1

Name: ____________ Date: ________ Period: ________

Restrict the Domain and Range

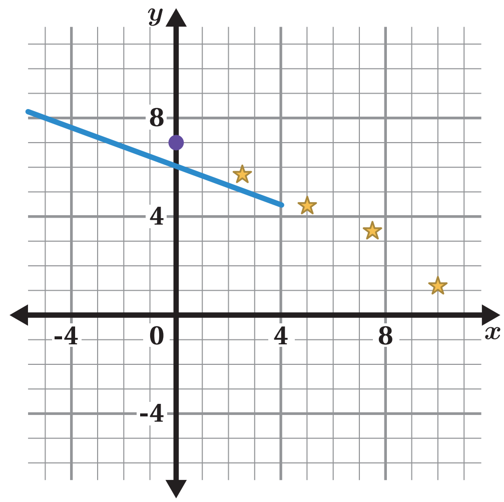
Use the digital screens for the next two problems.

2 Change one number to capture all the stars.

Original equation:
$y = -0.4x + 6 \; \{x < 4\}$

Your equation:

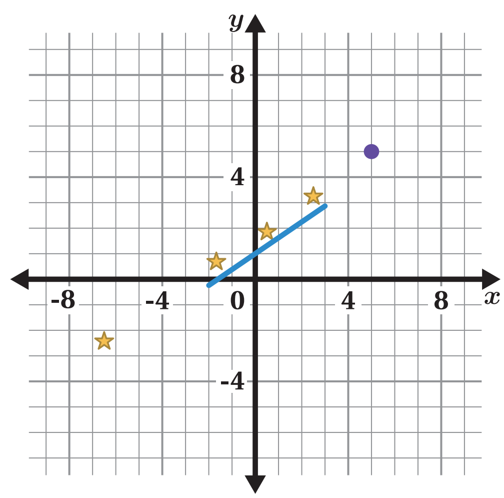
Responses vary.
$y = -0.4x + 6\{x < 7\}$



3 Change the domain to fix the Marbleslide.

Original equation:
$y = 0.6x + 1 \; \{-2 < x < 3\}$

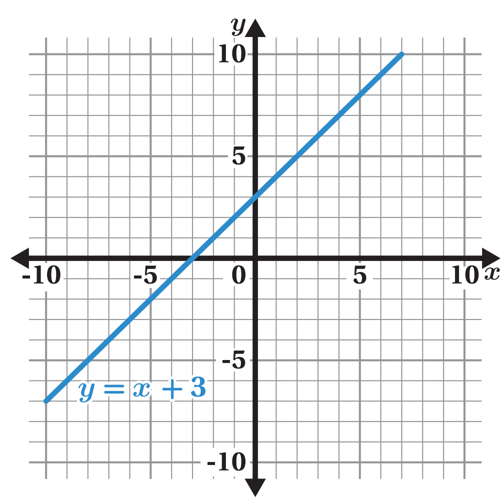
Your equation:
Responses vary.
$y = 0.6x + 1 \; \{ -5 < x < 6 \}$



4 If we included the range restriction $\{-2 < y < 4\}$, what would happen to the graph?

Draw on the graph if it helps with your thinking.

Responses vary. **We would only see the y-values that appear between -2 and 4 on the graph.**

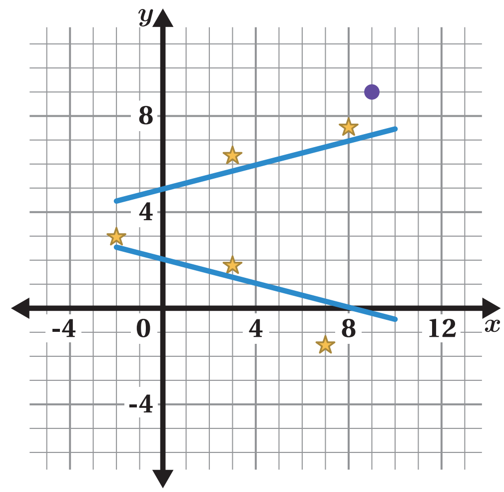


Name: Date: Period:

Restrict the Domain and Range (continued)

Use the digital screens for the next two problems.

5 Change the domain, range, or equations to collect all the stars.



Original equations:

$y = \frac{1}{4}x + 5$ {$-2 < x < 10$}

$y = -\frac{1}{4}x + 2$ {$-0.5 < y < 2.5$}

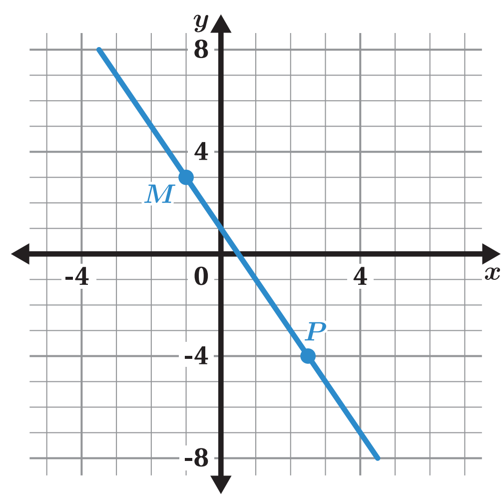
Your equations:

Responses vary.

$y = \frac{1}{4}x + 5$ {$2 < x < 10$}

$y = -\frac{1}{4}x + 2$ {$1 < y < 5$}

6 **a** Change the domain or range so that the line only appears from point M to point P.



Original equation:

$y = -2x + 1$

Your equation:

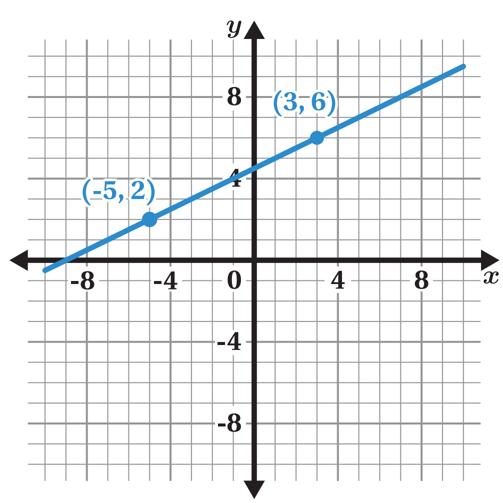
$y = 2x + 1$ {$-1 < x < 2.5$}

or $y = 2x + 1$ {$-4 < y < 3$}

b **Discuss:** What was your strategy?

Responses vary. **I saw that the x-values for M and P went from -1 to 2.5, so I added the domain restriction {$-1 < x < 2.5$}.**

7 Two students are discussing how to restrict the graph of $y = \frac{1}{2}x + 4.5$ between (-5, 2) and (3, 6).



Hailey says to include {$-5 < x < 3$}.
Ricardo says to include {$2 < y < 6$}.

Whose thinking is correct? Circle one.

Hailey's Ricardo's Both Neither

Explain your thinking. ***Explanations vary.***

Hailey and Ricardo are both correct. Hailey created a restriction for the domain, and Ricardo gave a restriction for the range.

Note: Students who select Hailey, Ricardo, or Both will be marked correct.

Activity 2

Name: ______ Date: ______ Period: ______

More Challenges!

Use the digital screens for this activity.

8 Create as many equations as you need to collect all the stars.

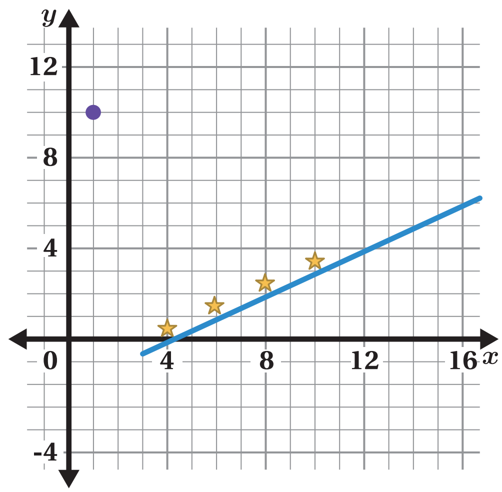
We have included the equation of a line that might help you start.

$$y = \frac{1}{2}x - 2.1\{x > 3\}$$

Responses vary.

$y = \frac{1}{2}x - 2.1\{x > 3\}$

$y = -\frac{1}{2}x + 10\{x < 10\}$

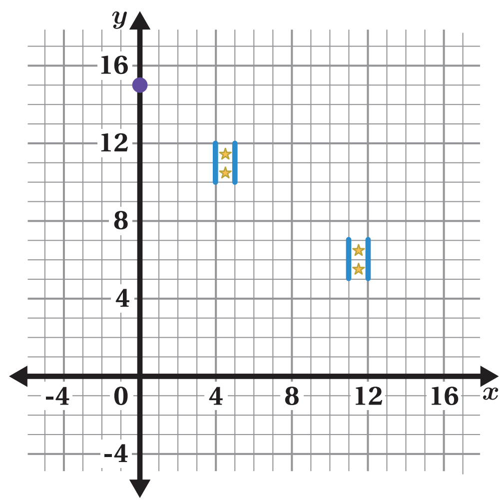


9 Create as many equations as you need to collect all the stars.

Responses vary.

$y = -\frac{1}{4}x + 13\{x < 4\}$

$y = -\frac{1}{4}x + 10\{x < 10\}$

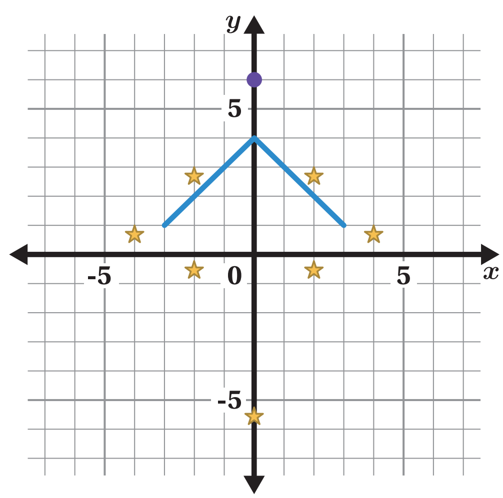


10 Create as many equations as you need to collect all the stars.

Responses vary.

$y = \frac{2}{3}x - 2\{y > -1.8\}$

$y = -\frac{2}{3}x - 2\{y > -1.8\}$



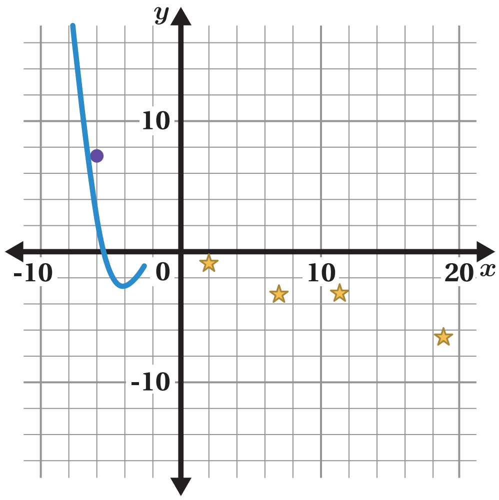
Name: .... Date: .... Period: ....

More Challenges! (continued)

11 Create as many equations as you need to collect all the stars.

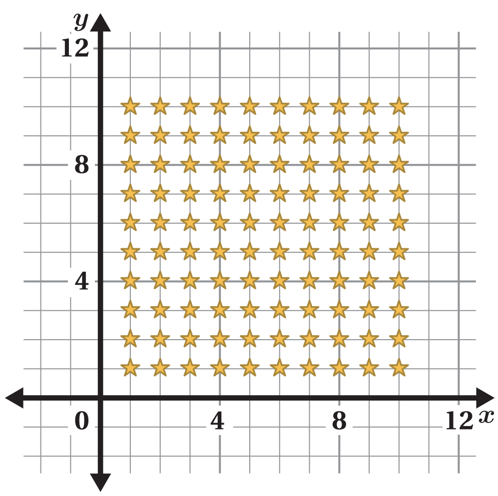
Responses vary.

$y = -1 \{-2 < x < 3\}$
$y = -3.8 \{x < 17\}$



12 Challenge yourself to collect as many stars as you can!

Responses vary.



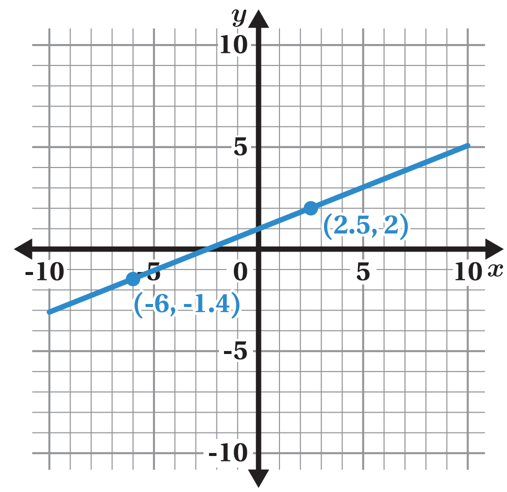
13 Synthesis

a How can you use the domain to restrict the graph from (-6, -1.4) to (2.5, 2)?

Responses vary. **To restrict the domain, I can look at the x-values in the ordered pairs to graph all of the x-values between -6 and 2.5.**

b How can you use the range?

Responses vary. **I can look at the y-values in the ordered pairs to graph all of the y-values between -1.4 and 2.**



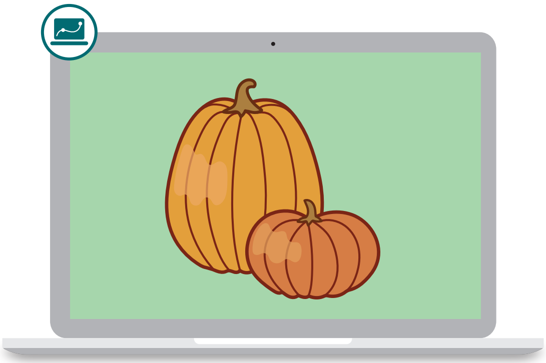
Things to Remember:

Unit 4

Lesson

11

Name: .... Date: .... Period: ....



Pumpkin Prices

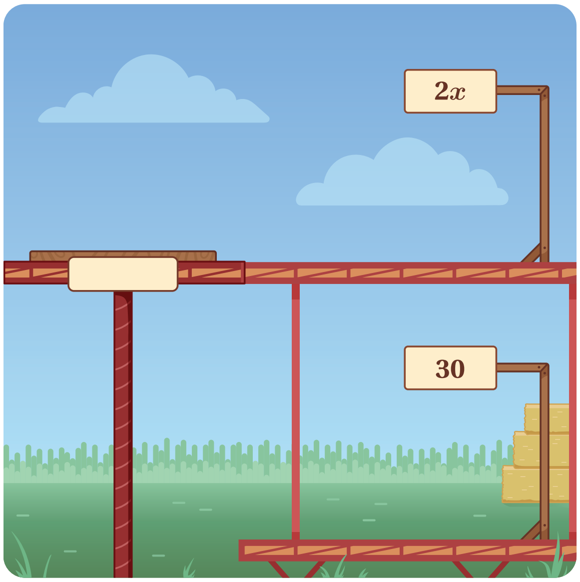
Let's make sense of piecewise-defined functions.

Warm-Up

1 Fran's Farm sells different sizes of pumpkins.

a Let's look at the prices of different pumpkins. Record the pumpkin weight and prices. ***Tables vary.***

Pumpkin Weight (lb)	Price ($)



b **Discuss:** What do you notice and wonder?
Responses vary.
- **The maximum price is $30.**
- **Lighter pumpkins cost double their weight.**
- **A 2,000-pound pumpkin costs the same as a 15-pound pumpkin.**
- **I wonder what the weight of the biggest pumpkin ever was.**

Activity 1

Name: .... Date: .... Period: ....

Piecewise-Defined Functions

2 This table shows the prices of different pumpkins.

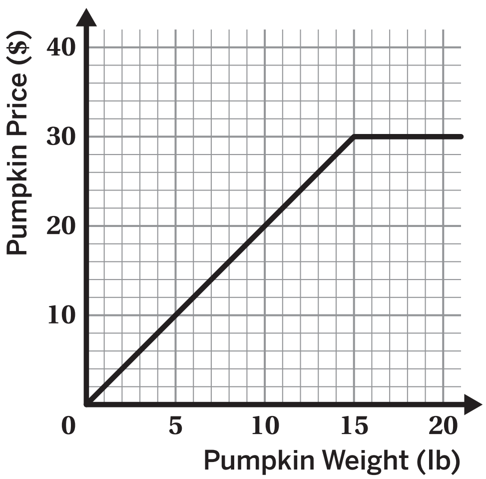
Pumpkin Weight (lb)	3	7	10	14	15	18	20	97
Price ($)	6	14	20	28	30	30	30	30

How can you determine the price of *any* size pumpkin?

Responses vary. **Pumpkins greater than 15 pounds cost $30, and pumpkins less than or equal to 15 pounds cost double their weight.**

3 The sign, the **piecewise-defined function** equation, and the graph each show pumpkin prices at Fran's Farm.

$$f(x) = \begin{cases} 2x & 0 \le x \le 15 \\ 30 & x > 15 \end{cases}$$



Discuss: Where do you see the same information in each representation?

Responses vary.

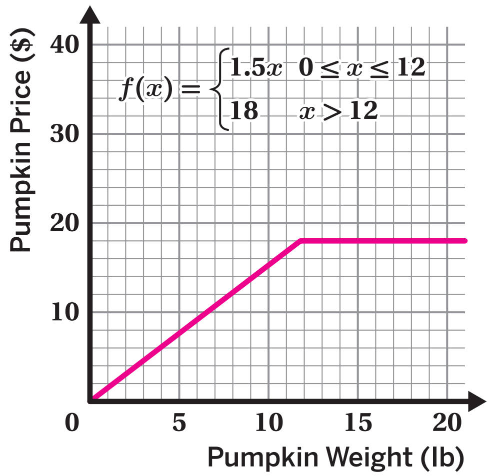
- **I see two different parts in each representation.**
- **I see the greater-than symbol in the second part of the function equation and in the second part of the sign.**
- **I see a 2 as the slope in the first part of the equation and that matches the $2 per pound in the sign.**
- **The first part of the graph is linear and the second part is constant, which matches the two parts of the equations.**

Activity 2

Name: .... Date: .... Period: ....

Pricing Plans

4 Fran's Farm is having a sale on pumpkins. Draw a graph of the sale prices.



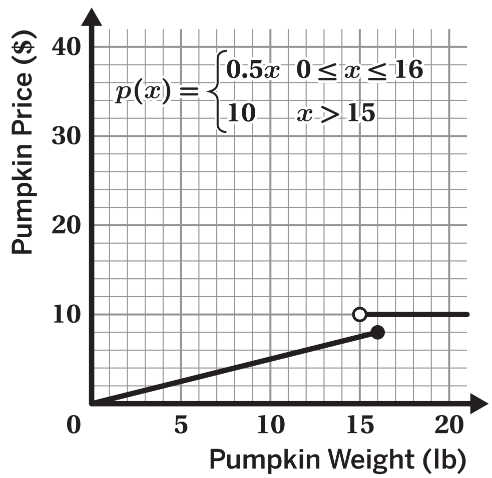
5 Pumpkins-a-Plenty also sells pumpkins.

The piecewise-defined function $p(x)$ represents the price of a pumpkin that weighs x pounds.

Customers are complaining that these prices are confusing.

Discuss: Why might someone think these prices are confusing?

***Responses vary.* These prices are confusing because pumpkins that are over 15 pounds and up to 16 pounds could be two different prices. How do customers and employees know which price to use?**

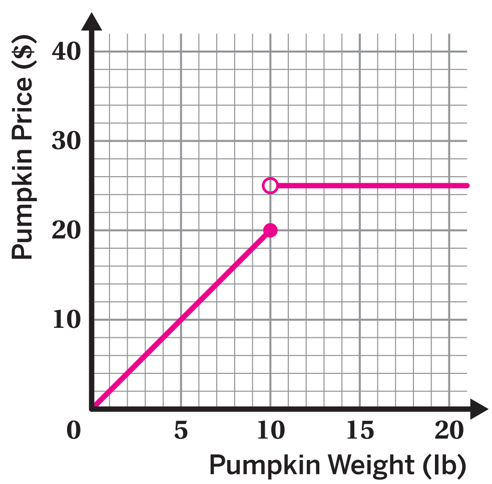


6 Pumpkins-a-Plenty wants you to fix the pumpkin prices.

a Draw a new graph so that each pumpkin only has one price.
Graphs vary.

b Explain what your graph says about the price of pumpkins.

***Responses vary.* Pumpkins up to 10 pounds cost $2 per pound. Pumpkins over 10 pounds cost $25.**



Activity 3

Name: ____________ Date: ________ Period: ________

Evaluating Piecewise-Defined Functions

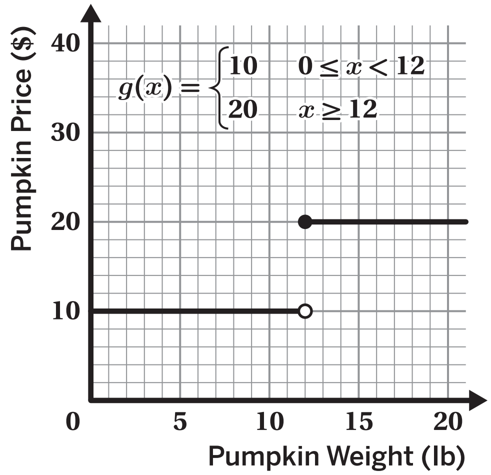
7 $g(x)$ represents the price of a pumpkin that weighs x pounds at Gourds, Gourds, Gourds.

Discuss:

- Where do you see $g(12)$ in the graph?
- Where do you see $g(12)$ in the equation?

Responses vary.

- **I see $g(12)$ on the graph at the closed circle at $(12, 20)$. The open circle at $(12, 10)$ means that point is not included.**
- **I see $g(12)$ in the equation in the second row because the second row includes 12 in the domain. The first row doesn't include 12.**



8 $g(x)$ represents the price of a pumpkin that weighs x pounds at Gourds, Gourds, Gourds.

$$g(x) = \begin{cases} 10 & 0 \le x < 12 \\ 20 & x \ge 12 \end{cases}$$

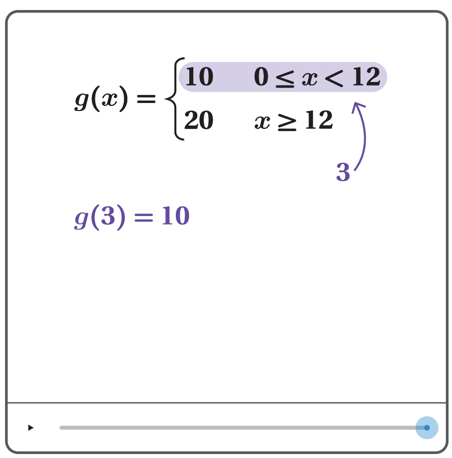
What is $g(3)$?
$10

9 Let's watch what Abdel did to determine $g(3)$.

a **Discuss:** How would you describe Abdel's strategy?
***Responses vary.* Abdel saw that 3 was included in the domain of the first piece of the equation, so he used the first row to determine that the price would be $10.**

b Describe how to use Abdel's strategy to determine $g(35)$.
***Responses vary.* Since 35 is in the domain of the second piece of the equation, $g(35)$ would be $20.**

Abdel



Activity 3

Name: .... Date: .... Period: ....

Evaluating Piecewise-Defined Functions (continued)

10 Here are some piecewise-defined functions.

$f(x) = \begin{cases} 3 & 0 < x \leq 5 \\ 8 & x > 5 \end{cases}$ What is $f(2)$? 3

$g(x) = \begin{cases} 4 & 0 \leq x \leq 8 \\ 6 & 8 < x < 16 \end{cases}$ What is $g(8)$? 4

$h(x) = \begin{cases} 5 & 0 < x < 9 \\ 12 & x \geq 9 \end{cases}$ What is $h(4)$? 5

$a(x) = \begin{cases} 1.5x & 0 < x < 9 \\ 12 & x \geq 9 \end{cases}$ What is $a(9)$? 12

$b(x) = \begin{cases} x & 0 \leq x < 3 \\ x + 2 & x \geq 4 \end{cases}$ What is $b(5)$? 7

$c(x) = \begin{cases} x + 3 & 0 \leq x < 4 \\ 3x & 4 \leq x < 7 \end{cases}$ What is $c(4)$? 12

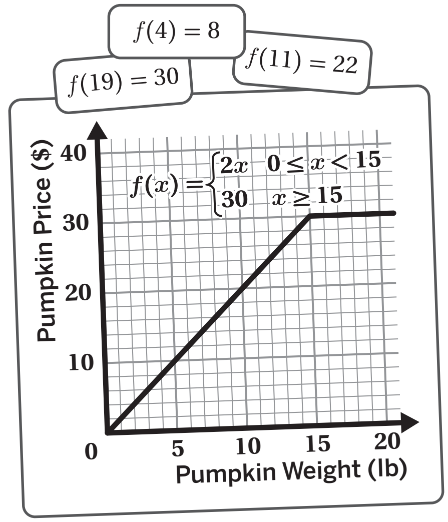
11 Synthesis

a How can you use a graph to evaluate a piecewise-defined function?

***Responses vary.* You can use a graph to find the input you're looking for on the x-axis and then trace up until you hit the function. If there is a closed and open point, use the closed point and determine the y-value of that point.**

b How can you use an equation to evaluate a piecewise-defined function?

***Responses vary.* First, determine which part of the equation to use by seeing which part of the domain the x-value you want is in. Then, plug that x-value into that part of the equation.**



Things to Remember:

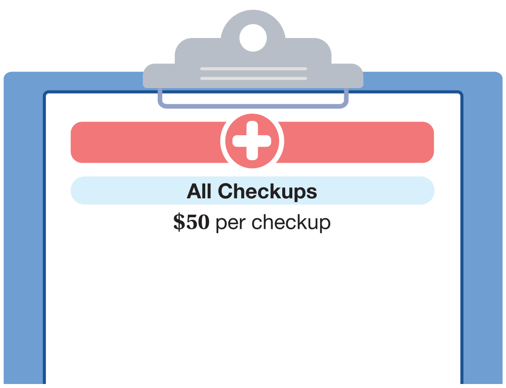
Name: ______________ Date: ________ Period: ________

Doctor Doctor

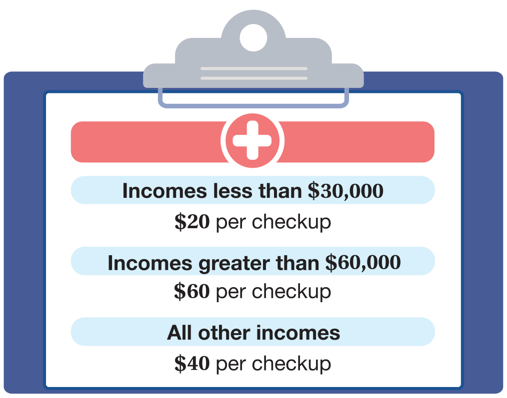
Let's explore ways to represent pricing plans.

Warm-Up

Doctor Remy's Office



Doctor Dyani's Office



1. What do you notice? What do you wonder?

I notice:

Responses vary.

- **I notice both doctors do checkups, but have different pricing plans.**
- **I notice that at Dr. Dyani's office, the price of a checkup is different for people with different incomes.**

I wonder:

Responses vary.

- **I wonder why they have different pricing plans.**
- **I wonder how they came up with their pricing plans.**
- **I wonder how patients feel about each pricing plan.**

Activity 1

Name: ______ Date: ______ Period: ______

Dr. Dyani

Dr. Remy is interested in learning more about how Dr. Dyani's office charges for checkups.

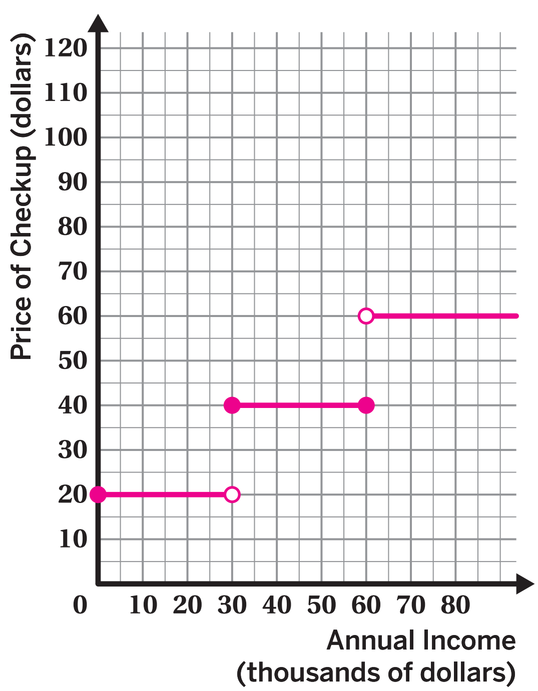
This is a summary of the checkup prices at Dr. Dyani's office.

Annual Incomes Less Than $30,000	Annual Incomes Greater Than $60,000	All Other Income
$20 per checkup	$60 per checkup	$40 per checkup

2. Complete the table.

Annual Income ($)	Price of Checkup ($)
20,000	**20**
40,000	**40**
60,000	**40**
75,000	**60**

3. Create a graph that shows the price of a checkup for any annual income.



4. The graph that represents the pricing plan for Dr. Dyani's office is an example of a special type of piecewise-defined function, called a **step function**. Why do you think it is called a step function?

***Responses vary.* It might be called a step function because the graph looks like steps on a staircase.**

Activity 1

Name: ________ Date: ________ Period: ________

Dr. Dyani (continued)

Hoang wrote a function to describe the price of a checkup at Dr. Dyani's office, $f(x)$, for an annual income of x dollars.

$$f(x) = \begin{cases} 30000 & 0 \le x \le 20 \\ 40 & 30000 < x < 60000 \\ 60 & x > 60000 \end{cases}$$

5. Some of Hoang's work is correct and some of Hoang's work is incorrect.

Discuss: What might be the mistake in Hoang's work?

Responses vary.

- **Hoang's first line says the price of a visit is \$30,000.**
- **Hoang's function doesn't include the price of a visit for people who have an annual income of exactly \$30,000 or \$60,000.**

6. Write a function that correctly describes the price of a checkup at Dr. Dyani's office.

$$f(x) = \begin{cases} 20 & 0 \le x < 30000 \\ 40 & 30000 \le x \le 60000 \\ 60 & x > 60000 \end{cases}$$

7. Neena says that the pricing plan at Dr. Remy's office is more fair than the plan at Dr. Dyani's office because everyone pays the same amount for a checkup.

Ichiro disagrees.

Discuss: Who do you agree with? Why?

Responses and explanations vary.

- **I agree with Neena because one way to treat everyone fairly is by charging the same amount of money no matter their situation.**
- **I agree with Ichiro because not everyone's situation is the same. \$50 might be a lot for someone and a little for someone else, and their financial situation should be considered.**
- **I don't agree with either of them. You shouldn't have to tell people your income to go to the doctor, so I think the price shouldn't be based on income.**

Activity 2

Name: ________ Date: ________ Period: ________

Dr. Remy

Dr. Remy's office currently charges a flat fee of \$50 per checkup. They are considering changing to one of these three pricing plans for checkups.

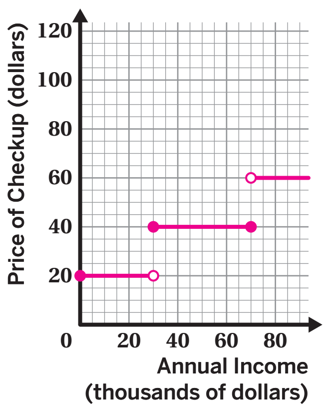
8. Fill in the missing graphs and piecewise-defined functions.

Plan A

Incomes less than \$30,000
\$20 per checkup

Incomes greater than \$70,000
\$60 per checkup

All other incomes
\$40 per checkup



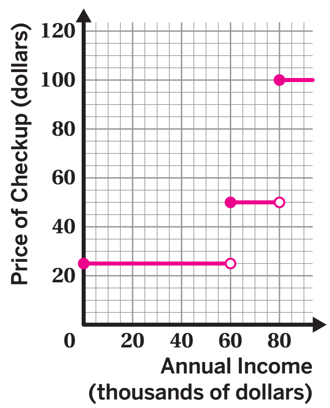
$$a(x) = \begin{cases} 20 & 0 \le x < 30000 \\ 40 & 30000 \le x \le 70000 \\ 60 & x > 70000 \end{cases}$$

Plan B

Incomes less than \$60,000
\$25 per checkup

Incomes greater than or equal to \$80,000
\$100 per checkup

All other incomes
\$50 per checkup



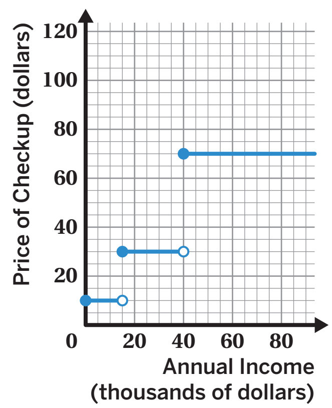
$$b(x) = \begin{cases} 25 & 0 \le x < 60000 \\ 50 & 60000 \le x < 80000 \\ 100 & x \ge 80000 \end{cases}$$

Plan C

Incomes less than \$15,000
\$10 per checkup

Incomes greater than or equal to \$40,000
\$70 per checkup

All other incomes
\$30 per checkup



$$c(x) = \begin{cases} 10 & 0 \le x < 15000 \\ 30 & 15000 \le x < 40000 \\ 70 & x \ge 40000 \end{cases}$$

Name: .... Date: .... Period: ....

Dr. Remy (continued)

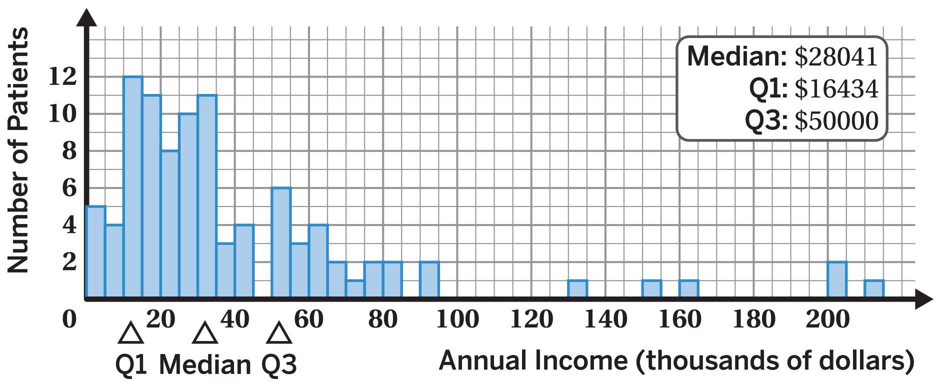
9. Choose one plan to analyze further. Circle one. ***Choices vary.***

Plan A Plan B Plan C

According to the plan you selected, what is the price of a checkup for someone who has a median annual income of $28,041?

Plan A: $20, Plan B: $25, Plan C: $30

10. This histogram shows the annual incomes of Dr. Remy's patients.



Use the histogram to help you describe how switching from a $50 flat fee to the plan you chose could impact patients of different annual incomes and Dr. Remy's office.

Responses vary.

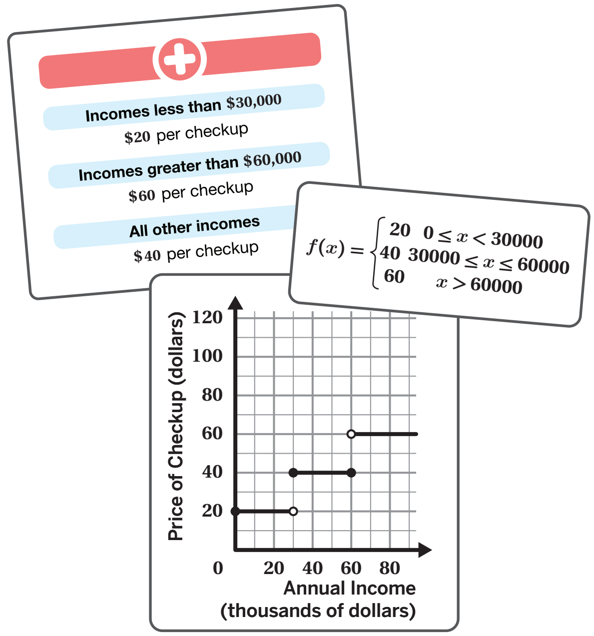
- **Plan A: This plan offers checkups cheaper than $50 for patients who make less than or equal to $70,000 per year. Patients who make over $70,000 have to pay $10 more than they did with the original plan. Dr. Remy's office will need to make sure it can still operate while possibly making less money with this plan since over 75% of patients will have cheaper checkups and only some will have a small increase.**
- **Plan B: The majority of Dr. Remy's patients will be charged the same price as before or a cheaper price. About 10% of Dr. Remy's patients will be charged double what they paid for in the original plan. I wonder how this price increase will be explained to this group of patients. This increase could help the office stay open, but might also cause some patients with higher incomes to leave.**
- **Plan C: This plan offers the lowest minimum price for a checkup of any of the plans. The majority of Dr. Remy's patients will pay less than what they paid before with the original plan. People who earn $50,000 or more will pay $20 more for a checkup, but this is only about 25% of Dr. Remy's patients. Will Dr. Remy's office make enough money to stay open if 75% of their patients are paying less than they did with the original plan?**

Synthesis

11. What is important to remember when graphing and writing equations of piecewise-defined functions?

Use the example if it helps with your thinking.

Responses vary. **When graphing piecewise-defined functions, it's important to look at the endpoints of each domain. That will help you know where each piece of the graph stops and starts, and whether the endpoint is included or not. When writing piecewise-defined function equations, it's important to use the correct inequality symbol and endpoints to define the domain of each part of the function.**



Things to Remember:

Unit 4

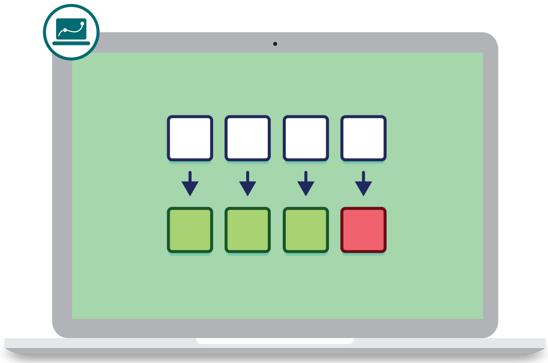
Lesson

13

Name: ______ Date: ______ Period: ______

Recursion Excursion

Let's write a recursive definition for a sequence using function notation.



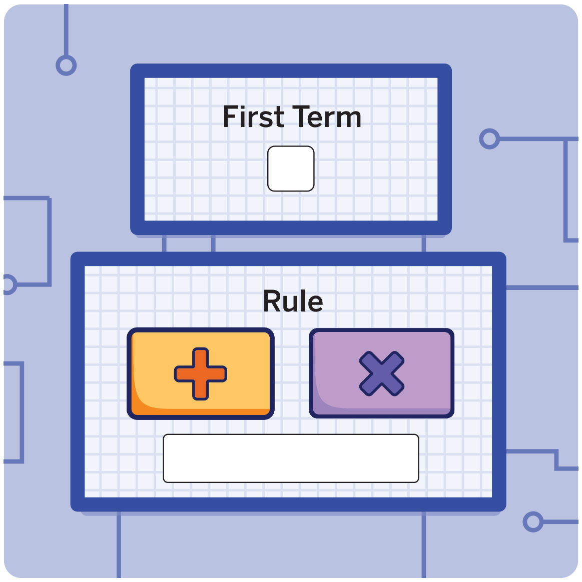
Warm-Up

1 Let's play with the first term and rule, and notice what happens to the *sequence*, graph, and table.

Discuss: What do you remember about sequences?

Responses vary.

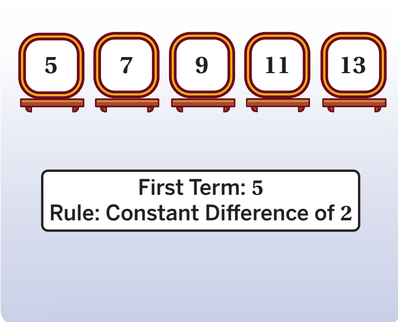
- **Sequences can be arithmetic and have a constant difference or they can be geometric and have a constant ratio.**
- **You can use sequences to model situations, like the number of trees in a city.**

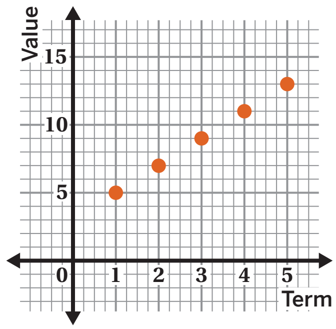


Name: ________ Date: ________ Period: ________

Sequences as Functions

2 Ariel made this sequence during the Warm-Up.





Term	Value
1	5
2	7
3	9
4	11
5	13

Ariel said: *If the domain is the term number, then all sequences are functions.*

Explain what Ariel might have been thinking.

Responses vary. **If you think of the domain as the term numbers, then each input will only have one possible output. For example, the second term in Ariel's sequence will always be 7. Since the domain of every sequence is the term number, then all sequences are functions with the same domain.**

3 Let $f(n)$ represent the value of term n in this sequence.

a **Discuss:** What does $f(4) = 11$ mean?

Responses vary. $f(4) = 11$ **means that the fourth term in the sequence is 11.**

b What is the value of $f(7-1)$?

15

4 A sequence's *recursive definition* is made up of its first term and rule.

a Here are some recursive definitions.

5, 7, 9, 11, 13	10, 6, 2, -2, -6	80, 40, 20, 10, 5
First Term: 5 Constant Difference: 2	First Term: 10 Constant Difference: -4	First Term: 80 Constant Ratio: 0.5
$f(n) = \begin{cases} 5 & n = 1 \\ f(n-1) + 2 & n \geq 2 \end{cases}$	$f(n) = \begin{cases} 10 & n = 1 \\ f(n-1) + (\text{-}4) & n \geq 2 \end{cases}$	$f(n) = \begin{cases} 80 & n = 1 \\ f(n-1) \cdot 0.5 & n \geq 2 \end{cases}$

b **Discuss:** What does $f(n-1)$ mean?

Responses vary. $f(n-1)$ **means the value of the previous term. So** $f(n-1)+2$ **means you take the value of the term before it and add 2.**

Activity 1

Name: ______ Date: ______ Period: ______

Sequences as Functions (continued)

5 These two recursive definitions will make the same sequence.

$$f(n) = \begin{cases} 7 & n = 1 \\ f(n-1) + 2 & n \geq 2 \end{cases}$$

$$f(1) = 7$$
$$f(n) = f(n-1) + 2$$

How are the definitions alike? How are they different? ***Responses vary.***

Alike:

Both definitions have a 7 and $f(n-1) + 2$.

Different:

The first definition is a piecewise-defined function. The second definition is two equations.

6 Match each recursive definition to the sequence that it makes. One sequence will have two matches.

Recursive Definition

a. $d(n) = \begin{cases} 3 & n = 1 \\ d(n-1) + n & n \geq 2 \end{cases}$

b. $h(1) = 5$
$h(n) = h(n-1) \cdot 3$

c. $k(n) = \begin{cases} 3 & n = 1 \\ k(n-1) + 5 & n \geq 2 \end{cases}$

d. $m(1) = 5$
$m(n) = m(n-1) + 3$

e. $v(1) = 3$
$v(n) = v(n-1) + 5$

Sequence

a $3, 5, 8, 12, \ldots$

e, c $3, 8, 13, 18, \ldots$

d $5, 8, 11, 14, \ldots$

b $5, 15, 45, 135, \ldots$

Activity 2

Name: ______ Date: ______ Period: ______

Sequence Challenges

7 Here is the recursive definition for a sequence:

$h(1) = 5$

$h(n) = h(n - 1) \cdot 2$

Write the first five terms of the sequence.

Term, n	Value
1	5
2	10
3	20
4	40
5	80

8 Complete the recursive definition so that it makes the sequence 7, 12, 17, 22, 27.

$g(1) =$ 7

$g(n) = g(n - 1)$ $+ 5$

Term, n	Value
1	7
2	12
3	17
4	22
5	27

Name: ______ Date: ______ Period: ______

Sequence Challenges (continued)

9 Here's a recursive definition Nyanna wrote for the sequence $7, 12, 17, 22, 27$.

Nyanna's Recursive Definition

$g(1) = \boxed{7}$

$g(n) = \boxed{5(n-1)}$

a What did Nyanna do well?

Responses vary. **Nyanna correctly wrote that $g(1) = 7$ because 7 is the first term in the sequence. Also, it looks like Nyanna noticed the sequence increases by 5.**

b What would you recommend Nyanna change to get a recursive definition that creates the sequence?

Responses vary. **Nyanna's sequence is using the previous term number, not the term value. I would recommend Nyanna change $g(n) = 5(n-1)$ to $g(n) = g(n-1) + 5$.**

10 Write a recursive definition that will make the sequence $448, 224, 112, 56, 28$.

$f(1) =$ **448**

$f(n) =$ **$f(n-1) \cdot 0.5$**

Term, n	Value
1	448
2	224
3	112
4	56
5	28

Activity 3

Name: ______ Date: ______ Period: ______

Choose Your Own Sequence

11 The Fibonacci sequence is a sequence in which each term is the sum of the two terms that come before it.

A common Fibonacci sequence is $1, 1, 2, 3, 5, 8, 13$.

Write a recursive definition for it.

$f(1) =$ **1**

$f(2) =$ **1**

$f(n) =$ **$f(n-1) + f(n-2)$**

12 You will be designing a challenge for your classmates to solve.

a **Make It! Create a sequence.** ***Sequences vary.***

- Write the first five terms of your sequence.
- Write a recursive definition that will make your sequence. Your definition can include $f(n-1)$, $f(1)$, and/or n.

My Challenge	Recursive Definition
___, ___, ___, ___, ___	$f(1) =$ $f(n) =$

b **Solve It!**

- Share your sequence with a classmate. Keep your recursive definition a secret!
- Write a recursive definition that will make their sequence.

Challenges	Recursive Definition
___'s Sequence ___, ___, ___, ___, ___	$f(1) =$ $f(n) =$
___'s Sequence ___, ___, ___, ___, ___	$f(1) =$ $f(n) =$
___'s Sequence ___, ___, ___, ___, ___	$f(1) =$ $f(n) =$

13 Synthesis

Explain the different parts of a recursive definition and what they tell you about a function.

Use one or both examples if they help with your thinking.

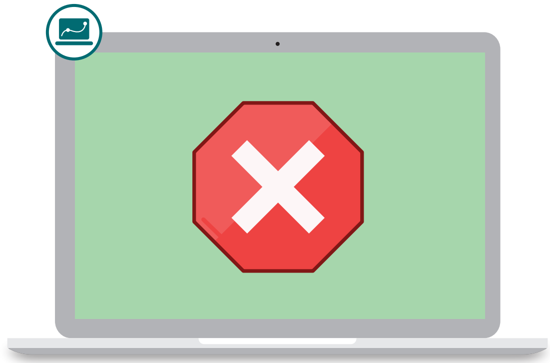
$$f(n) = \begin{cases} 25 & n = 1 \\ f(n-1) + 5 & n \geq 2 \end{cases}$$

$$f(1) = 25$$
$$f(n) = f(n-1) + 5$$

Responses vary. **A recursive definition has (at least) two parts: a first term and a rule for generating all other terms. In both of the recursive definitions here, the top line says that the first term is 25. The next line says the rule, which is "take the previous term and add 5." That means that this function is an arithmetic sequence and its graph will be linear.**

Things to Remember:

Name: ____________ Date: ____________ Period: ____________



What's Your Score?

Let's make sense of absolute value functions.

Warm-Up

1 Which one doesn't belong?

A. $x = |\text{-}3|$

B. $|x| = 3$

C. $x = |9| - |12|$

D. $|9 - 12| = x$

Explain your thinking.

Responses and explanations vary.

- **A: It's the only one with a negative number inside the absolute value signs.**
- **B: It's the only one where x can equal two different values.**
- **C: It's the only one where x must be -3.**
- **D: It's the only one that has subtraction inside the absolute value signs.**

Name: ______________________ Date: __________ Period: __________

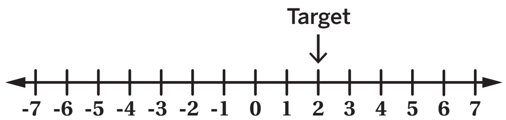
Target Numbers

2 Let's play a game. On the digital screen, press "Stop" to stop the arrow and get a score.

a Play up to five times.

b **Discuss:** How are the scores determined?

Numbers and scores vary. **I noticed that the score is how far away from the target number I was when I pressed stop.**



3 Here are Adriana's scores.

Number	Score
5	3
1	1
2	0
-4	6

Adriana got a score of 4 on her next try.

What number do you think she stopped on? Why?

Responses vary.

- **-2 or 6 because they both are 4 units away from 2.**
- **-2 because it is 4 units to the left of the target, 2.**
- **6 because it is 4 bigger than the target, 2.**

4 Now there is a mystery target in this game!

a Play several rounds of the game on the digital activity. Record your number and score in the table.

Number	Score
6	**2**
4	**0**
-6	**10**

b **Discuss:** What do you think the target is? Why?

Responses vary.

- **4 because I got a score of 2 when I landed on 6 and when I landed on 2 . 4 is two numbers away from both 2 and 6.**
- **4 because when I landed on 4, I got a score of 0.**

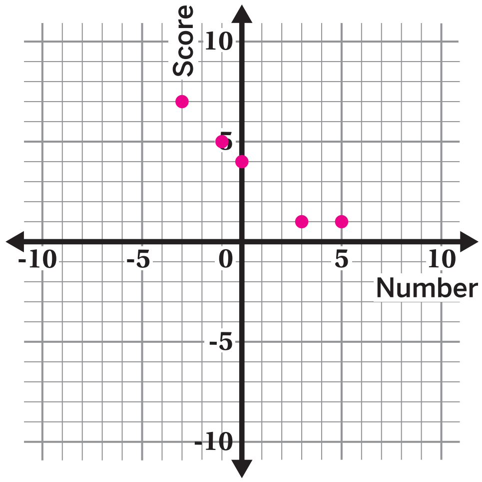
Activity 1

Name: .... Date: .... Period: ....

Target Numbers (continued)

5 **a** Plot the scores on the graph.

Number	Score
5	1
-1	5
3	1
0	4
-3	7



b **Discuss:**

- What do you think the graph of all possible scores looks like?
- Where can you see the mystery target?

Responses vary.

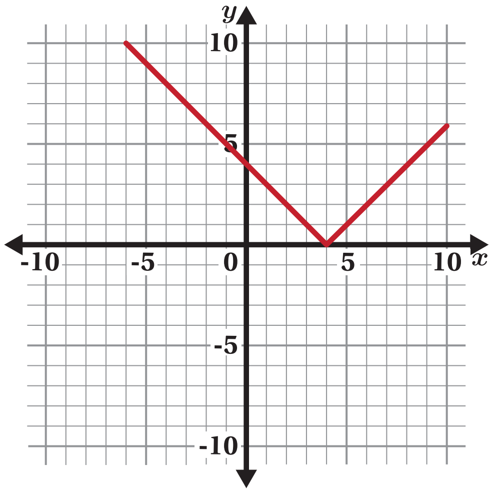
- **I think the graph would look like two lines and the mystery target is where the two lines meet.**
- **I think the graph would look like a V and the mystery target is the minimum point.**

6 The function $f(x) = |x - 4|$ is an example of an **absolute value function**.

This particular function tells you how far away you are from a target value of 4.

What is the value of $f(-2)$?

6



Activity 2

Name: Date: Period:

Absolute Value Functions

7 Use the digital screen to play another game. In this game, your score is how far away your guess is from a mystery number.

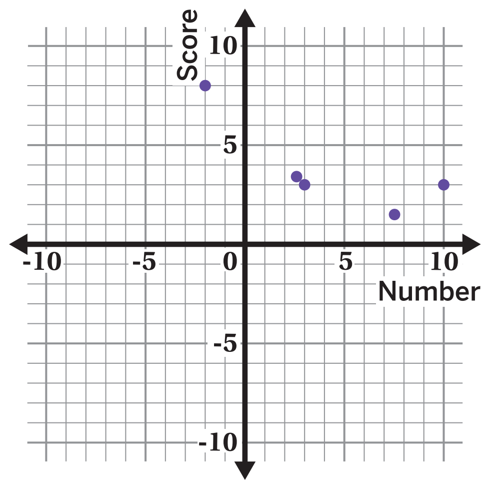
a Use the digital screen to enter up to five guesses.

b Tell a partner what the mystery number is and why.

Responses vary.

- **6 because the score was 0.**
- **6 because 6 is where the lowest point of the V shape on the graph is located.**

8 Here are some guesses and scores.
Which function gives the score for each guess in this game?



A. $a(x) = |x| + 6$

B. $b(x) = |x + 6|$

C. $c(x) = |x| - 6$

D. $d(x) = |x - 6|$

Explain your thinking.

Explanations vary. **Since the mystery number is 6, I should get a score of 0 when I input a 6 for x, so it could be function $c(x)$ or $d(x)$. The graph also has a point at $(0, 6)$. It must be $d(x)$ because an input of 0 would only give me an output of 6 for $d(x)$.**

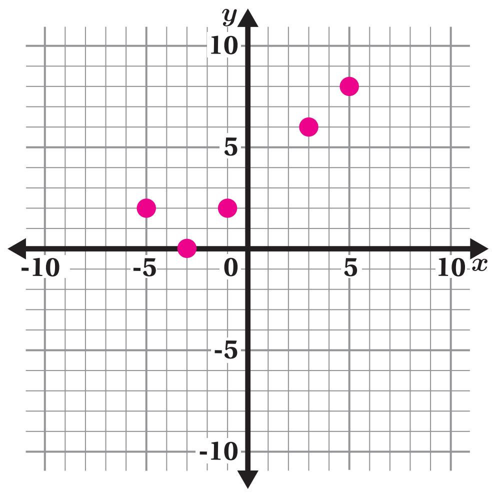
Name: ______ Date: ______ Period: ______

Absolute Value Functions (continued)

9 There is a new mystery number. The function $f(x) = |x + 3|$ gives the score for each guess, x.

Complete the table and plot the ordered pairs.

x	$f(x)$
5	**8**
-1	**2**
-5	**2**
-3	**0**
3	**6**



Explore More

10 Here are some guesses and scores for a new mystery number. Can these be scores for the same mystery number? Circle one.

Guess	Score
1	4
6	2

Yes No

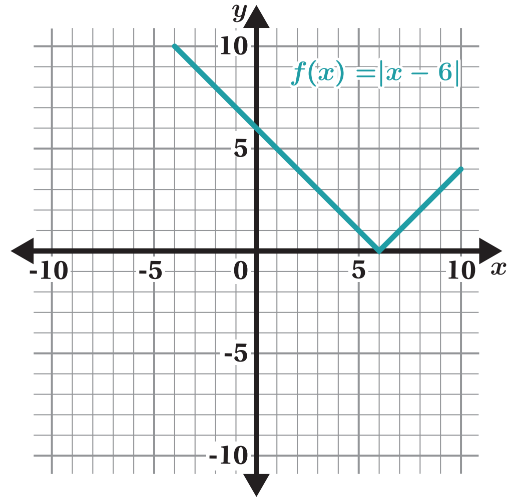
Explain your thinking.

***Explanations vary.* If a guess of 1 gives a score of 4, the mystery number could be 5 or -3. If a guess of 6 gives a score of 2, the mystery number could be 4 or 8. Since there is no mystery number that works for both of these guesses, these can't be scores for the same mystery number.**

11 Synthesis

How is an absolute value function related to the distance from a number?

Use the graph and equation if they help with your thinking.



Responses vary.

- **The absolute value function takes an input and gives an output that equals the distance from a certain number. In this case, $f(x)$ gives the distance of x from 6.**
- **You can determine the distance from a number by subtracting, but sometimes subtracting gives you a negative, so the absolute value will make it positive.**
- **The outputs repeat because there are two different values that are the same distance from any number.**

Things to Remember:

Name: ____________ Date: ________ Period: ________

Absolute Value Machines

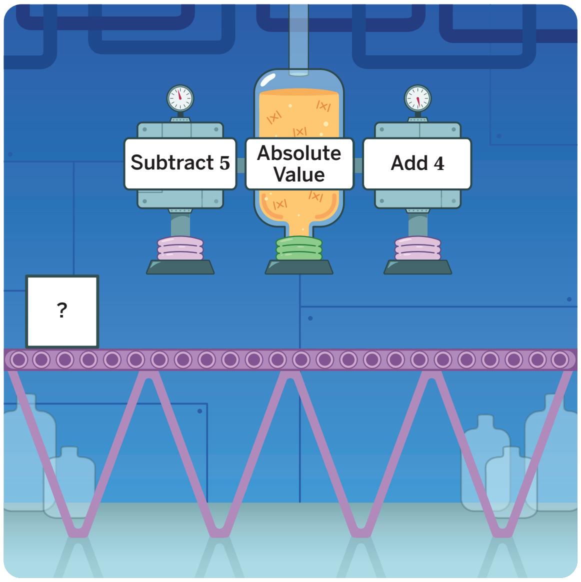
Let's graph absolute value functions.

Warm-Up

1 Here is a machine for $f(x) = |x - 5| + 4$.

a Let's watch how the machine works. Write down what happens to each input value at each solving step.

x	$x - 5$	$\lvert x - 5\rvert$	$\lvert x - 5\rvert + 4$



b **Discuss:** What do you notice and wonder?

Responses vary.

- I notice that each part of the machine updates the value in the box by performing a step in the operation. First you subtract 5, then you take the absolute value of the number, and finally you add 4.
- I wonder if it's possible to have a final answer of 0 using this machine.

Activity
1

Name: ______ Date: ______ Period: ______

Features of Absolute Value Functions

2 Kiri tried the numbers in the table.

She says: *The minimum value the machine can make is* 4.

Do you agree? Circle one.

Yes No Not enough information

Explain your thinking.

x	$x-5$	$\lvert x-5\rvert$	$\lvert x+5\rvert+4$
-1	-6	6	10
2	-3	3	7
5	0	0	4
6	1	1	5

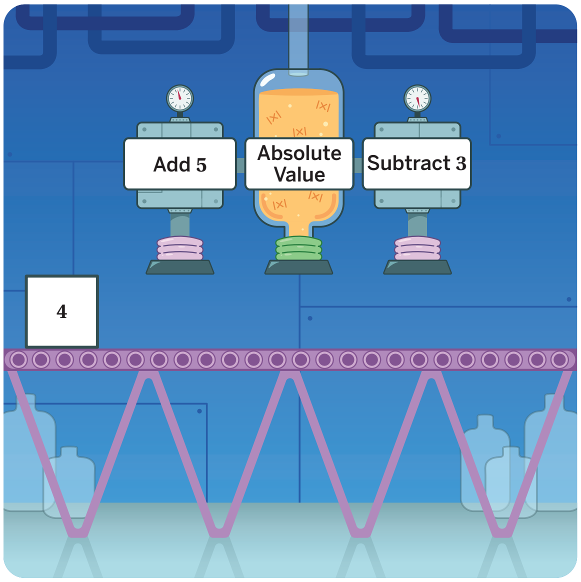
Responses and explanations vary.
Yes. The smallest absolute value you can get is 0, which occurs when you input 5. Then you add 4, which is the minimum value.

Not enough information. You can enter any number into the machine, but since you can never try them all, you can't be sure that the minimum value is 4.

3 Here is a machine for $g(x) = |x + 5| - 3$.

What number will come out of the machine if we enter 4, -1, and -6?

Use the table if it helps with your thinking.



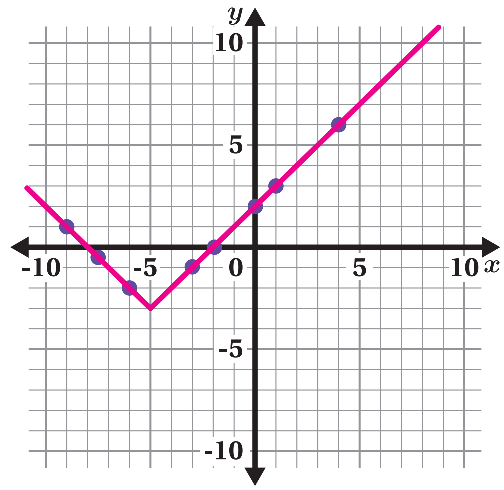
	x	$x+5$	$\lvert x+5\rvert$	$\lvert x+5\rvert-3$
a	4	**9**	**9**	**6**
b	-1	**4**	**4**	**1**
c	-6	**-1**	**1**	**-2**

Name: .... Date: .... Period: ....

Features of Absolute Value Functions (continued)

4 Here are some points on the graph of $g(x) = |x + 5| - 3$.

a Draw a sketch that shows what all the points look like.



b Describe your sketch using some of these terms:

positive	maximum	increasing	domain
negative	minimum	decreasing	range
symmetry		piecewise-defined function	

***Responses vary.* The minimum value is at (-5, -3). The graph is increasing when $x >$ -5 and decreasing when $x <$ -5. The domain is all numbers, and the range is $g(x) \geq$ -3.**

5 Here are descriptions of $g(x) = |x + 5| - 3$ from other students.

Select *all* the descriptions that are true.

- [x] **A.** The domain is all numbers.
- [ ] **B.** The minimum is at (-6, -2).
- [x] **C.** The range is $g(x) \geq$ -3.
- [ ] **D.** $g(x)$ is increasing when $x >$ -6.
- [x] **E.** The minimum is at (-5, -3).

Activity
2

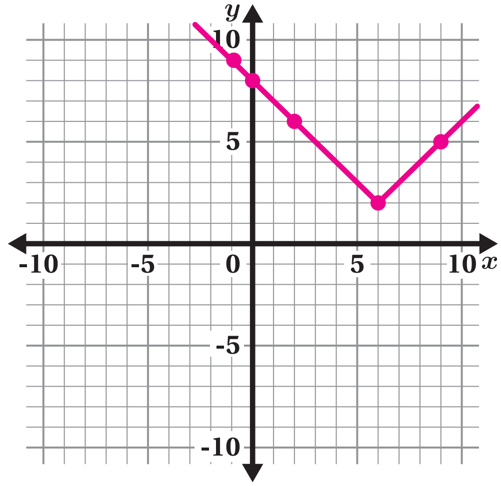
Name: Date: Period:

Graphing Absolute Value Functions

6 Here is a new function: $f(x) = |x - 6| + 2$.

Complete the table and then plot the graph of $f(x)$.

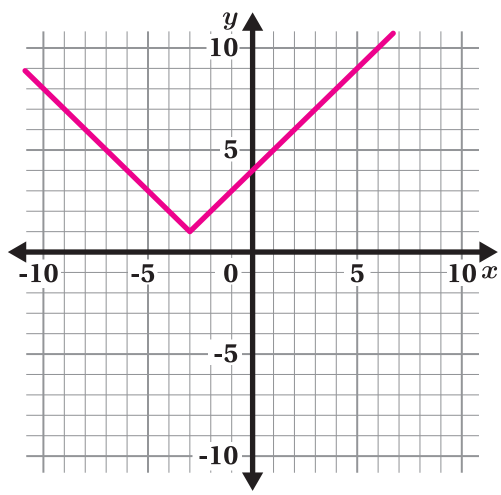
x	$f(x)$
9	5
2	6
0	8
6	2
-1	9



7 Draw a graph of $j(x) = |x + 3| + 1$.

Use the table if it helps with your thinking.

x	$j(x)$



Name: .... Date: .... Period: ....

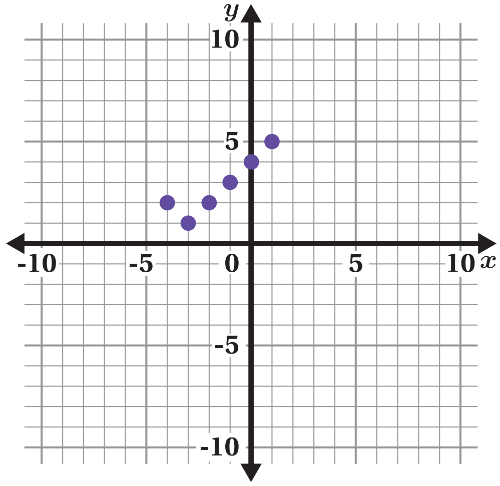
Graphing Absolute Value Functions (continued)

8 Here are the points that Tiana plotted for $j(x) = |x + 3| + 1$.

Tiana says: *I can use symmetry to plot more points on the graph.*

Show or describe what you think this means.

Responses vary. **Tiana knew that the minimum value for the function is at (-3, 1), so there is a line of symmetry at x = -3 that she can use to mirror the ordered pairs on the other side.**

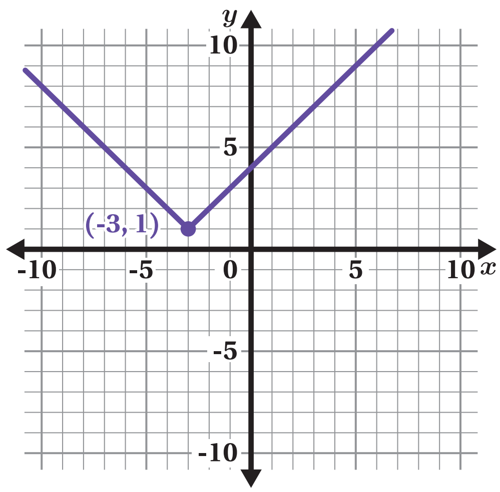


9 Here is the graph of $j(x) = |x + 3| + 1$.

The minimum value is shown.

How can you see the minimum value in the equation?

Responses vary. **I can see the minimum value in the equation by looking for the x-value that makes the absolute value expression equal to zero. In this case, $-3 + 3 = 0$, so the x-value of the minimum is -3. When the input is -3, the output is 1, so the minimum is (-3, 1).**



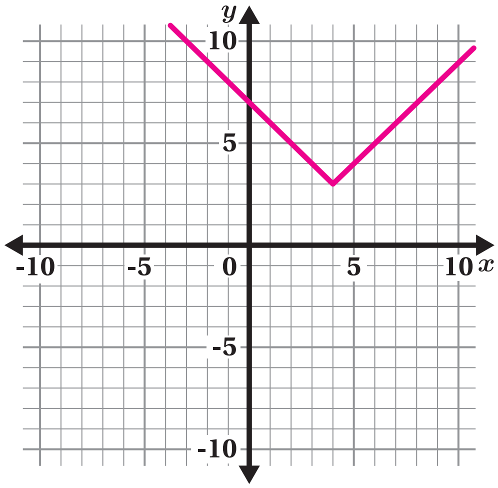
Activity 3

Name: ________ Date: ________ Period: ________

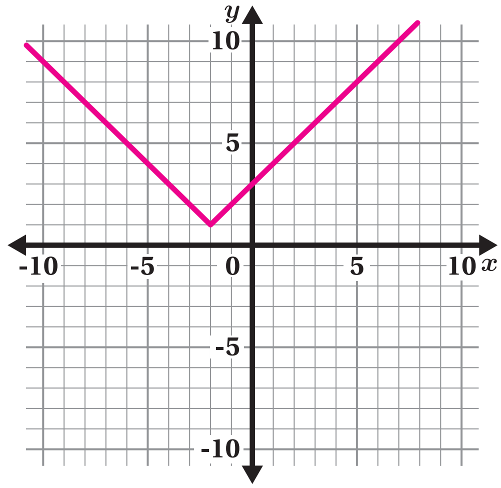
Repeated Challenges

10 Draw the graph of each function.

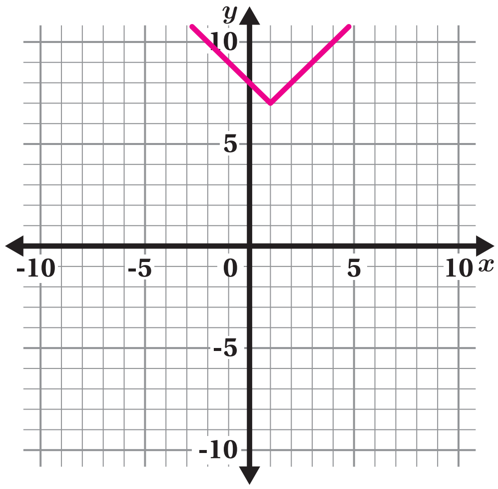
a $f(x) = |x - 4| + 3$



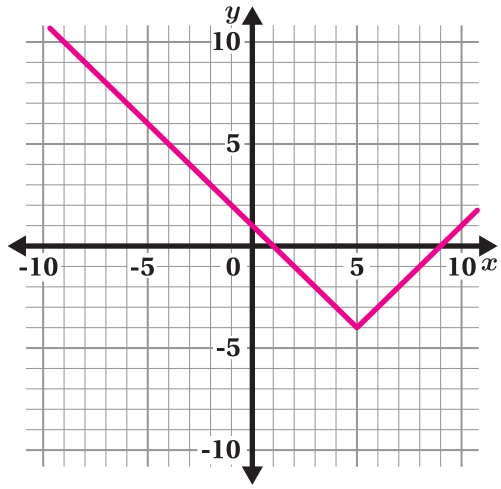
b $f(x) = |x + 2| + 1$



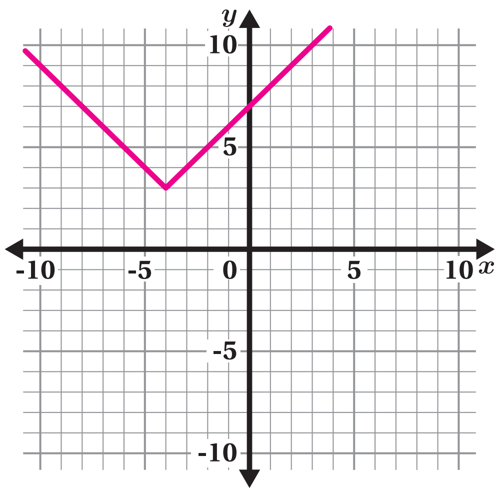
c $f(x) = |x - 1| + 7$



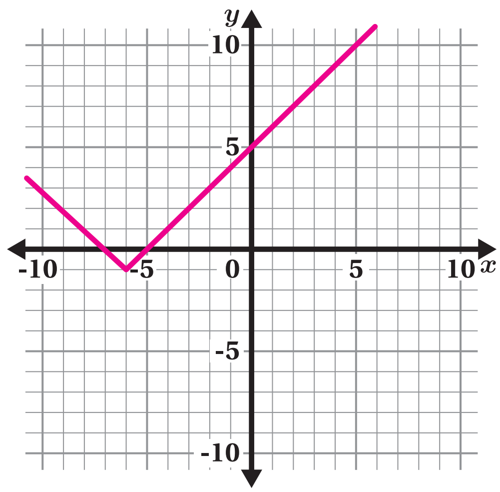
d $f(x) = |x - 5| - 4$



e $f(x) = |x + 4| + 3$



f $f(x) = |x + 6| - 1$



11 Synthesis

What can you know about the graph of an absolute value function by looking at its table or equation?

Use the example if it helps with your thinking.

$f(x) = |x - 4| + 3$

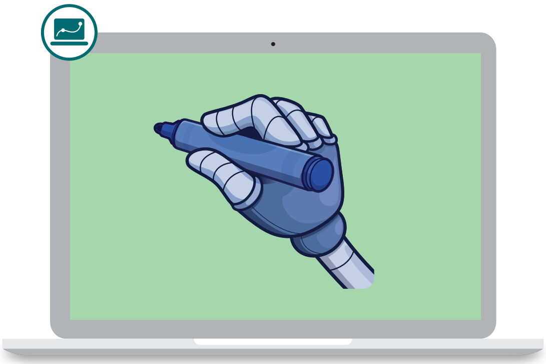
x	$f(x)$
-2	9
0	7
2	5
4	3
6	5

Responses vary. **I can know where the absolute value graph is increasing or decreasing by comparing values in a table.**
I can know where the minimum value of the absolute value function will be by looking at its equation. Just plug in the number that makes the absolute value expression equal to zero. Then use the input and output values to plot a point.

Things to Remember:

Unit 4
Lesson
18

Name: ______ Date: ______ Period: ______



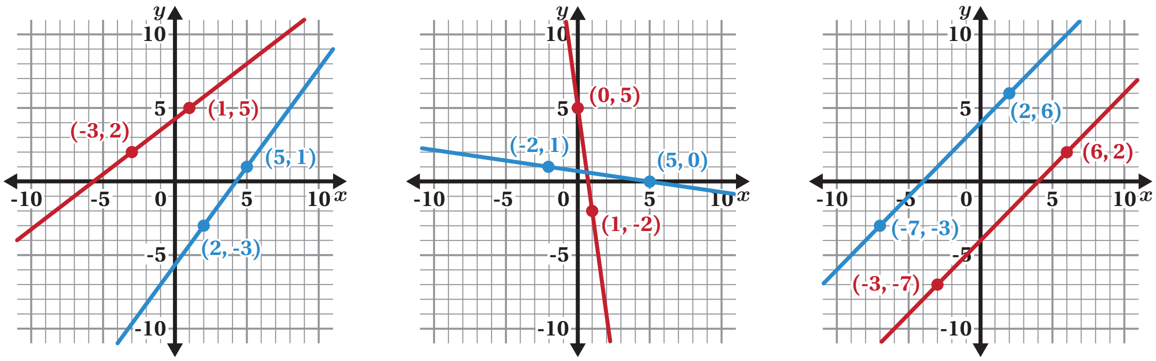
Chip the Robot

Let's make connections between a function and its inverse using graphs.

Warm-Up

1 Let's play a game with Chip the Robot!

When you create a red line, Chip draws a blue line using a rule.



Explain how Chip's rule works.

Responses vary.

- **Chip plots a line that reverses all of the points on my line.**
- **Chip switches the x- and y-values of the points on my line.**
- **Chip reflects my line across a diagonal line.**

Activity 1

Name: Date: Period:

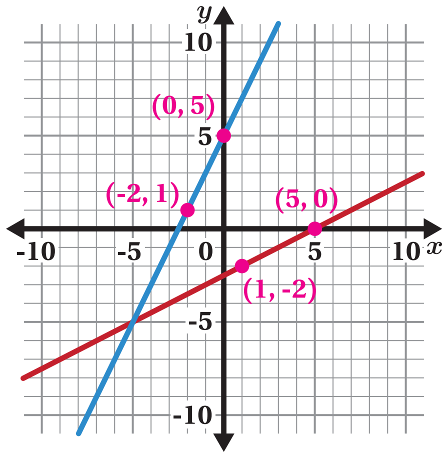
Chip's Rule

2 Here is a line Vihaan plotted, along with Chip's copy.

Vihaan says: *All of the points on Chip's line are reversed from my points.*

Show or describe what you think this means.

***Responses vary.* Vihaan's line goes through the points $(1, -2)$ and $(5, 0)$, so Chip's line goes through the points $(-2, 1)$ and $(0, 5)$.**

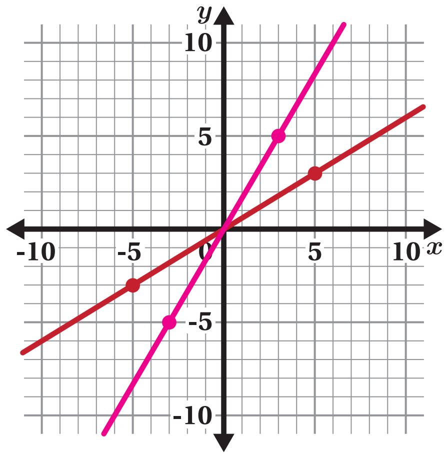


3 Here is a line that Aki plotted.

a Sketch the line you think Chip would draw.

b **Discuss:** What is your strategy?

***Responses vary.* I swapped the x- and y-values of the two points on Aki's line to draw two points on Chip's line. I drew a line through those two points to make Chip's line.**

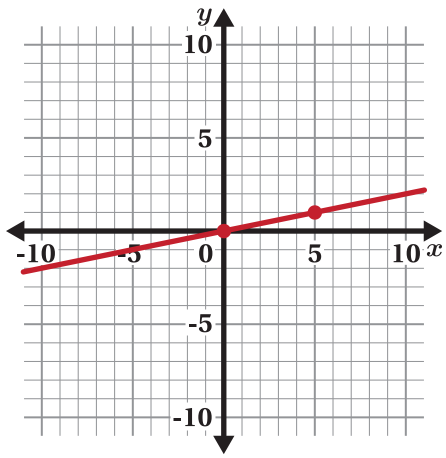


4 Chip can also use function notation to draw the line.

Mariam plotted the function $f(x) = \frac{1}{5}x$.

What is the function you think Chip will plot?

$g(x) = \mathbf{5x}$



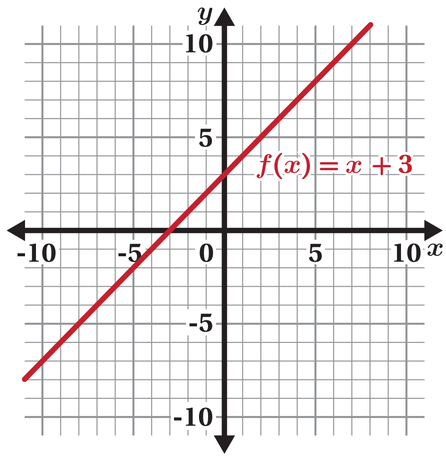
Name: ______ Date: ______ Period: ______

Chip's Rule (continued)

5 Chip the Robot plots **inverse functions**.

In general, if a function has a point at (h, k), then the inverse function has a point at (k, h).

Watch the screen to see the inverses of each of the different functions plotted on the graph.



Which function is the inverse of $f(x) = x + 3$?

A. $a(x) = x$

B. $b(x) = 3x$

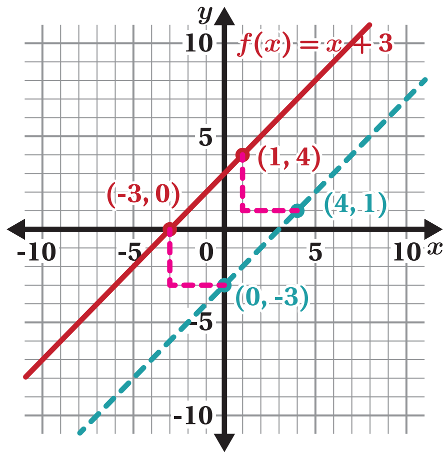
C. $c(x) = \text{-}x + 3$

D. $d(x) = x - 3$

6 Fatima says that the inverse of $f(x) = x + 3$ goes through the points $(0, \text{-}3)$ and $(4, 1)$.

Show or describe how you can write the equation of the inverse function using these two points.

***Responses vary*. You can use the two points to determine the slope and y-intercept of the line. The slope of the line is $\frac{4}{4} = 1$ and the y-intercept is -3. So the equation of the inverse function is $g(x) = x - 3$.**



Activity
2

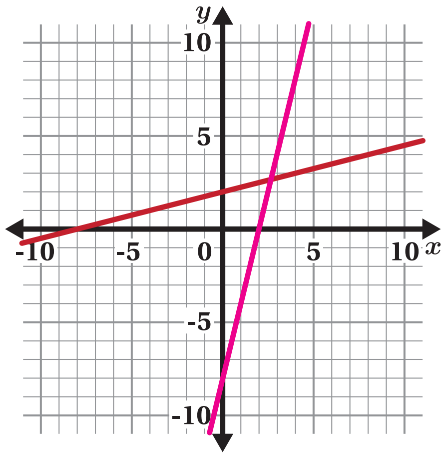
Name: .... Date: .... Period: ....

Inverse Challenges, Challenging Inverses

7 Here is the function $f(x) = \frac{1}{4}x + 2$.

Use the graph of the function to write an equation for the inverse function.

$g(x) =$ $\mathbf{4x - 8}$



8 Here is the function $f(x) = -3x + 6$.

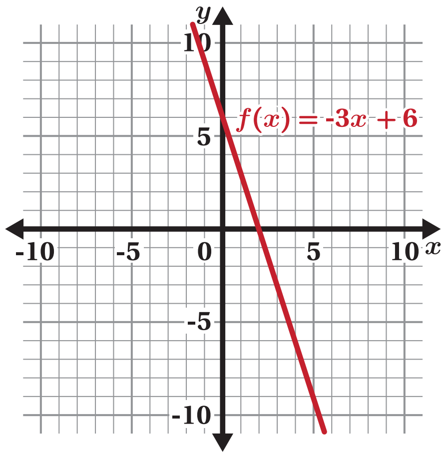
Jin says the inverse of $f(x)$ will pass through $(0, -6)$.

Is Jin correct? Circle one.

Yes No I'm not sure

Explain your thinking.

Explanations vary. **The original line has an x-intercept at $(2, 0)$, so the y-intercept of the inverse will be $(0, 2)$.**



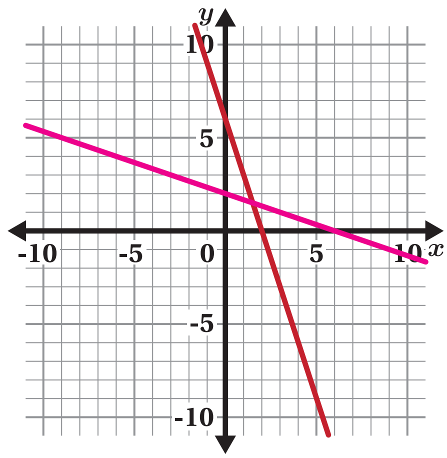
Name: ________ Date: ________ Period: ________

Inverse Challenges, Challenging Inverse (continued)

9 Here is the function $f(x) = -3x + 6$.

Use the graph of the function to write an equation for the inverse function.

$g(x) = -\frac{1}{3}x + 2$

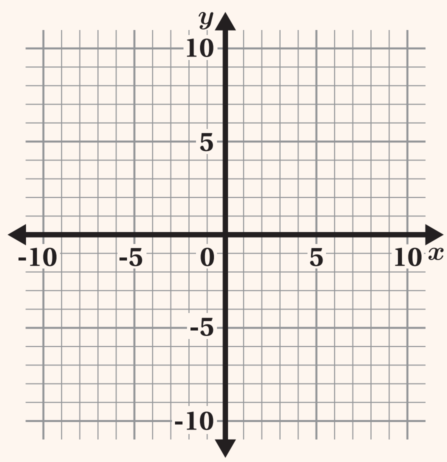


Explore More

10 Find as many pairs of linear inverse functions as you can. Try to find a pair that you think no one else in your class will find!

Responses vary.

Function	Inverse Function
$f(x) = x$	$g(x) = x$
$f(x) = x + 5$	$g(x) = x - 5$
$f(x) = 2x$	$g(x) = \frac{x}{2}$
$f(x) = 5x - 10$	$g(x) = \frac{1}{5}x + 2$

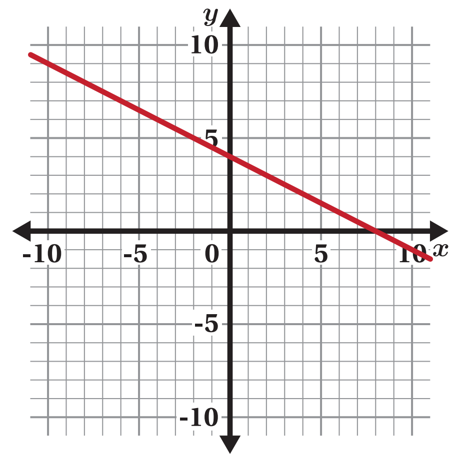


11 Synthesis

Describe a strategy for determining the inverse of a linear function.

Use the example if it helps with your thinking.

Responses vary. **To determine the inverse of a linear function, you can start by choosing two points on the original function and swapping the x- and y-values of each point. Draw a line that goes through those points and determine the slope and y-intercept of that line to write an equation in $y = mx + b$ form.**



Things to Remember: