desmos

Science Mom Lesson 65

Unit A1.7, Lesson 8: What's My Graph?

Answer Key

Warm-Up

$a(x) = x^2 + 3x + 1$

x	$a(x)$
–2	–1
–1	–1
0	1
1	5
2	11

$b(x) = (x + 3)(x + 1)$

x	$b(x)$
–2	–1
–1	0
0	3
1	8
2	15

Warm-Up Synthesis

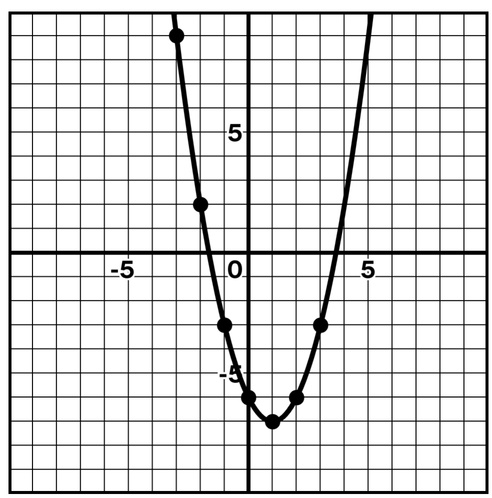
1. *Responses vary*.
   - The table in $a(x)$ separates $x^2 + 3x + 1$ into three parts, x^2, $3x$, and 1. Once each part is separately evaluated, they are added back together to equal 11.
   - The table in $b(x)$ separates $(x + 3)(x + 1)$ into two parts. Once the factors are separately evaluated, they are multiplied to equal 15.

2. *Responses vary*. Both of the tables break up the function into its parts. The table for $a(x)$ adds up three parts while the table for $b(x)$ multiplies two parts.

Activity 1: Coordinate Co-Op

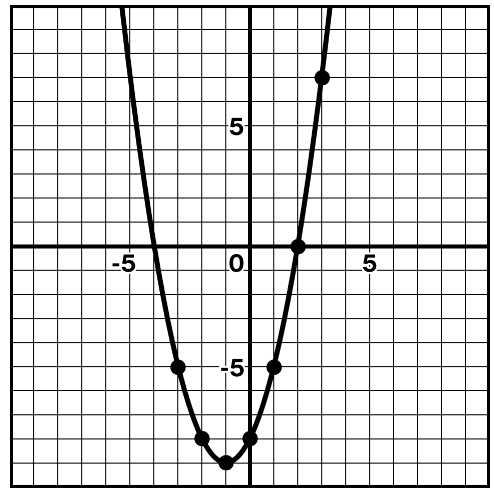
1.

x	x^2	$-2x$	-6	$x^2 - 2x - 6$
–3	9	6	–6	9
–2	4	4	–6	2
–1	1	2	–6	–3
0	0	0	–6	–6
1	1	–2	–6	–7
2	4	–4	–6	–6
3	9	–6	–6	–3



2.

x	$(x + 4)$	$(x - 2)$	$(x + 4)(x - 2)$
–3	1	–5	–5
–2	2	–4	–8
–1	3	–3	–9
0	4	–2	–8
1	5	–1	–5
2	6	0	0
3	7	1	7



Explore

Responses vary.

Activity 2 Launch

1. *Responses vary.* Jordan correctly multiplied $-3 \cdot 3$ and added $6 + -9 + 5$ to equal 2.
2. *Responses vary.* 3^2 does not equal 6. Jordan might have thought 3^2 was the same as $3 \cdot 2$.
3. *Responses vary.* Jordan should change the x^2 column to 9 because $3 \cdot 3 = 9$. Once they add that up again, the point $(3, 5)$ will be on the graph.

Activity 2: Fix It!

Responses vary.

Amir

- **Correct:** Amir correctly added the parts of the equation.
- **Mistake:** Amir started by multiplying 5 by 2. He should have started by squaring the 5.
- **Second draft:** The point $(5, 56)$ is on the graph!

Brielle

- **Correct:** Brielle correctly multiplied $-5 \cdot -2 = 10$.
- **Mistake:** Brielle might have thought that $(-2)^2 = -4$. But $-2 \cdot -2 = 4$. So $-(-2)^2 = -4$.
- **Second draft:** The point $(-2, 9)$ is on the graph!

Juliana

- **Correct:** Juliana correctly calculated $2x + 6$ and $x + 10$ when $x = -3$.
- **Mistake:** Instead of multiplying 0 and 7, Juliana added 0 and 7.
- **Second Draft:** The point $(-3, 0)$ is on the graph!

Lesson Synthesis

Responses vary. Graphing quadratic functions can be difficult when you try to calculate everything at once. A table can help you organize your thinking and calculate each piece of the quadratic function separately. Once you have broken it down into pieces, you can combine the pieces together with addition or multiplication.

Cool-Down

C. The point $(2, 8)$ is on the graph.

Science Mom Lesson 75

Unit A1.8, Lesson 3: X-Factor

Answer Key

Warm-Up

1. B
2. D
3. A
4. C

Activity 1: Diagram Puzzles

	Diagram	Standard Form	Factored Form
1	$\begin{array}{c|c|c} & 3x & -5 \\ \hline 4x & 12x^2 & -20x \\ \hline -3 & -9x & 15 \end{array}$	$12x^2 - 29x + 15$	$(3x - 5)(4x - 3)$ (or equivalent)
2	$\begin{array}{c|c|c} & 2x & 3 \\ \hline 2x & 4x^2 & 6x \\ \hline -3 & -6x & -9 \end{array}$	$4x^2 - 9$ or $4x^2 + 0x - 9$	$(2x + 3)(2x - 3)$ (or equivalent)
3	$\begin{array}{c|c|c} & 2x & -3 \\ \hline x & 2x^2 & -3x \\ \hline 4 & 8x & -12 \end{array}$	$2x^2 + 5x - 12$	$(2x - 3)(x + 4)$ (or equivalent)
4	$\begin{array}{c|c|c} & 3x & 4 \\ \hline x & 3x^2 & 4x \\ \hline 5 & 15x & 20 \end{array}$	$3x^2 + 19x + 20$	$(3x + 4)(x + 5)$ (or equivalent)

5	$\begin{array}{ccc} & x & 2 \\ x & x^2 & 2x \\ -5 & -5x & -10 \end{array}$ (or equivalent)	$x^2 - 3x - 10$	$(x + 2)(x - 5)$ (or equivalent)
6	$\begin{array}{ccc} & 3x & 1 \\ x & 3x^2 & x \\ 1 & 3x & 1 \end{array}$ (or equivalent)	$3x^2 + 4x + 1$	$(3x + 1)(x + 1)$ (or equivalent)
7	$\begin{array}{ccc} & x & 5 \\ x & x^2 & 5x \\ 4 & 4x & 20 \end{array}$ (or equivalent)	$x^2 + 9x + 20$	$(x + 5)(x + 4)$ (or equivalent)
8	$\begin{array}{ccc} & 2x & 1 \\ 3x & 6x^2 & 3x \\ 2 & 4x & 2 \end{array}$ (or equivalent)	$6x^2 + 7x + 2$	$(2x + 1)(3x + 2)$ (or equivalent)

Activity 2 Launch

Responses vary. Tameeka might try to brainstorm pairs of numbers that multiply to 7.

Activity 2: Next Steps

1. *Responses vary.*
   - Tameeka's work is incorrect because $14x + x = 15x$, not $9x$.
   - Tameeka did well choosing factors of 7 and she multiplied correctly.
   - Next, she should try to switch the places of her 1 and her 7.

2.1 *Responses vary.*
-1 and 12, -12 and 1, 2 and -6, -2 and 6, -3 and 4, -4 and 3

2.2 *Responses vary.*
   - Sneha's work is incorrect because $2x - 12x = -10x$, not $23x$.
   - Sneha did well choosing factors of -12 and she multiplied correctly.
   - Next, she could try to switch the places of the -6 and 2, switch which constant is positive, or try a different pair of numbers that multiply to -12.

2.3 $(2x - 1)(x + 12)$ (or equivalent)

3.1 *Responses vary.* Because the a-value is not prime, Ariana needs to think about possible pairs of numbers that multiply to get both the a- and the c-value. There is more than one pair of numbers, so Ariana has multiple factors of 10 to try and multiple factors of -12.

3.2 $(5x + 4)(2x - 3)$ (or equivalent)

4.1 $(x + 4)(x - 3)$ (or equivalent)

4.2 $(3x + 2)(x - 6)$ (or equivalent)

4.3 $(3x + 4)(2x - 3)$ (or equivalent)

Lesson Synthesis

Responses vary. You can use a diagram to help you by putting the ax^2 in the top-left corner of the diagram and c in the bottom right. Then try different pairs of numbers that multiply to the terms on the inside of the diagram until you get an expression equivalent to the given standard-form expression.

Cool Down

$(x + 7)(x - 4)$ or equivalent

	x	7
x	x^2	$7x$
-4	$-4x$	-28

(or equivalent)

Warm-Up

1. *Responses vary.* She should try $7, -8$ or $8, -7$ first. Since these numbers are close together, they are more likely to combine to x.
2. $(x + 8)(x - 7)$ or equivalent
3. $(x + 28)(x - 2)$ or equivalent

Activity 1: Spotting Similarities

1. *Responses vary.*
   Group 1: These expressions only have two terms. Every term is a perfect square.
   Group 2: These expressions all have $a \neq 1$. Each expression has three terms. All of the terms share a common factor.
   Group 3: These expressions all have $a = 1$.

2. *These or equivalent factored forms.*

Group 1	Group 2	Group 3
$(2x + 5)(2x - 5)$	$8(x + 3)(x + 1)$	$(x - 9)(x + 3)$
$(x - 6)(x + 6)$	$4(x - 4)(x + 2)$	$(x + 10)(x - 8)$
$(x - 10)(x + 10)$	$10(x + 1)(x + 1)$	$(x - 10)(x - 3)$
$(5x - 7)(5x + 7)$	$2(x - 6)(x - 5)$	$(x + 9)(x - 7)$

3.1 *Explanations vary.* They are equivalent because when I multiply 7 to all three terms, I get $7x^2 + 28x + 21$. Deiondre may have written $7(x^2 + 4x + 3)$ because he might find smaller a-, b-, and c-values easier to work with.

3.2 *Responses vary.* Deiondre's expression belongs in group 2 because it has three terms that all have a common factor and $a \neq 1$.

4.1 *Explanations vary.* Yasmine's expression belongs in group 1 because it has a b-value of 0 and each term is a perfect square.

4.2 *Responses vary.* $x^2 - 81$

4.3 *Responses vary.* $(x - 9)(x + 9)$

5. $3(x - 7)(x + 5)$ or equivalent
6. $(4x - 7)(4x + 7)$ or equivalent
7. $4(x + 10)(x + 3)$ or equivalent

Activity 2: Solve and Swap

These or equivalent factored forms.

A. $(x + 6)(x - 1)$
B. $(x + 5)(x - 2)$
C. $(2x + 1)(2x - 5)$
D. $(2x + 3)(x - 8)$
E. $(10x - 3)(10x + 3)$
F. $(3x - 1)(3x + 1)$
G. $-2(x - 2)(x + 1)$
H. $6(x + 2)(x - 3)$
I. $(x - 7)(x - 8)$
J. $(x + 10)(x + 8)$
K. $(4x + 5)(x + 2)$
L. $(3x - 4)(x + 4)$
M. $(5x - 8)(5x + 8)$
N. $(x - 4)(x + 4)$
O. $-3x(2x - 7)$
P. $5(x - 4)(x + 1)$
Q. $(x - 10)(x + 4)$
R. $(x + 9)(x + 2)$
S. $(3x + 4)(x + 3)$
T. $(2x + 3)(x + 6)$
U. $(x - 6)(x + 6)$
V. $(x - 2)(x + 2)$
W. $5(3x - 4)(x + 1)$
X. $10(x - 4)(x - 2)$

Lesson Synthesis

Responses vary.

- Remember to try 1 and c when testing pairs of numbers that multiply to c.
- Try to divide out a common factor first if possible.
- I check factored form by multiplying to see if I get the correct standard form.
- Be persistent. Factoring often takes multiple attempts.
- When standard form only has two terms, rewrite it with a 0 for the missing term.
- If the c-value is negative, the signs of my constants are different.

Cool Down

$2(x + 1)(x - 5)$ or equivalent

Activity 2: Solution Search

1.1 *Responses vary.*

- Equation: $2x^2 + 8x + 6 = 0$ Solutions: $x = \dfrac{-8 \pm \sqrt{16}}{4}$
- Equation: $1x^2 + 0x - 9 = 0$ Solutions: $x = \dfrac{0 \pm \sqrt{36}}{2}$

1.2 *Responses vary.*

- Equation: $1x^2 + 4x + 4 = 0$ Solutions: $x = \dfrac{-4 \pm \sqrt{0}}{2}$
- Equation: $9x^2 + 0x + 0 = 0$ Solutions: $x = \dfrac{0 \pm \sqrt{0}}{18}$

1.3 *Responses vary.*

- Equation: $7x^2 + 2x + 5 = 0$ Solutions: $x = \dfrac{-2 \pm \sqrt{-136}}{14}$
- Equation: $-2x^2 + 5x - 4 = 0$ Solutions: $x = \dfrac{-5 \pm \sqrt{-7}}{-4}$

2. *Responses vary*.
   - I notice that the number inside the square root can tell me how many solutions the equation will have.
   - I notice that when there is a square root of a negative number, there are no solutions.
   - I notice that if my b- and c-values are equal to 0, then my quadratic equation will always have one solution at $x = 0$.

Warm-Up

- $a = 3,\ b = -8,\ c = 15$
- $a = 1,\ b = 3,\ c = 4$
- $a = 5,\ b = 0,\ c = -20$
- $a = -1,\ b = 2,\ c = 12$

Activity 1 Launch

1. Troy substituted the $a = 1$, $b = -8$, and $c = 15$ into the quadratic formula.
2. Troy should calculate each part of the quadratic formula.

Activity 1: Form Over Function

1.1 *Work varies.* $x^2 - 8x + 15 = 0$ $x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(15)}}{2(1)}$ $x = \frac{8 \pm \sqrt{64 - 60}}{2}$ $x = \frac{8 \pm \sqrt{4}}{2}$ $x = \frac{8 \pm 2}{2}$ $x = \frac{10}{2}$ and $x = \frac{6}{2}$ $x = 5$ and $x = 3$	1.2 *Work varies.* $x^2 + 10x + 18 = 0$ $x = \frac{-10 \pm \sqrt{10^2 - 4(1)(18)}}{2(1)}$ $x = \frac{-10 \pm \sqrt{100 - 72}}{2}$ $x = \frac{-10 \pm \sqrt{28}}{2}$ $x = -5 \pm \frac{\sqrt{28}}{2}$
1.3 *Work varies.* $9x^2 - 6x = -1$ $9x^2 - 6x + 1 = 0$ $x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(1)}}{2(9)}$ $x = \frac{6 \pm \sqrt{36 - 36}}{18}$ $x = \frac{6 \pm \sqrt{0}}{18}$ $x = \frac{6 \pm 0}{18}$ $x = \frac{1}{3}$	1.4 *Work varies.* $2x^2 + 6x + 5 = 0$ $x = \frac{-6 \pm \sqrt{6^2 - 4(2)(5)}}{2(2)}$ $x = \frac{-6 \pm \sqrt{36 - 40}}{4}$ $x = \frac{-6 \pm \sqrt{-4}}{4}$ No solutions.

2. *Responses vary.* The quadratic formula is the best strategy for solving $9x^2 - 6x = -1$ and $2x^2 + 6x + 5 = 0$ because I did not know how to factor either equation. I would rather solve $x^2 - 8x + 15 = 0$ by factoring because I can rewrite it as $(x - 5)(x - 3) = 0$.

Activity 2: Error Analysis

1. *Discussions vary.*
    - Equation 1.1: $(-8)^2$ is equal to 64, not -64.
    - Equation 1.2: $\frac{-10 \pm \sqrt{28}}{2}$ is equivalent to $\frac{-10}{2} \pm \frac{\sqrt{28}}{2}$. $\frac{-10}{2}$ can be reduced to -5 but $\frac{\sqrt{28}}{2}$ cannot be reduced to $\sqrt{14}$.
    - Equation 1.3: The c-value should be 1 because the equation is $9x^2 - 6x + 1 = 0$ when written in standard form.
    - Equation 1.4: There are no numbers that would give you $\sqrt{-4}$ when squared, so $2x^2 + 6x + 5 = 0$ has no solutions.

2.1 *Errors vary.*

$3x^2 - 6x - 1 = 0$

$a = 3,\ b = -6,\ c = -1$

$$x = \frac{-6 \pm \sqrt{(-6)^2 - 4(3)(-1)}}{2(3)}$$

$$x = \frac{-6 \pm \sqrt{36 + 12}}{6}$$

$$x = \frac{-6 \pm \sqrt{48}}{6}$$

$$x = -1 \pm \frac{\sqrt{48}}{6}$$

2.2 *Responses vary.* $b = -6$, so when you substitute b into the quadratic formula, $-b$ is $-(-6)$, which is equal to 6.

3.1 *Responses vary.*
- I think it is easy to get my positive and negative values confused.
- I could forget to make my equation equal to 0 before using the quadratic formula.

3.2 *Responses vary.*
- When you substitute, use parentheses to keep track of what numbers should be negative and positive.
- Check your work by using a calculator to plug your solutions back into your equation.

Lesson Synthesis

Responses vary.
- Advantage: The quadratic formula can be used to solve any quadratic equation, so you don't need to think about which strategy to use.
- Advantage: The quadratic formula can solve equations that would be really difficult to solve using other strategies, like $2x^2 + 7x - 10 = 0$.
- Disadvantage: You could make a mistake while calculating your answers, so it might be a good idea to always check your work.
- Disadvantage: Some equations might be easier to solve by factoring, like $x^2 - 6x + 8 = 0$.

Cool Down

$x = -4$ and $x = 1.5$